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Optimal timing of CCS policies with heterogeneous energy consumption sectors

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Abstract

Using the Chakravorty et al. (2006) ceiling model, we characterize the optimal consumption paths of three energy resources: dirty oil, which is non-renewable and carbon emitting; clean oil, which is also non-renewable but carbon-free thanks to an abatement technology, and solar energy, which is renewable and carbon-free. The resulting energy-mix can supply the energy needs of two sectors. These sectors differ in the additional abatement cost they have to pay for consuming clean rather than dirty oil (sector 1 can abate its emissions at a lower cost than sector 2). We show that it is optimal to begin by fully capturing sector 1's emissions before the ceiling is reached. Also, there may exist optimal paths along which both capture devices have to be activated. In this case first sector's 1 emissions are fully abated before sector 2 abates partially. Finally, we discuss the effect of heterogeneity regarding the abatement cost on the uniqueness of the sectoral energy price paths.

Keywords: Energy resources; Carbon stabilization cap; Heterogeneity; Carbon capture and storage; Air capture.

JEL classifications: Q32, Q42, Q54, Q58.

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1 Introduction

Among all the alternative abatement technologies aiming at reducing the anthropogenic carbon dioxide emissions, Carbon Capture and Sequestration (CCS) is of particular interest (IPCC, 2005 and 2007). Even if the efficiency of this technology is still under assessment[1], current engineering estimates suggest that CCS could be a credible cost-effective approach for eliminating most of the emissions from coal and natural gas power plants (MIT, 2007). Along this line of arguments, Islegen and Reichelstein (2009) point out that CCS has considerable potential to reduce CO_2 emissions at a "reasonable" social cost, given the social cost of carbon emissions predicted for a business-as-usual scenario. CCS is also intended to play a major role in limiting the effective carbon tax, or the market price for CO_2 emission permits under a cap-and-trade system. The crucial point is then to estimate how far the CO_2 price would have to rise before the managers of power plants would find it advantageous to install the CCS technology rather then to buy emission permits or to pay a carbon tax. The International Energy Agency (2006) estimates such a break-even price in the range of \$30-90/tCO_2 (depending on the technology). However, when assuming reasonable technology advances, projected CCS cost would drop at around \$25/tCO_2 by 2030.

The deployment capacity of CCS strongly depends upon the type of the energy users/carbon emitters. Obviously, capturing emissions from a power plant running with gas will be cheaper than capturing emissions from vehicles powered by this fossil energy source. More generally, CCS technology is proved to be better adapted to large point sources of pollution such as power plants or huge manufacturing rather than to small and scattered emitters such as transportation, individual residence heating or agricultural activities. Although in this last case filtering CO_2 flows would be technically possible by using e.g. air capture, this technology is still prohibitively costly. Keith (2009) underlined that while this technology costs more than CCS, it allows to treat small and mobile emission sources,

[1] CCS technology consists in filtering CO_2 fluxes at the source of the emissions. For this purpose, in fossil energy-fueled power plants for instance, scrubbers are installed next to the top of chimney stacks. Carbon is next sequestered in reservoirs, such as depleted oil and gas fields or deep saline aquifers. However, as mentioned by Herzog (2011), the idea of separating and capturing CO_2 from the flue gas of power plants did not originate out climate change concerns. The first commercial CCS plants that have been built in the late 1970s in the United States aimed at achieving enhanced oil recovery operations where CO_2 is injected into oil reservoirs to increase the pressure and thus the output of the reservoir.

an advantage that may compensate for the intrinsic difficulty of capturing carbon from the air. Estimates of marginal cost of chemical air capture[2] range from \$100-200/tCO_2, which is larger than the cost of alternatives for emissions reduction such as CCS. They are also larger than current estimates of the social cost of carbon, which range from about \$7-85/tCO_2. But, as concluded by Barrett (2009), bearing the cost of chemical air capture can become profitable in the future under constraining cap-and-trade scenarios. For the time being, air capture is somewhat an extreme example. However, even when considering the CCS technology, abatement costs can differ among energy users, depending upon the location of the storage site and the type of reservoirs (Hamilton et al., 2009).

This paper addresses the question of the heterogeneity of energy users regarding their abatement costs. It examines how this heterogeneity affects the optimal energy consumption and price paths as well as the timing of abatement policies. To tackle this issue, we use the "ceiling model" developed by Chakravorty et al. (2006) and extended to the specific CCS abatement device by Lafforgue et al. (2008-a and 2008-b).

The sketch of the model is the following. We consider two sectors of energy consumption which differ in the cost of the abatement technology they can use. Their energy needs can be supplied by three types of energy resources that are perfect substitutes. The first type is depletable and carbon-emitting (dirty oil), the second is also depletable but it does not contribute to climate change thanks to a specific abatement device (clean oil). The last energy source is renewable and carbon-free (solar). The problem is to characterize the optimal path of the energy-mix of each sector, given that the atmospheric carbon stock should not exceed some critical ceiling. This energy-mix choice results from the comparison of the respective full marginal cost of each energy option. Both the marginal extraction cost of oil and the marginal cost of solar energy are constant and the same in each sector, but the former is assumed to be lower than the latter. Producing clean oil requires an additional cost of carbon capture which varies among the two sectors. This cost is assumed to be

[2] Currently, chemical air capture is probably the most credible process to capture carbon directly from the atmosphere (Barrett, 2009). This technology consists in bringing air into contact with a chemical "sorbent" (an alkaline liquid). The sorbent absorbs CO_2 in the air, and the chemical process then separates the CO_2 and recycles the sorbent. The captured CO_2 is stored in geologic deposits just as it is done by means of the CCS technology in power plants. Otherwise, the most obvious way to reduce the atmospheric carbon concentration would be to exploit the process of photosynthesis by increasing the forest areas or changing the agricultural processes. However, this is not the type of device we want to consider in the present paper.

larger in sector 2 than in sector 1, and constant in both cases. Furthermore, since the patterns of the optimal paths strongly depends upon the level of the solar energy cost as compared with the full cost of clean oil, we examine various possibilities depending on whether the solar cost is high, intermediate or low. Last, we assume that when a sector abates its emissions, carbon is stockpiled into very large reservoirs. As in Chakravorty et al. (2006), this suggests a generic abatement scheme of unlimited capacity.[3]

The important point is that the ceiling constraint can be relaxed owing to two mitigation options. The first one consists in substituting clean oil for dirty oil and the second one in substituting solar energy for dirty oil. Each option allows both to delay the (endogenous) point in time at which the ceiling constraint begins to be effective and to relax this constraint once it is binding.

The key results of the paper are: i) Irrespective of sector 2's ability to capture its emissions, it may be optimal to begin sector 1's abatement before the atmospheric carbon concentration cap is attained.[4] ii) Due to the abatement cost differential among the sectors, it is also optimal to capture sector 1's entire emissions before the ceiling is reached. These two first results, obtained for any level of solar energy cost, differ from Chakravorty et al. (2006), Lafforgue et al. (2008-a and 2008-b) who consider a single type of energy user and a single abatement technology. iii) In the optimal scenarios where both sectors have to consume clean oil, sector 2 must start to abate its emissions when the ceiling constraint begins to be active and it has to abate only partially. iv) The sectoral prices of the energy-mix may be different.

The paper is organized as follows. Section 2 presents the model. In Section 3 we lay down the social planner program and derive the optimality conditions. Section 4 examines the case in which only sector 1 consumes clean oil along the optimal path and discusses the optimal scenarios depending on the level of the solar energy cost. In Section 5 we characterize the policies when it is optimal to consume clean oil for both sectors. In Section 6 we focus on the specific problem of air capture. In this case, instead of having access

[3]See Lafforgue et al. (2008-a, 2008-b) for the case of limited capacities.

[4]This result is in accordance with Coulomb and Henriet (2010) who show that in a model with a single abatement technology when technical constraints make it impossible to capture emissions from all energy consumers, CCS should be used before the ceiling is reached if emissions that cannot be captured are large enough.

to the CCS technology, sector 2 can capture the carbon directly from the atmosphere. Finally, we briefly conclude in the last section.

2 The model

Let us consider a stationary economy with two sectors, indexed by $i = 1, 2$, in which the instantaneous gross surpluses derived from the energy consumption are the same.[5] For an equal energy consumption q in both sectors, $q_1 = q_2 = q$, the sectoral surplus $u_1(q)$ and $u_2(q)$ are thus equal: $u_1(q) = u_2(q) = u(q)$. We assume that this common function u is twice continuously differentiable, strictly increasing, strictly concave, with $\lim_{q\downarrow 0} u'(q) = +\infty$ and $\lim_{q\uparrow +\infty} u'(q) = 0$. We denote by $p(q)$ the sectoral marginal gross surplus function $u'(q)$ and by $q(p) = p^{-1}(q)$, the direct demand function of the sector.

Energy can be supplied by two primary resources, a potentially polluting non-renewable resource (oil) and a carbon-free renewable resource (solar).

Clean and dirty oil

Let $X(t)$ be the available stock of oil at time t and X^0 be the initial endowment. Each sector can consume either "dirty oil" or "clean oil".

Consuming dirty oil implies some carbon emissions that are proportional to its use. Let ζ be the unitary pollutant content of dirty oil so that the emission flow of sector i amounts to ζx_{id}, where x_{id} is the dirty oil consumption of this sector. We denote by c_x the average delivery cost of oil, assumed to be constant and the same in both sectors. This cost includes the extraction cost of the resource, the cost of industrial processing (crude oil refining) and the transportation cost.

The consumption of clean oil is carbon-free but at the same time it is also costlier than the consumption of dirty oil. We denote by s_i the average cost that has to be borne by sector i in addition to c_x for using clean rather than dirty oil. This cost is assumed to be constant and smaller in sector 1 than in sector 2: $0 < s_1 < s_2$.[6] In other words, sector 1

[5]Since the focus of the paper is on the effect of heterogeneity on the abatement costs, all the other sectoral characteristics are assumed to be the same in order to highlight the effects of this only difference in the sectoral characteristics.

[6]s_i is an average cost per unit of oil and may be seen as a cost of carbon capture and storage. The CCS cost per unit of carbon captured in sector i amounts to s_i/ζ. It is assumed to be constant. For non-linear

has access to a cheaper abatement technology than sector 2. In both sectors we assume that carbon emissions are stockpiled into reservoirs whose capacities are unlimited so that no additional rent has to be charged.[7]

Denoting by x_{ic} the consumption of clean oil in sector i, the dynamics of X must satisfy:

$$\dot{X}(t) = -\sum_{i=1,2}[x_{ic}(t) + x_{id}(t)] \tag{1}$$

$$X(0) = X^0 \quad \text{and} \;\; X(t) \geq 0 \tag{2}$$

$$x_{ik}(t) \geq 0, \;\; i = 1,2 \;\; \text{and} \;\; k = c, d. \tag{3}$$

Pollution stock

Let $Z(t)$ be the stock of carbon within the atmosphere at time t, and Z^0 its initial level. The atmospheric pollution stock is fed by the emissions coming from dirty oil consumption. Moreover, we assume that this stock is self-regenerating at a constant proportional rate α, $\alpha > 0$.

The pollution damage may be neglected if the pollution stock does not exceed some critical level $\bar{Z}$. Above this threshold, the damage is supposed to be infinitely high.[8] Put differently, we assume that a carbon cap policy is prescribed to prevent catastrophic damages which would be infinitely costly and that this policy consists in forcing the atmospheric stock to stay under $\bar{Z}$. Thus the dynamics of Z must satisfy:

$$\dot{Z}(t) = \zeta[x_{1d}(t) + x_{2d}(t)] - \alpha Z(t) \tag{4}$$

$$Z(0) = Z^0 < \bar{Z} \quad \text{and} \;\; \bar{Z} - Z(t) \geq 0 \tag{5}$$

When the atmospheric carbon stock reaches its critical level, that is when $Z(t) = \bar{Z}$, the total dirty oil consumption is constrained to be at most equal to $\bar{x}_d = \alpha\bar{Z}/\zeta$, where $\bar{x}_d = x_{1d} + x_{2d}$. Since the two sectors have the same gross surplus function, each of them

cost functions, see Amigues et al. (2012).

[7] In order to focus on the abatement options for each sector and their respective costs, we dispense from considering reservoirs of limited capacity. The question of the size of carbon sinks and of the time profile of their filling up is addressed by Lafforgue et al. (2008-a) and (2008-b).

[8] Taking into account non negligible damages for $Z < \bar{Z}$ would not change the main qualitative properties of the optimal paths as shown in Amigues et al. (2011).

must consume the same quantity of dirty oil $x_{id} = \bar{x}_d/2$, for $i = 1, 2$, when the ceiling constraint (5) is binding and when none of them uses clean oil simultaneously.

We assume that it may be optimal to use clean oil in each sector – and therefore to abate carbon emissions – in order to delay the point of time at which the ceiling begins to constrain the oil consumption and/or to relax this constraint once it is active, that is: $c_x + s_1 < c_x + s_2 < u'(\bar{x}_d)$.

Solar energy

Solar energy is a perfect substitute for oil. It is available at a constant average cost c_y which is assumed to be the same for each sector and to be larger than c_x. Hence denoting by y_i its consumption in sector i, the sectoral aggregate energy consumption amounts to $q_i = x_{ic} + x_{id} + y_i$.

As we shall see, the structures of the optimal paths strongly depend upon the solar energy cost. Thus three intervals of average cost have to be distinguished: high, when $c_y > u'(\bar{x}_d/2)$; intermediate, when $u'(\bar{x}_d/2) > c_y > u'(\bar{x}_d)$; and low, when $u'(\bar{x}_d) > c_y$.

We denote by $\tilde{y}$ the solar consumption rate solving $u'(y) = c_y$. This rate $\tilde{y}$ reads as the optimal sectoral consumption of solar energy when oil is exhausted and absent any constraint on its use. That is, assuming that its natural supply is large enough, at least as large as $2\tilde{y}$, in which case no rent has ever to be charged for its use. The only physical constraint on the y_i's are then the non-negativity constraints:

$$y_i(t) \geq 0, \quad i = 1, 2 \tag{6}$$

Social welfare and discounting

If constraint (5) is satisfied, the social welfare function W writes as the sum of the sectoral net surpluses discounted at some constant social rate ρ, $\rho > 0$. It is equal to $-\infty$ otherwise (that is if the critical threshold $\bar{Z}$ is overshot).

3 Social planner problem and optimality conditions

The problem of the social planner consists in maximizing W subject to the various constraints introduced above. Denoting by S_i the instantaneous net surplus of sector i, $S_i(x_{ic}, x_{id}, y_i) = u(x_{ic} + x_{id} + y_i) - [c_x + s_i]x_{ic} - c_x x_{id} - c_y y_i$, the social planner has to solve the following program (P):

$$(P): \quad \max_{\{x_{ic}, x_{id}, y_i\}} \int_0^\infty \left\{ \sum_{i=1,2} S_i(x_{ic}(t), x_{id}(t), y_i(t)) \right\} e^{-\rho t} dt$$

subject to (1)-(6).

We denote by λ_X the costate variable of X, by λ_Z minus the costate variable of Z, by γ's the Lagrange multipliers associated with the non-negativity constraints on the control variables, by ν_X the multiplier associated with the non-negativity constraint on X and by ν_Z the multiplier associated with the ceiling constraint on Z. Dropping out the time index for notational convenience, the current value Lagrangian $\mathcal{L}$ of program (P) is:

$$\begin{aligned} \mathcal{L} &= \sum_{i=1,2} S_i(x_{ic}, x_{id}, y_i) - \lambda_X \sum_{i=1,2} \sum_{k=c,d} x_{ik} - \lambda_Z \left[\zeta \sum_{i=1,2} x_{id} - \alpha Z \right] \\ &\quad + \nu_X X + \nu_Z [\bar{Z} - Z] + \sum_{i=1,2} \sum_{k=c,d} \gamma_{ik} x_{ik} + \sum_{i=1,2} \gamma_{iy} y_i \end{aligned} \tag{7}$$

The first-order conditions for optimality are:

$$u'(x_{ic} + x_{id} + y_i) = c_x + s_i + \lambda_X - \gamma_{ic}, \quad i = 1, 2 \tag{8}$$

$$u'(x_{ic} + x_{id} + y_i) = c_x + \zeta\lambda_Z + \lambda_X - \gamma_{id}, \quad i = 1, 2 \tag{9}$$

$$u'(x_{ic} + x_{id} + y_i) = c_y - \gamma_{iy}, \quad i = 1, 2 \tag{10}$$

$$\dot{\lambda}_X = \rho\lambda_X - \nu_X \tag{11}$$

$$\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \tag{12}$$

together with the associated complementary slackness conditions and the following transversality conditions:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \quad \text{and} \quad \lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{13}$$

Remarks

Since c_x is constant, the shadow marginal cost of the stock of oil must grow at the social rate of discount. Defining $\lambda_{X0} \equiv \lambda_X(0)$, from (11), we get the following well known result:

$$X(t) > 0 \Rightarrow \lambda_X(\tau) = \lambda_{X0}e^{\rho\tau}, \quad \tau \in [0,t] \tag{14}$$

The transversality conditions (13) imply that if oil has some positive initial value $\lambda_{X0} > 0$, then it must be exhausted along the optimal path, that is $\lim_{t\uparrow\infty} X(t) = 0$.

Next, since $Z^0 < \bar{Z}$ there exists some initial maximum time interval $[0, \underline{t}_Z)$ during which the ceiling constraint is not active, hence $\nu_Z = 0$, so that from (12):

$$\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}, \quad t \in [0, \underline{t}_Z) \tag{15}$$

where $\lambda_{Z0} \equiv \lambda_Z(0)$ and $\underline{t}_Z$ is the first date at which $Z(t) = \bar{Z}$. Clearly, once the ceiling constraint is not active anymore, λ_Z must be nil:[9]

$$\lambda_Z(t) = 0, \quad t \in [\bar{t}_Z, \infty) \tag{16}$$

where $\bar{t}_Z$ is the last date at which $Z(t) = \bar{Z}$.

Solving strategy of the social planner

In order to characterize the optimal paths, the first problem to solve consists in determining which sector, if any, has to consume clean oil. Note that from (8) and (9), and under the assumption that oil has to be consumed, each sector i must use either only dirty oil or only clean oil at any time t, depending on whether $\zeta\lambda_Z(t)$ is lower or higher than s_i. This suggests the following test.

[9] This characteristic is standard under the assumption of a linear natural regeneration process of the atmospheric carbon stock. For non-linear decay functions, see e.g. Toman and Withagen (2000).

First, solve a modified social planner program in which the use of clean oil is not possible in any sector. Let $\lambda_Z^1(t)$, for any $t \geq 0$, be the trajectory of the shadow marginal cost of the pollution stock of this program, and $\bar{\lambda}_Z^1$ be the maximum value of λ_Z^1 along its trajectory. Consequently, either $\zeta\bar{\lambda}_Z^1 > s_1$ and it would be preferable to use clean oil in sector 1 during some time interval, or $\zeta\bar{\lambda}_Z^1(t) \leq s_1 < s_2$ and clean oil would never be used in any sector.

Assuming that $\zeta\bar{\lambda}_Z^1 > s_1$, the next step consists in solving a second modified program in which consuming clean oil is possible in sector 1 but not in sector 2. Let $\lambda_Z^2(t)$, $t \geq 0$, be the new trajectory of λ_Z and $\bar{\lambda}_Z^2$ its maximum value. Applying now the test for sector 2 we conclude that either $\zeta\bar{\lambda}_Z^2 \leq s_2$ and only sector 1 uses clean oil, or $\zeta\bar{\lambda}_Z^2 > s_2$ and clean oil is used simultaneously in both sectors.

In the following sections, we will successively characterize the case where only sector 1 has consumes clean oil (section 4) and next, the case where the both sectors have to abate their emissions (section 5).

Notations

For better reading, we introduce the following additional notations. We first denote by $p^F(t, \lambda_{X0})$ the common component of the clean and dirty oil full marginal cost: $p^F(t, \lambda_{X0}) \equiv c_x + \lambda_{X0}e^{\rho t}$, where F stands for free of tax and/or CCS cost.

In the figures to come, $p_i(t)$ denotes the energy full marginal cost for sector i, that is: $p_i(t) \equiv \min\left\{p^F(t, \lambda_{X0}) + \min\left\{\zeta\lambda_Z(t), s_i\right\}, c_y\right\}$, $i = 1, 2$.

Last, we use the following generic notations for the critical dates in the different scenarios:

- $\underline{t}_Z$ and $\bar{t}_Z$ are the dates at which the ceiling constraint begins and ends to be active respectively.

- $\underline{t}_{ic}$ and $\bar{t}_{ic}$, $i = 1, 2$, are the dates at which sector i begins and ends to use clean oil respectively, or equivalently, begins and ends to capture either some part or the totality of its potential carbon emissions.

- $\tilde{t}$ is the time at which $p^F(t, \lambda_{X0}) + s_1 = u'(\bar{x}_d)$, if it exists.

- $\bar{t}_x$ is the time at which the initial oil endowment X^0 is exhausted.

- t_y is the date from which on solar energy is exploited.

Note that in some scenarios several critical dates might be confused. For instance, when the solar energy cost is high, formally when $c_y > u'(\bar{x}_d/2)$, then $\bar{t}_x = t_y$ as we shall see in the next section.

4 Optimal policies with abatement only in sector 1

This case arises when the solving strategy tests introduced above result in $\zeta\bar{\lambda}_Z^1 > s_1$ and $\zeta\bar{\lambda}_Z^2 \leq s_2$. Several types of optimal paths may occur depending on whether $\underline{t}_{1c}$ is smaller or equal to $\underline{t}_Z$ and depending on the cost level of the solar substitute. We examine first the family of scenarios where sector 1 should deploy carbon capture before the time at which the ceiling constraint begins to be active, that is $\underline{t}_{1c} < \underline{t}_Z$. These scenarios imply that the sectoral energy consumer prices p_1 and p_2 are distinct during some phases. Next, we consider the scenarios where sector 1 begins to use clean oil at the precise time $\underline{t}_Z$ at which the pollution stock reaches its critical level. In such a case the sectoral energy consumer prices are always identical. Note that the case where carbon capture would be deployed after $\underline{t}_Z$ cannot be optimal with constant marginal costs (see Lafforgue et al., 2008-a, p.593).

4.1 Sector 1's abatement starts before $\underline{t}_Z$

Let us consider successively the cases of high, intermediate and low average solar energy costs. We show that the results of the former case strongly contrast with those of the two latter cases as far as the aggregate consumption of dirty oil has to be shared out among the sectors during some phases at the ceiling when solar energy is competitive. In the high solar cost case this allocation is always strictly determined, whereas in the two other cases the global constraint on dirty oil consumption may give rise to an infinite number of feasible allocations when solar energy is competitive and when the constraint on the pollution stock is active at the same time.

4.1.1 The high solar cost case: $u'(\bar{x}_d/2) < c_y$

To proceed as simply as possible, we reason graphically by considering Figure 1 below that plots the paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_1$ and $p^F(t, \lambda_{X0}) + s_2$. In this figure, each path can be obtained from the other by a vertical translation. Moreover λ_{X0} is set small enough such that the trajectories $p^F(t, \lambda_{X0})$ and $p^F(t, \lambda_{X0}) + s_1$ cross the horizontal lines $u'(\bar{x}_d)$, $u'(\bar{x}_d/2)$ and c_y at some finite dates. Furthermore, $\zeta\lambda_{Z0} < s_1$ in such a way that the path $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ starts below the path $p^F(t, \lambda_{X0}) + s_1$, but crosses this last path at a time $\underline{t}_{1c}$ which is earlier than $\underline{t}_Z$ at which it crosses the horizontal line $u'(\bar{x}_d)$. A last feature of Figure 1 is that, at time $\underline{t}_Z$, $p^F(t, \lambda_{X0}) + s_2 > u'(\bar{x}_d)$.

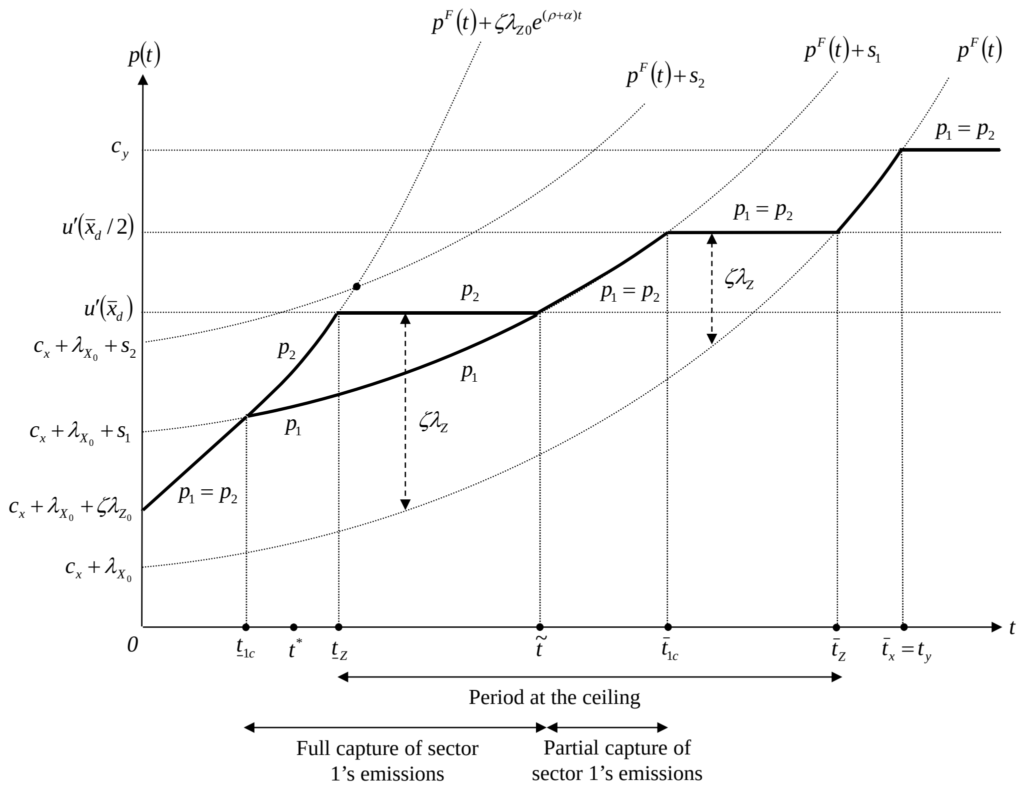

Figure 1: Optimal path supporting scenarios where clean oil is used only in sector 1, with an abatement beginning before $\underline{t}_Z$. The high solar cost case: $u'(\bar{x}_d/2) < c_y$

The optimal scenario suggested by Figure 1 is a seven-phases scenario.

- Phase 1, before the ceiling and without any clean oil use: $[0, \underline{t}_{1c})$

During this phase, $\zeta\lambda_Z(t) = \zeta\lambda_{Z0}e^{(\rho+\alpha)t} < s_1 < s_2$, hence dirty oil and only dirty oil is used in both sectors. The phase is ending at time $\underline{t}_{1c}$ when $\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_{1c}} = s_1$, that is when the marginal shadow cost of the pollution induced by dirty oil use equals the additional marginal cost of abatement in sector 1.

Note that during this phase the energy consumer price is the same for each sector: $p_i(t) = p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$, $i = 1, 2$. Moreover, $x_{id}(t) = q\left(p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right) > \bar{x}_d$, $i = 1, 2$, and since $Z^0 < \bar{Z}$, the pollution stock must increase during the phase. However, the existence of such a phase requires that, at time $\underline{t}_{1c}$, the critical level is not attained yet: $Z(\underline{t}_{1c}) < \bar{Z}$.

- Phase 2, before the ceiling with full abatement of sector 1's emissions: $[\underline{t}_{1c}, \underline{t}_Z)$

During this phase, $s_1 < \zeta\lambda_{Z0}e^{(\rho+\alpha)t} < s_2$ hence sector 1 uses clean oil exclusively while sector 2 still uses only dirty oil. Consequently, the two sectoral energy consumer prices now differ, $p_1(t) = p^F(t, \lambda_{X0}) + s_1 < p_2(t) = p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ resulting in a larger energy consumption in sector 1 than in sector 2. Since at $\underline{t}_{1c}$ the pollution stock is lower than $\bar{Z}$ and $x_{2d} = q\left(p^F(\underline{t}_{1c}, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_{1c}}\right) > \bar{x}_d$, it is still increasing at least at the beginning of the phase. The phase is ending at time $\underline{t}_Z$ when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = u'(\bar{x}_d)$ and, simultaneously, the pollution stock reaches the stabilization cap $\bar{Z}$.

- Phase 3, at the ceiling with full abatement of sector 1's emissions: $[\underline{t}_Z, \tilde{t})$

During this first phase at the ceiling, $\zeta\lambda_Z(t) = u'(\bar{x}_d) - p^F(t, \lambda_{X0}) > s_1$ and also $\zeta\lambda_Z(t) < s_2$, hence sector 1 uses only clean oil while sector 2 consumes only dirty oil, as during the previous phase. However, since the ceiling constraint is active, the dirty oil consumption by sector 2 is bounded from above by $\bar{x}_d$. Consequently this sector is the only one that has to bear the burden of the constraint: $x_{2d}(t) = \bar{x}_d$. The shadow marginal cost of pollution $\lambda_Z(t)$ is decreasing as $p^F(t, \lambda_{X0})$ is increasing and the phase is ending at time $\tilde{t}$ when $\zeta\lambda_Z(t) = s_1$.

- Phase 4, at the ceiling with partial abatement of sector 1's emissions: $[\tilde{t}, \bar{t}_{1c})$

During this second phase at the ceiling, $\zeta\lambda_Z(t) = s_1$. Since $\zeta\lambda_Z(t) < s_2$, sector 2 consumes only dirty oil while sector 1, being indifferent, uses a mix of clean and dirty oil. Now the burden of the ceiling constraint is borne simultaneously by both sectors: $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. Both sectors also consume the same amount of energy: $x_1(t) = x_{1c}(t) + x_{1d}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right) = x_{2d}(t) = x_2(t)$, implying that $x_{1c}(t) = 2q\left(p^F(t, \lambda_{X0}) + s_1\right) - \bar{x}_d$ is decreasing, $x_{1d}(t) = \bar{x}_d - q\left(p^F(t, \lambda_{X0}) + s_1\right)$ is increasing and $x_{2d}(t)$ is decreasing. Sector 1 thus substitutes gradually dirty for clean oil.

The phase is ending at time $\bar{t}_{1c}$ when $p^F(t, \lambda_{X0}) + s_1 = u'(\bar{x}_d/2)$. At this time, $x_{1d}(t) = x_{2d}(t) = \bar{x}_d/2$ and $x_{1c}(t) = 0$. From this time onwards, sector 1 must in turn use only dirty oil, as clean oil becomes too costly in relative terms.

- Phase 5, at the ceiling and without abatement of sector 1's emissions: $[\bar{t}_{1c}, \bar{t}_Z)$

This is the last phase at the ceiling. Since now $\zeta\lambda_Z(t) = u'(\bar{x}_d/2) - p^F(t, \lambda_{X0}) < s_1 < s_2$, both sectors use only dirty oil and share equally the burden of the ceiling constraint: $x_{1d}(t) = x_{2d}(t) = \bar{x}_d/2$. The phase is ending at time $\bar{t}_Z$ when $p^F(t, \lambda_{X0}) = u'(\bar{x}_d/2)$ that is when $\lambda_Z(t) = 0$, which indicates the end of the period at the ceiling.

- Phase 6, post-ceiling phase of oil exhaustion: $[\bar{t}_Z, t_y)$

This phase is a pure Hotelling regime during which only oil is consumed by both sectors like in the initial phase, but now with $\lambda_Z(t) = 0$. Since $p^F(t, \lambda_{X0}) > u'(\bar{x}_d/2)$, we get $x_{1d}(t) + x_{2d}(t) = 2q\left(p^F(t, \lambda_{X0})\right) < \bar{x}_d$ and the ceiling constraint is not active anymore. The phase is ending at time t_y when $p^F(t, \lambda_{X0}) = c_y$ and the oil stock is exhausted at the same time.

- Phase 7, permanent solar energy consumption: $[t_y, +\infty)$

From t_y onwards the solar energy is competitive and $y_1(t) = y_2(t) = \tilde{y}$.

The following proposition states the existence of such a path.

Proposition 1 *Assume that $u'(\bar{x}_d/2) < c_y$, that λ_{X0} and λ_{Z0} generate the full marginal cost paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_i$, $i = 1, 2$, and that $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ has the properties plotted in Figure 1. Furthermore the carbon stabilization cap $\bar{Z}$ is attained when $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} = u'(\bar{x}_d)$ and the initial oil endowment is exhausted when $p^F(t, \lambda_{X0}) = c_y$. Under these conditions the above seven phases scenario is the solution of the social planner problem.*

Proof: Clearly, there exist non-negative multipliers $\gamma_{ik}(t)$, $i = 1, 2$, $k = c, d$, $\nu_X(t)$ and $\nu_Z(t)$, $t \geq 0$, such that the first-order conditions (8)-(12) and the transversality conditions (13) are all satisfied. This is trivial for the Lagrange multipliers γ_{ik} associated with the control variables. Next, we can show that $\nu_X(t) = 0$, $t \geq 0$ is the right candidate and that the optimal trajectory of $\nu_Z(t)$ is given by:

$$\nu_Z(t) = \begin{cases} 0 & , t \in [0, \underline{t}_Z) \\ \left\{(\rho+\alpha)[u'(\bar{x}_d) - c_x] - \alpha\lambda_{X0}e^{\rho t}\right\}/\zeta & , t \in [\underline{t}_Z, \tilde{t}) \\ (\rho+\alpha)s_1/\zeta & , t \in [\tilde{t}, \bar{t}_{1c}) \\ \left\{(\rho+\alpha)[u'(\bar{x}_d/2) - c_x] - \alpha\lambda_{X0}e^{\rho t}\right\}/\zeta & , t \in [\bar{t}_{1c}, \bar{t}_Z) \\ 0 & , t \in [\bar{t}_Z, \infty) \end{cases} \tag{17}$$

Last, since the program (P) is convex, the first-order conditions (8)-(12) are sufficient and have a unique solution. ■

Since the proofs of all the other forthcoming propositions are basically the same, they will not be repeated in the next sections.

Discussion

As far as abatement is concerned, it would also be possible to have sector 1's full abatement starting from the initial date. Then, the first phase of the scenario illustrated in Figure 1 would be similar to the second one, with exclusive clean oil consumption in sector 1 and dirty oil consumption in sector 2.

Assume for instance that the social planner is facing the initial conditions Z^*, $Z^0 < Z^* < \bar{Z}$, and X^*, $X^* < X^0$, corresponding respectively to the pollution stock level and the available oil stock at time t^*, $\underline{t}_{1c} < t^* < \underline{t}_Z$, and starting from $Z(0) = Z^0$ and $X(0) = X^0$ as initially considered. Then the optimal scenario associated with these new initial conditions is proved to be the continuation of the initial scenario from t^* onwards: At time t, any variable in the scenario corresponding to the new initial conditions takes its value at time $t + t^*$ in the original scenario.

The same remark applies to all the cases we will examine hereafter. We have chosen to systematically present the longest possible scenario corresponding to the case under study, beginning by a phase of dirty oil consumption in both sectors.

The pattern of the optimal scenario results from two main logics. The first one is the Herfindahl least cost principle which delays the introduction of solar energy only when oil has been exhausted. More generally this least cost principle gives priority to the cheapest energy source, that is dirty oil, whence the ceiling constraint does not bind anymore. The second driving force results from the dynamics of energy prices under the Hotelling rule. The energy price never decreases through time, implying that if carbon capture has to be deployed, it has to be at the maximum rate initially. The result is a full capture of emissions by sector 1 once the profitability threshold condition over price and cost is verified. The progressive depletion of the resource stock makes increase the scarcity cost of oil consumption, $\lambda_X(t)$, an incentive for sector 1 to cut its abatement cost and revert to dirty oil gradually. Such an outcome could not arise if oil is infinite supply, that is if $\lambda_X(t) = 0$. With an infinite oil endowment, sector 1 should never stop to fully capture its emissions, the ceiling constraint being forever binding. The pattern of carbon capture in this scenario is thus the consequence of the Hotelling scarcity effect when combined with the optimal pollution accumulation pattern resulting from a global constraint over atmospheric carbon concentration. The same logic is at work in the scenarios that are examined below although with different consequences.

4.1.2 The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$

This case is illustrated in Figure 2.

The optimal path is now a six-phases scenario. The first four phases are similar to the first four phases depicted in Figure 1, meaning that sector 1's emissions begin again to be captured before the ceiling is reached.

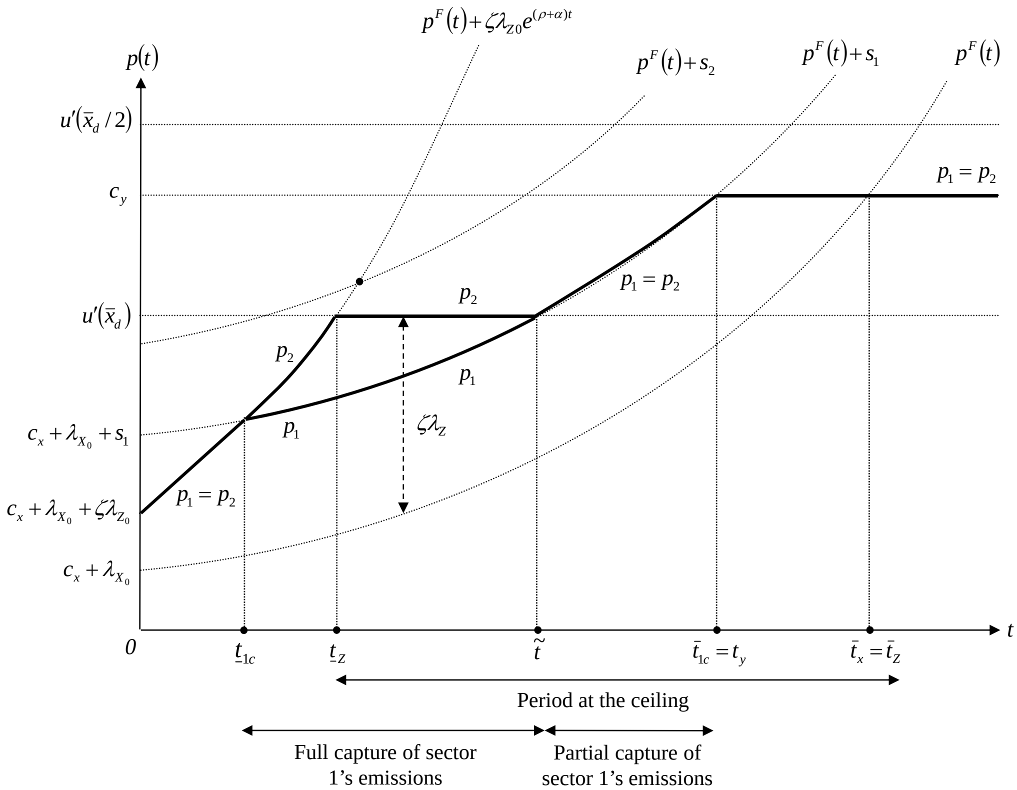

Figure 2: Optimal path supporting scenarios where clean oil is used only in sector 1, with an abatement beginning before $\underline{t}_Z$. The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$

The differences between these two scenarios occur at the end of phase 4. In the present case at $\bar{t}_{1c}$, $p^F(t, \lambda_{X0}) + s_1 = c_y$ contrary to what has been observed at the end of the fourth phase in the previous scenario where we found $p^F(t, \lambda_{X0}) + s_1 = u'(\bar{x}_d/2)$ at time $\bar{t}_{1c}$. Remember that during this fourth phase at the ceiling the aggregate consumption of dirty oil is constant and equal to $\bar{x}_d$, while the aggregate total consumption of oil is larger than $\bar{x}_d$: $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. Sector 2 uses only dirty oil: $x_2(t) = x_{2d}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right) > \bar{x}_d/2$ and sector 1 uses a mix of clean and dirty oil: $x_{1c}(t) = 2q\left(p^F(t, \lambda_{X0}) + s_1\right) - \bar{x}_d > 0$ and $x_{1d}(t) = \bar{x}_d - q\left(p^F(t, \lambda_{X0}) + s_1\right) > 0$. Since now at the end the phase $p^F(t, \lambda_{X0}) + s_1 = c_y < u'(\bar{x}_d/2)$, we have $x_{1c}(\bar{t}_{1c}) > 0$ contrary to the case illustrated in Figure 1 where $x_{1c}(\bar{t}_{1c})$ is nil.

The fifth phase $[\bar{t}_{1c}, \bar{t}_x)$ is a phase at the ceiling during which the aggregate consumption of dirty oil is locked at $\bar{x}_d$: $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. Since $c_y < u'(\bar{x}_d/2)$, the aggregate consumption of energy amounts to $2\tilde{y}$, which is larger than $\bar{x}_d$. The difference $2\tilde{y} - \bar{x}_d$ is supplied by solar energy since the marginal cost of clean oil in sector 1 is larger than the marginal cost of solar energy: $p^F(t, \lambda_{X0}) + s_1 > c_y$.[10] The distribution of the dirty oil aggregate consumption among the sectors, hence the correlative distribution of the solar energy aggregate consumption, is a matter of indifference.

The shadow marginal cost of the pollution stock remains positive, $\lambda_Z(t) = \left[c_y - p^F(t, \lambda_{X0})\right]/\zeta > 0$, which reflects the counterpart of the tightness persistency of the ceiling constraint. This fifth phase is ending when $\lambda_Z(t) = 0$. At this point in time, the initial oil endowment must be completely exhausted, and the ceiling must be definitively left: $\bar{t}_x = \bar{t}_Z$.

The sixth and last phase $[\bar{t}_x, \infty)$ is the phase of exclusive and definitive use of solar energy: $q_i(t) = y_i(t) = \tilde{y}$, $i = 1, 2$.

The following proposition concludes the examination of this intermediate solar cost case.

Proposition 2 *Assume that $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$, that λ_{X0} and λ_{Z0} generate the full*

[10] Since both c_x and c_y are constant, dirty oil and solar energy may only be simultaneously used during a phase at the ceiling. A generalization of this result to the case of a damage function that is increasing with the atmospheric carbon stock can be found in Hoel and Kverndokk (1996) and Tahvonen (1997). In particular, using a stock-dependent marginal extraction cost, but a constant marginal cost of the backstop, Tahvonen (1997) shows that there can exist a multiplicity of simultaneous energy use scenarios.

marginal cost paths $p^F(t, \lambda_{X0})$*,* $p^F(t, \lambda_{X0}) + s_i$*,* $i = 1, 2$*, and that* $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ *has the properties plotted in Figure 2. Moreover the carbon stabilization cap* $\bar{Z}$ *is attained when* $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} = u'(\bar{x}_d)$ *and the initial oil endowment is exhausted when* $p^F(t, \lambda_{X0}) = c_y$*. Under these conditions the above six phases scenario is optimal.*

4.1.3 The low solar cost case: $c_y < u'(\bar{x}_d)$

The case is illustrated in Figure 3. The new important feature of the figure is that we have $\zeta\lambda_{Z0}e^{(\rho+\alpha)t_y} < s_2$ at time t_y at which $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_y - p^F(t, \lambda_{X0})$.

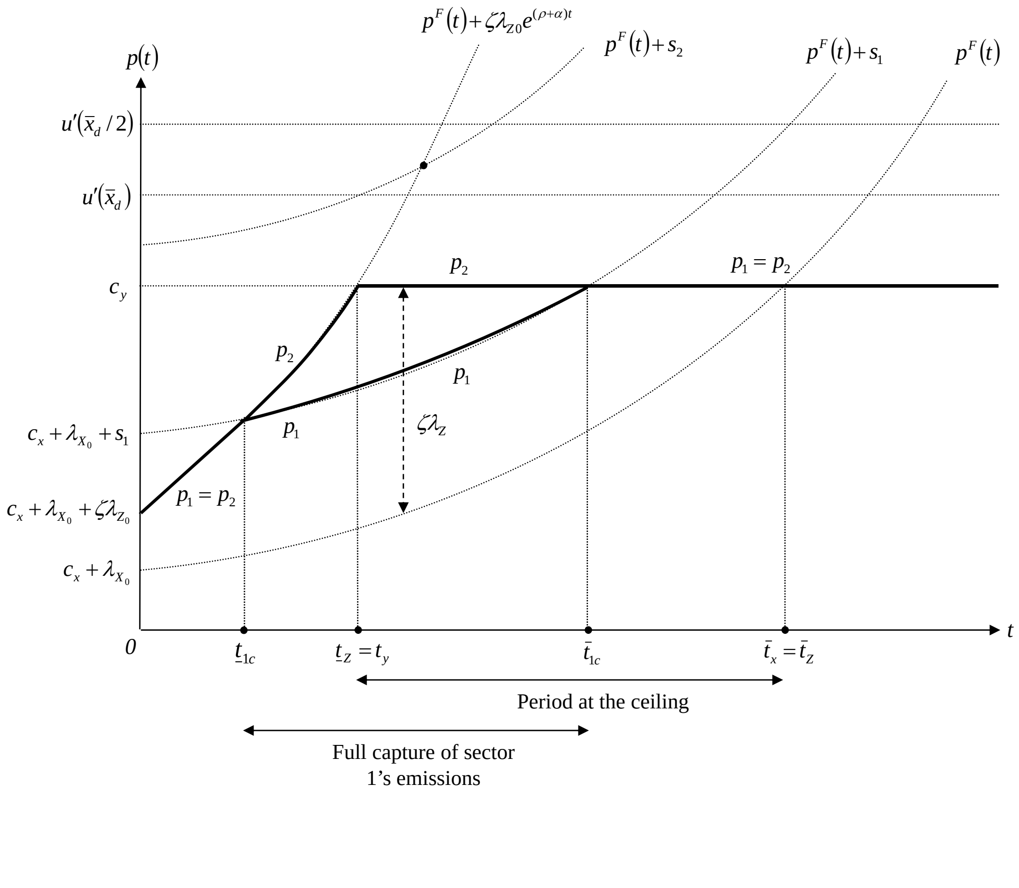

Figure 3: Optimal path supporting scenarios where clean oil is used only in sector 1, with an abatement beginning before $\underline{t}_Z$. The low solar cost case: $c_y < u'(\bar{x}_d)$

The optimal scenario is a sequence of five phases. The first two phases are similar to the phases obtained in the previous scenarios. Once again, sector 1 starts the capture of its emissions at time $\underline{t}_{1c}$ before the ceiling constraint begins to be active. However, the present scenario diverges from the previous ones at the end of the second phase during which sector 1 fully abates and sector 2 uses only dirty oil. In the present case at t_y, the end of phase 2, the full marginal cost of dirty oil matches the level c_y and simultaneously the ceiling constraint begins to be tight, so that $t_y = \underline{t}_Z$. Since $c_y < u'(\bar{x}_d)$ the dirty oil consumption rate of sector 2 amounts to $\tilde{y}$ being larger than $\bar{x}_d$: $x_d(t_y) = \tilde{y} > \bar{x}_d$.

The third phase $[t_y, \bar{t}_{1c})$ is a phase at the ceiling where sector 1 consumes only clean oil, while sector 2 combines dirty oil with $x_{2d}(t) = \bar{x}_d$ and solar energy with $y_2(t) = \tilde{y} - \bar{x}_d$, since it bears the burden of the ceiling constraint alone. During this phase, the shadow marginal cost of the pollution stock is still positive as the constraint is still active: $\lambda_Z(t) = \left[c_y - p^F(t, \lambda_{X0})\right]/\zeta > s_1$. Because $\lambda_Z(t)$ is decreasing, the phase ends at time $\bar{t}_{1c}$ when $\zeta\lambda_Z(t) = s_1$. From this date onwards, capturing sector 1's carbon emissions becomes too costly.

The fourth phase $[\bar{t}_{1c}, \bar{t}_Z)$ is similar to the fifth phase in the previous scenario, as illustrated in Figure 2. The ceiling constraint is still active and no sector may use clean oil because it is too costly, hence $x_{1d}(t) + x_{2d}(t) = \bar{x}_d$. The rest of the energy needs is supplied by solar energy: $y_1(t) + y_2(t) = 2\tilde{y} - \bar{x}_d$. Again, the share out of dirty oil and solar energy among the two sectors is a matter of indifference.

The shadow marginal cost of the pollution stock is positive, $\lambda_Z(t) = \left[c_y - p^F(t, \lambda_{X0})\right]/\zeta$, and it is declining to 0 at the end of the phase. At this closing time, the stock of oil must be exhausted: $\bar{t}_Z = \bar{t}_x$.

The fifth and last phase $[\bar{t}_Z, \infty)$ is the usual infinite phase of exclusive solar energy consumption.

Proposition 3 *Assume that $c_y < u'(\bar{x}_d)$, that λ_{X0} and λ_{Z0} generate the full marginal cost paths $p^F(t, \lambda_{X0})$, $p^F(t, \lambda_{X0}) + s_i$, $i = 1, 2$, and that $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ has the properties plotted in Figure 3. Furthermore the critical pollution stock $\bar{Z}$ is reached when $p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_y$ and the stock of oil is exhausted when $p^F(t, \lambda_{X0}) = c_y$. Under these conditions then the above five phases scenario is optimal.*

4.2 Sector 1's abatement starts at $\bar{t}_Z$

Such policies may exist, provided that at the time where $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_1$, we have $\min\{u'(\bar{x}_d/2), c_y\} > p^F(t,\lambda_{X0}) + s_1 > u'(\bar{x}_d)$ and, simultaneously, the ceiling is attained. Figure 4 illustrates the high solar cost case $c_y > u'(\bar{x}_d/2)$, and Figure 5 the intermediate solar cost case $u'(\bar{x}_d/2) > c_y > u'(\bar{x}_d)$. This scenario may not occur under the low solar cost assumption $c_y < u'(\bar{x}_d)$, which will be explained later.

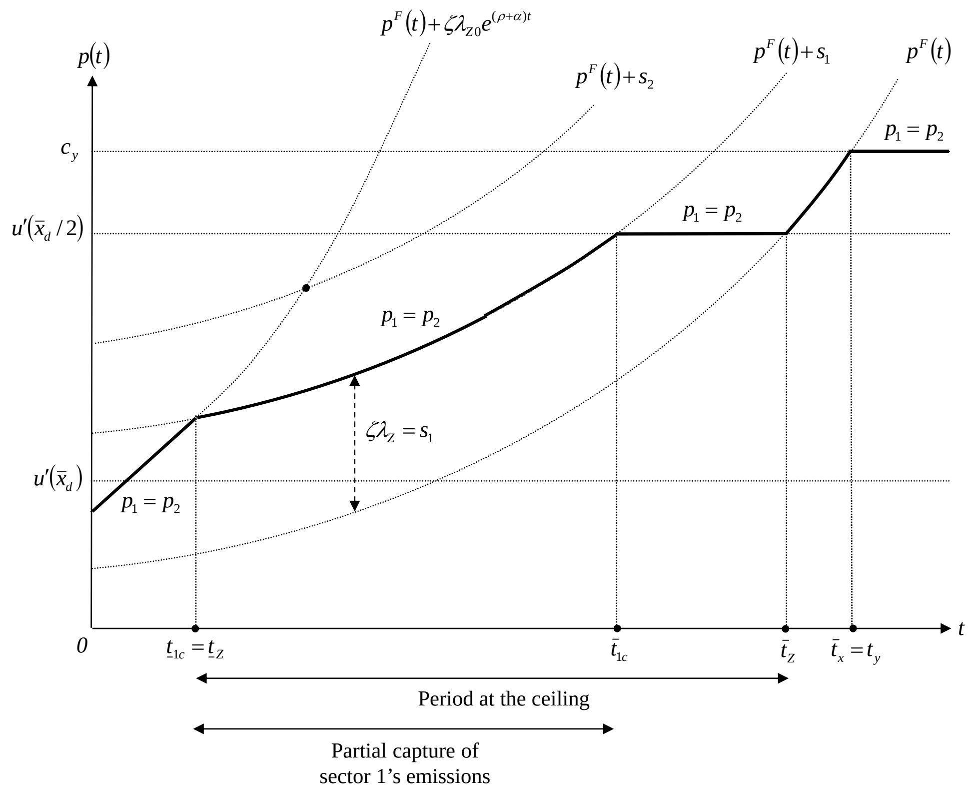

Figure 4: Optimal path supporting scenarios where clean oil is used only in sector 1, with an abatement beginning at $\underline{t}_Z$. The high solar cost case: $u'(\bar{x}_d/2) < c_y$

In both Figures 4 and 5 the first two phases of the optimal scenarios are the same. First, each sector consumes exclusively dirty oil up to the time $\underline{t}_Z = \underline{t}_{1c}$ where the atmospheric carbon stock hits the cap $\bar{Z}$. At the same time $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_1$, which implies that abatement may now be a competitive option for sector 1. Now, $x_{1d}(\underline{t}_{1c}) = x_{2d}(\underline{t}_{1c}) = q\left(p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1\right) < \bar{x}_d$ since $p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1 > u'(\bar{x}_d)$.

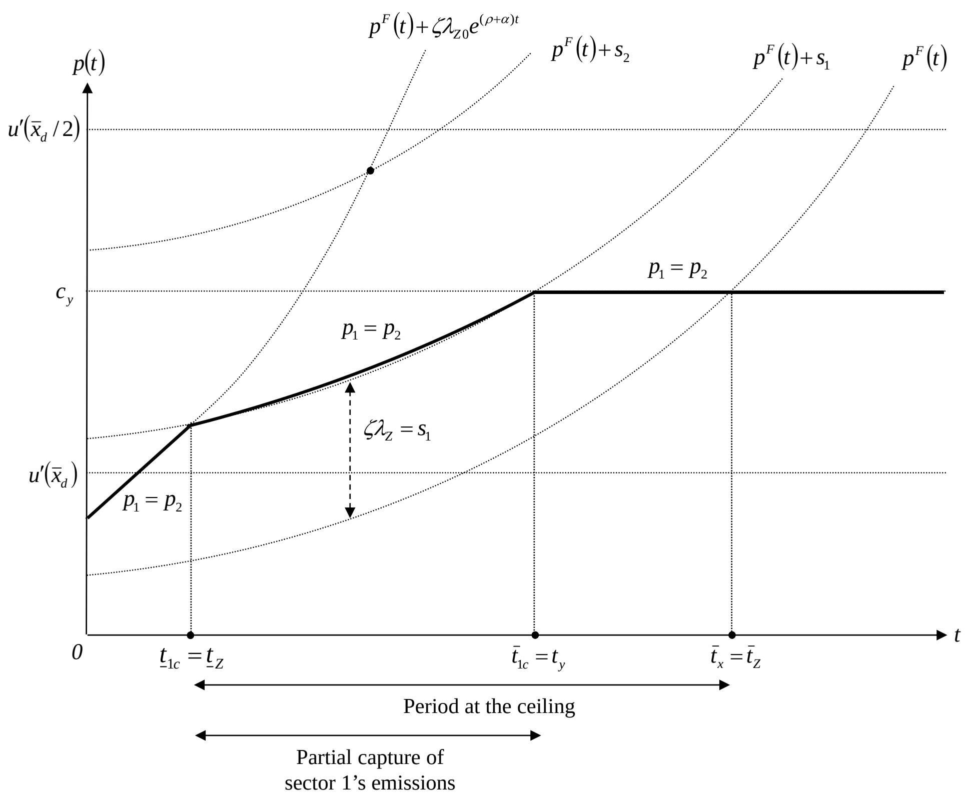

Figure 5: Optimal path supporting scenarios where clean oil is used only in sector 1, with an abatement beginning at $\underline{t}_Z$. The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$

It has to be clear that the assumption $p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1 = p^F(\underline{t}_{1c}, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_{1c}} > u'(\bar{x}_d)$ on which Figures 4 and 5 are drawn is crucial. If $p^F(\underline{t}_{1c}, \lambda_{X0}) + s_1$ is lower than $u'(\bar{x}_d)$, the second phase of the above two scenarios cannot occur.

The second phase is a phase at the ceiling. Because $p^F(t, \lambda_{X0}) + s_1 < \min\{u'(\bar{x}_d/2), c_y\}$ the burden of the ceiling constraint has to be borne by both sectors. This result comes from the fact that $q\left(p^F(t, \lambda_{X0}) + s_1\right) > \bar{x}_d/2$ and also that $q\left(p^F(t, \lambda_{X0}) + s_1\right) < \bar{x}_d$, resulting in $x_{1c}(t) = 2q\left(p^F(t, \lambda_{X0}) + s_1\right) - \bar{x}_d$, $x_{1d}(t) = \bar{x}_d - q\left(p^F(t, \lambda_{X0}) + s_1\right)$, $x_{2c}(t) = 0$ and $x_{2d}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right)$.

The contrasting features between Figures 4 and 5 are the same as those distinguishing Figure 1 and Figure 2. The phases occurring after the date $\bar{t}_{1c}$ at which sector 1 stops to abate its emissions in Figure 4 (respectively Figure 5) are the same as in Figure 1 (resp. Figure 2).

Finally, note that both sectors permanently face the same full marginal cost of energy.

Proposition 4 *For the optimal scenarios in which sector 1 is the only sector using clean oil, two cases can occur:*

i) Sector 1 begins to abate its emissions before the ceiling is reached. In this case its full marginal cost of energy is lower than sector 2's during the first two phases of sector 1's clean oil consumption;

ii) Sector 1 begins to abate when the ceiling is attained and then the full marginal cost of energy is the same in both sectors during any phase of the scenario.

5 Optimal policies with abatement in both sectors

The case of abatement in both sectors arises when the solving strategy test in Section 3 results in $\zeta\bar{\lambda}_Z^2 > s_2$. In this case the sectoral full marginal costs of energy are necessarily distinct during the phases of simultaneous abatement. This comes from the fact that the additional marginal abatement cost is smaller in sector 1 than in sector 2, which means that sector 1 will necessarily abate if sector does so.

The existence of such scenarios requires now to assume that when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_2$, $p^F(t, \lambda_{X0}) + s_2 < \min\{u'(\bar{x}_d), c_y\}$, as illustrated in Figures 6, 7 and 8 below for the high, intermediate and low solar energy cost cases. This characteristic contrasts with the one identifying the previous scenarios developed in Section 4, where abatement in sector 2 was never optimal, since the above inequality was reversed (see Figures 1 to 5).

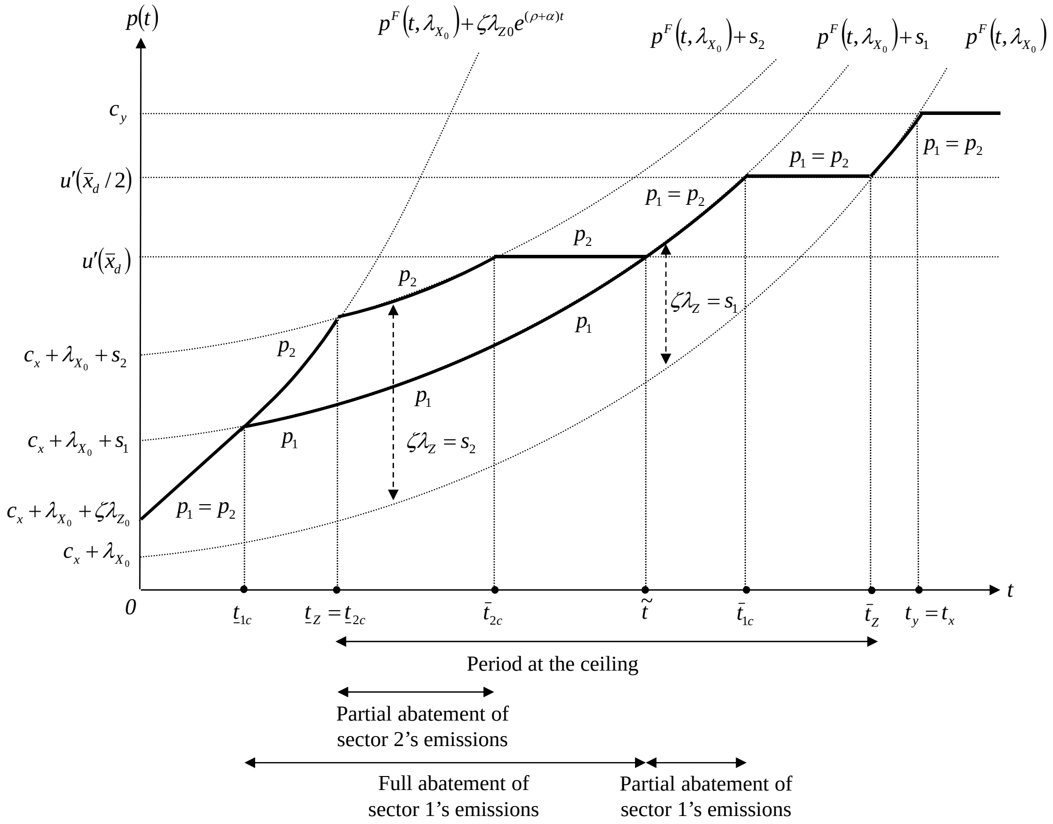

Figure 6: Optimal path supporting scenarios where clean oil is used in both sectors, with abatement beginning before $\underline{t}_Z$ in sector 1 and at $\underline{t}_Z$ in sector 2. The high solar cost case: $u'(\bar{x}_d/2) < c_y$

Whatever the cost of the solar substitute, the three first phases of the scenarios are the same. The distinguishing features of the following phases are similar to the differences observed in the scenarios depicted by Figures 1, 2 and 3 when sector 1 is the only sector using clean oil. For this reason we focus the analysis on these three first phases.

Phase 1, for $t \in [0, \underline{t}_{1c})$, is the usual initial phase during which both sectors use only dirty oil and the pollution stock increases since $x_{id}(t) = q\left(p^F(t,\lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right) > \bar{x}_d$, $i = 1,2$. The phase ends when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_1$, and abatement becomes a competitive

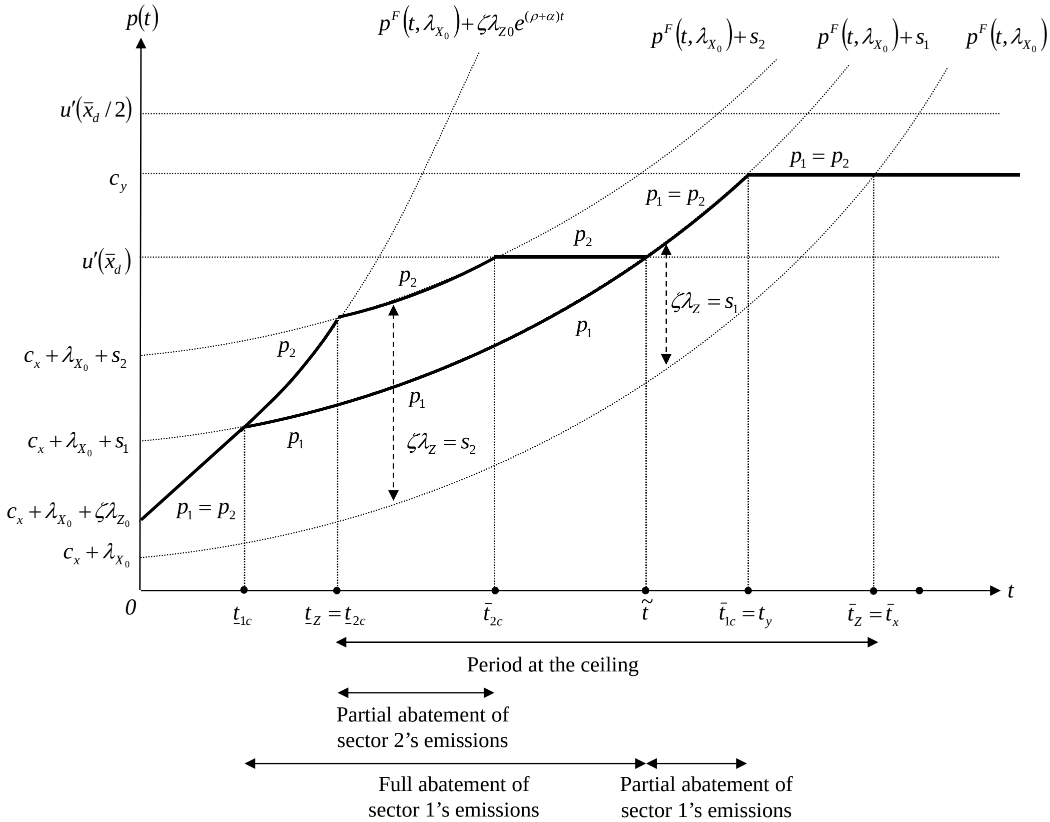

Figure 7: Optimal path supporting scenarios where clean oil is used in both sectors, with abatement beginning before $\underline{t}_Z$ in sector 1 and at $\underline{t}_Z$ in sector 2. The intermediate solar cost case: $u'(\bar{x}_d) < c_y < u'(\bar{x}_d/2)$

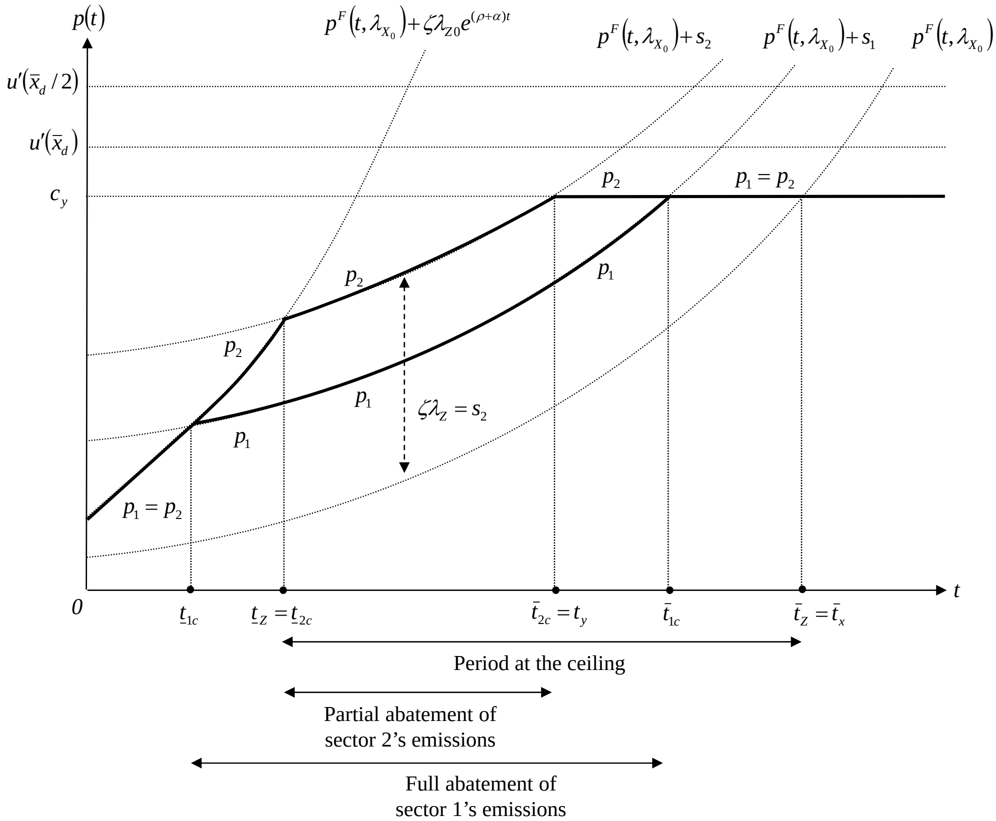

Figure 8: Optimal path supporting scenarios where clean oil is used in both sectors, with abatement beginning before $\underline{t}_Z$ in sector 1 and at $\underline{t}_Z$ in sector 2. The low solar cost case: $c_y < u'(\bar{x}_d)$

option for sector 1. The pollution stock stays below the cap $\bar{Z}$.

During the second phase, for $t \in [\underline{t}_{1c}, \underline{t}_{2c})$, sector 1 uses only clean oil and sector 2 only dirty oil. Since $x_{2d} = q\left(p^F(t, \lambda_{X0}) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}\right) > \bar{x}_d$ and initially $Z(\underline{t}_{1c}) < \bar{Z}$, the atmospheric carbon stock is still increasing. The phase ends when $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = s_2$ and, simultaneously, $Z(t) = \bar{Z}$, implying $\underline{t}_{2c} = \underline{t}_Z$. Given the characteristic that has been emphasized above, that is $p^F(t, \lambda_{X0}) + s_2 < \min\{u'(\bar{x}_d), c_y\}$, we get $x_{2d}(\underline{t}_{2c}) = q\left(p^F(t, \lambda_{X0}) + s_2\right) > \bar{x}_d$ at the beginning of the next phase.

During phase 3, for $t \in [\underline{t}_{2c}, \bar{t}_{2c})$, the economy is constrained by the carbon stabilization cap. The abatement option being comparatively cheap for sector 1, this sector uses only clean oil: $x_{1d}(t) = 0$ and $x_{1c}(t) = q\left(p^F(t, \lambda_{X0}) + s_1\right)$. Sector 2 bears the burden of the ceiling constraint alone and consumes a mix of clean and dirty oil: $x_{2d}(t) = \bar{x}_d$ and $x_{2c}(t) = q\left(p^F(t, \lambda_{X0}) + s_2\right) - \bar{x}_d$. Moreover, since $p_2(t) = p^F(t, \lambda_{X0}) + s_2 = u'(x_{2c}(t) + \bar{x}_d)$ is increasing, the clean oil consumption of sector 2 is decreasing during this phase. Time differentiating this last equality, we get $\dot{x}_{2c}(t) = \rho\lambda_{X0}e^{\rho t}/u''(x_{2c}(t) + \bar{x}_d) < 0$. The phase is ending when $p^F(t, \lambda_{X0}) + s_2 = u'(\bar{x}_d)$ in the high and intermediate solar energy cost cases (see Figures 6 and 7, respectively), or when $p^F(t, \lambda_{X0}) + s_2 = c_y$ in the low solar energy cost case (see Figure 8).

The subsequent phases are:

- the same phases 4 to 7 as the phases described in paragraph 4.1.1 when the solar energy cost is high;

- the same phases 4 to 6 as the phases described in paragraph 4.1.2 when the solar energy cost is intermediate;

- the same phases 4 to 5 as the phases described in paragraph 4.1.3 when the solar energy cost is low.

We conclude as follows:

Proposition 5 *In the optimal scenarios where both sectors have to consume clean oil, for any level of solar energy cost, sector 1 must begin to capture its carbon emissions before the ceiling is attained. On the other hand, sector 2 begins to partially abate when the ceiling constraint begins to be active.*

6 Direct capture from the pollution stock: The air capture option

Let us now assume that sector 2 is not able to capture its potential emissions at their source – hence it cannot directly use clean oil – but that the society can directly capture the carbon from the atmospheric pollution stock. We denote by $a(t)$ the instantaneous carbon capture rate from the atmosphere and by c_a the associated average capture cost assumed to be constant.

The dynamics of the oil and pollution stocks are now:

$$\begin{aligned}
\dot{X}(t) &= -x_{1c}(t) - \sum_i x_{id}(t) && (18)\\
\dot{Z}(t) &= \zeta \sum_i x_{id}(t) - a(t) - \alpha Z(t) && (19)\\
a(t) &\geq 0 && (20)
\end{aligned}$$

Define the instantaneous net surplus S_1 of sector 1 as in Section 3 and the surplus S_2 of sector 2 by:

$$S_2(x_{2d}(t), y_2(t)) = u(x_{2d}(t) + y_2(t)) - c_x x_{2d}(t) - c_y y_2(t)$$

The new social planner program becomes:

$$\max_{\{x_{id}, y_i, x_{1c}, a\}} \int_0^\infty \{S_1(x_{1c}(t), x_{1d}(t), y_1(t)) + S_2(x_{2d}(t), y_2(t)) - c_a a(t)\} e^{-\rho t} dt$$

subject to the constraints (18)-(20), (2), (3), (5) and (6).

The optimality conditions (8), (9) and (10) corresponding respectively to sector 1's energy choices x_{1c}, x_{1d} and y_1, remain the same, as well as the conditions (11), (12) and (13). Concerning the optimality conditions belonging to sector 2's choices, (8) does not exist anymore and (9) and (10) must be rewritten as:

$$\begin{aligned}
u'(x_{2d} + y_2) &= c_x + \lambda_X + \zeta \lambda_Z - \gamma_{2d} && (21)\\
u'(x_{2d} + y_2) &= c_y - \gamma_{2y} && (22)
\end{aligned}$$

Finally, denoting by $\gamma_a(t)$ the Lagrange multiplier associated with the non-negativity constraint on a, the optimality condition related to this last command variable is:

$$c_a = \lambda_Z(t) + \gamma_a(t) \tag{23}$$

together with the corresponding complementary slackness condition.

Assume that $a(t) > 0$ during some time interval. Then $\gamma_a(t) = 0$, $c_a = \lambda_Z(t)$ implying that $\dot{\lambda}_Z(t) = 0$ and, from (12), we get: $\nu_Z(t) = (\rho + \alpha)c_a > 0$. This situation is possible if and only if $Z(t) = \bar{Z}$. Thus, direct capture from the atmospheric pollution stock is proved to occur only during some phases at the ceiling, implying that $\dot{Z}(t) = 0$ and, equivalently, that $a(t) = \zeta \sum_i x_{id}(t) - \alpha\bar{Z}$.

Assume furthermore that $s_1 < \zeta c_a$, that is CCS in sector 1 is cheaper than the air capture technology. Then sector 1 must consume only clean oil and $a(t) = \zeta x_{2d}(t) - \alpha\bar{Z}$. Since sector 2 consumes only dirty oil and $\lambda_Z = c_a$, it follows that $x_{2d}(t) = q\left(p^F(t, \lambda_{X0}) + \zeta c_a\right)$. To make the analogy with the initial model, sector 2's oil consumption clearly reads as the amount of oil that sector 2 should consume if it had access to clean oil at an additional marginal cost $s_2 = \zeta c_a$. In this case it would use $\bar{x}_d$ units of dirty oil and $q\left(p^F(t, \lambda_{X0}) + s_2\right) - \bar{x}_d$ units of clean oil as during all the phases $[\underline{t}_{2c}, \bar{t}_{2c})$ in Figures 6, 7 and 8 in Section 5. The flow of potential emissions that must be captured to meet this clean oil consumption rate amounts to $\zeta\left[q\left(p^F(t, \lambda_{X0}) + s_2\right) - \bar{x}_d\right] = \zeta q\left(p^F(t, \lambda_{X0}) + s_2\right) - \alpha\bar{Z}$. This is precisely the flow of direct atmospheric carbon capture when sector 2 has only access to air capture at the cost $c_a = s_2/\zeta$. Thus, the economy behaves exactly as if sector 2 had access to clean oil at an additional marginal cost $s_2 = \zeta c_a$.

Proposition 6 *When sector 2 has only access to air capture at a constant average cost c_a, $\zeta c_a > s_1$, the optimal paths of the full marginal costs of clean and dirty oil in sector 1 and the optimal path of the full marginal cost of energy in sector 2 are the same as in the case where sector 2 has access to clean oil at an average additional cost $s_2 = \zeta c_a$. The sectoral energy consumption paths and the atmospheric pollution stock are the same in both cases.*

7 Conclusion

Using the Chakravorty et al. (2006) model, we have determined the optimal timing of CCS policies for an economy composed of two kinds of energy users differing in the cost of the abatement technology they have access to. In any case the marginal cost of CCS is constant, but capturing carbon emissions is more costly in sector 2 than in sector 1. Both sectors face a global maximal atmospheric carbon concentration constraint.

In this framework we have shown that carbon sequestration carried out by sector 1 must begin strictly before the atmospheric carbon stock reaches its critical threshold. Furthermore sector 1's emissions have to be fully abated during a first time phase with constant marginal cost of abatement and a stationary demand schedule. This result stands in contrast with the findings of Chakravorty et al. (2006) who showed that abatement should begin only when the atmospheric ceiling has been attained in a model with only one energy using sector and a single abatement technology.

The difference appears to be a consequence of the heterogeneity of the abatement costs of the energy users. This heterogeneity constrains the potential of CCS to be at most equal to the sole emissions of sector 1 and thus to be always smaller than the total carbon emissions of fossil energy consumers. In a constant CCS cost setting there is no limitation on the amount of abated emissions below the gross emission level. In a case where sector 2's emissions alone would drive atmospheric concentration up to its maximum threshold, full emission abatement by sector 1 appears to be the only optimal choice for the economy. Furthermore, with or without abatement possibility in sector 2, delaying CCS beyond the time where the atmospheric carbon stock reaches its maximum level is dominated by an earlier development of CCS by sector 1. However, even with abatement in sector 2, the total carbon emission flow from the two sectors remains only partially abated, resulting in a time phase during which the atmospheric carbon constraint binds over the fossil fuel consumption possibilities of the two sectors.

Note also that when both sectors have to capture their emissions, abatement in sector 2 is undertaken only after the beginning of the atmospheric carbon ceiling phase and that this abatement effort is always smaller than its gross contribution to carbon emissions. This result is now in accordance with Chakravorty et al. (2006).

For the sake of computational convenience, we have assumed constant marginal cost. In a similar ceiling model with a single sector of energy consumption, Amigues et al. (2012) explore more sophisticated CCS cost functions that depend either on the flow of sequestration or on the cumulated sequestration. Considering first a flow-dependent and increasing marginal cost, they show that optimal abatement must begin during the pre-ceiling phase. In this case, carbon sequestration allows both to delay the time at which the ceiling constraint begins to be active and to relax this constraint once active. Moreover the optimal sequestration flow is first increasing during the pre-ceiling phase and next decreasing during the phase at the ceiling. Next, they investigate the case of stock-dependent cost functions, which gives rise to two contrasting effects. The scarcity effect that is obtained by assuming an increasing marginal cost function conveys the idea that it becomes more and more costly to store carbon emissions as the stock already sequestered increases. On the opposite, the learning effect, obtained if the marginal cost is decreasing, implies that the deployment of the CCS technology improves as the installed capacity increases. In both cases, they show that it is never optimal to deploy CCS before the ceiling is reached. However, these two effects have contrasting effects on the pattern of the energy price. Under the learning effect, the price trajectory can exhibit declining phases while it is always increasing under the scarcity effect.

It is interesting to observe that the economy may experience a rather complex dynamic pattern of energy prices while being constrained by the atmospheric carbon ceiling. With constant abatement unit cost, the energy price at the consumer stage is composed of a sequence of constant price phases separated by increasing price phases. This complex shape translates into the time profile of the carbon tax implemented to meet the atmospheric concentration objective.

The carbon tax must increase over time before the ceiling is reached. Note that sector 1 escapes the tax when fully abating its emissions and bears a comparatively lower sequestration cost. The environmental constraint burden is transferred over to sector 2. Such a discrepancy between sectors is justified by the fact that sector 2 benefits from the carbon sequestration efforts of sector 1, a sort of positive "external" effect of sector 1 upon sector 2. Of course this is not a real external effect, since it comes through the carbon price. But this observation opens interesting policy questions with regard to the use of carbon regulation in order to develop non polluting transportation devices, like the electric car when electricity comes from power generation plants that use the CCS technology.[11] During the ceiling phase, the carbon tax has an overall decreasing shape which goes down to zero at the end of the phase. However this general shape is actually composed of a complex sequence of phases with decreasing rates, separated by phases with constant rates. These latter phases correspond respectively to sector 2's abatement phase and to the partial abatement phase of sector 1 which succeeds its full carbon abatement phase.

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[11] See e.g. Chakravorty et al. (2011).

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