INSTITUT D'ECONOMIE INDUSTRIELLE

# RAPPORT DE RECHERCHE :

## *Séquestration des gaz à effet de serre, compétitivité relative des énergies primaires et diversité de leurs usages*

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# Avant-propos

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# Séquestration des gaz à effet de serre, compétitivité relative des énergies primaires et diversité de leurs usages

Rapport de recherche

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## SYNTHESE DE LA RECHERCHE

Parmi les nombreux facteurs qui déterminent la compétitivité des énergies carbonées fossiles par rapport à d'autres sources d'énergie, en particulier l'énergie nucléaire et les énergies renouvelables réputées propres, un facteur clé est la possibilité de maîtriser à coûts raisonnables les rejets de gaz à effet de serre qu'implique leur utilisation massive.

Les deux premières études annexées au présent rapport supposent donnée cette possibilité et caractérisent les sentiers d'exploitation optimale de ce type de ressource et les politiques de capture et de séquestration qui en résultent. La troisième étude est un essai de définition des politiques qu'il conviendrait de promouvoir pour obtenir les coûts raisonnables supposés déjà acquis dans les deux premiers essais.

Le plus simple pour aller à l'essentiel est de retenir comme modèle des dommages induits par la concentration atmosphérique de gaz à effet de serre le modèle dit « modèle à plafond » dans lequel les dommages en question sont « minimes » tant qu'un seuil critique de concentration n'a pas été dépassé mais sont incommensurablement élevés dès que ce seuil est franchi. Dans ce type de modèle, puisque les ressources carbonées fossiles sont abondantes et d'un coût de mobilisation relativement modeste, la contrainte de non-franchissement est nécessairement active. Dès lors la date à partir de laquelle la contrainte en question restreint la consommation d'énergie fossile et oblige peut-être à recourir aux moyens de capture et de séquestration apparaît comme une date phare. Le problème est alors de savoir s'il faut mobiliser ces moyens avant la date phare en question ou attendre d'être contraint. Une interprétation large du principe de précaution suggèrerait qu'il ne faudrait pas trop attendre, c'est-à-dire qu'il ne faudrait pas attendre d'être contraint.

Les deux premiers essais démontrent qu'une politique active de séquestration ne doit être mise en œuvre avant d'avoir atteint le seuil critique de concentration que dans deux cas :

- lorsque les possibilités de capture, et donc leurs coûts moyens, sont différents selon l'usage des ressources ;
- lorsque ces possibilités de capture sont les mêmes quels qu'en soient les usages, le coût moyen de capture dépendant alors du flux de rejets à traiter.

Un résultat fort de notre recherche est de montrer que dans ce dernier cas, même lorsque le coût moyen de capture décroît avec l'expérience accumulée dans cette activité et donc qu'on pourrait être tenté de croire qu'il faille démarrer assez tôt la politique de séquestration, il faut toujours attendre d'être au plafond pour commencer à capturer.

La diversité des structures des sentiers optimaux de substitution entre énergies carbonées fossiles et énergies renouvelables propres selon que la ressource fossile est plus ou moins abondante et que l'effet d'apprentissage, bien que dominant, est plus ou moins prégnant est un second résultat de la recherche qui mérite d'être d'autant plus souligné qu'il n'était pas attendu. Lorsque les effets d'apprentissage sont si faibles que, le stock des rejets capturés et séquestrés augmentant, le coût moyen de capture et de séquestration lui-même augmente, nous montrons qu'il n'y a plus alors qu'un seul type de sentier optimal, à quelques variations mineures près.

Pouvoir disposer de techniques de capture et de séquestration à des coûts raisonnables n'est pas un don du Ciel, mais le fruit soit de l'expérience, soit d'efforts de recherche conséquents, soit d'une conjugaison des deux.

La recherche présente deux avantages par rapport à l'apprentissage. Elle évite de mettre en œuvre trop tôt une technologie de coût initial par hypothèse excessivement élevé tant que l'on n'a pas suffisamment appris. Elle permet aussi l'exploration d'un éventail beaucoup plus large d'options techniques. Le troisième essai s'attache à préciser d'abord les politiques optimales permettant une percée technologique, une réduction drastique des coûts d'abattement, qui ne reposeraient que sur l'un ou l'autre des leviers permettant de la déclencher en supposant que chacun de ces leviers puisse être assez puissant pour provoquer un tel bouleversement des coûts.

Une pure politique de recherche devrait faire en sorte que la percée technologique a lieu lorsque la contrainte de plafond commence à restreindre la consommation d'énergie polluante et pas avant. Il ne sert à rien de disposer dès aujourd'hui d'une technologie que l'on n'aura à mettre en œuvre que demain. Mais il faut noter que la date à laquelle les effets de cette contrainte doivent être pris en compte est elle-même endogène.

Une mobilisation optimale des possibilités d'apprentissage suppose de combiner un prix des rejets dans l'atmosphère à une subvention en faveur de l'utilisation de technologies de dépollution. Tel n'est pas le cas lorsque la percée est obtenue par la R&D seule. La taxation des émissions est alors suffisante pour induire des efforts optimaux de recherche.

La possibilité de combiner apprentissage à partir des technologies initialement existantes et efforts de recherche conduit à élargir considérablement la perspective. L'accumulation du carbone dans l'atmosphère et le développement de technologies d'atténuation des émissions apparaissent alors comme deux processus dynamiques en interaction avec leurs propres logiques et contraintes. On montre ainsi qu'il est possible qu'il faille introduire l'abattement avant d'atteindre le plafond. On montre aussi que les politiques optimales de mobilisation combinée peuvent initialement s'appuyer uniquement sur de l'apprentissage ou uniquement sur des efforts de recherche. Dans des scénarios où il convient de recourir simultanément à la recherche et à l'apprentissage pour provoquer une percée technologique, on montre enfin que l'effort d'apprentissage doit croître à un rythme plus soutenu que celui des efforts de recherche. Ces derniers en effet ne produisent de résultats qu'au moment de la percée, ce qui n'est pas le cas des efforts d'apprentissage, l'abattement de la pollution qu'ils permettent contribuant à réduire le poids de la contrainte climatique.

# 1 INTRODUCTION

Les ressources carbonées fossiles sont des énergies primaires abondantes dont la mobilisation permet à la plupart d'entre elles de satisfaire les besoins en services énergétiques des usagers à des coûts relativement modestes. Leur compétitivité par rapport à d'autres ressources primaires, en particulier l'énergie nucléaire et ses différentes filières et les énergies renouvelables qui exploitent à court terme l'énergie solaire incidente, énergies réputées propres, semblerait donc assurée[1]. Cette perspective de développement risque cependant d'être compromise par les rejets conséquents de gaz à effet de serre (GES) qu'implique leur utilisation massive, gaz dont l'accumulation dans l'atmosphère, dès lors qu'elle est trop forte, peut déclencher des dommages difficilement supportables. Par difficilement supportable, il faut comprendre que les coûts qu'induit cette concentration sont sans commune mesure avec les bénéfices tirés de la consommation d'énergie fossile dont elle est issue.

Pour contourner un handicap qui serait susceptible de s'avérer à terme dirimant, un facteur clé est la possibilité de maîtriser à coûts raisonnables les rejets de GES qu'implique l'exploitation soutenue de ces ressources fossiles. Le GES d'origine anthropique le plus important est le $CO_2$ puisqu'il représente à lui seul entre 72 et 76% des émissions totales. L'un des moyens envisagés pour réduire ces rejets de $CO_2$ dans l'atmosphère est le captage et la séquestration géologique du carbone (CSC), solution préconisée par le GIEC (Groupe d'Experts Intergouvernemental sur l'Evolution du Climat) dans un rapport dédié à cette technologie (IPCC, 2005). Sans entrer dans les détails techniques, qui font par ailleurs l'objet d'une abondante littérature spécialisée, ce procédé d'abattement consiste à capter à la source les émissions de composés carbonées avant rejet dans l'atmosphère et à les injecter ensuite dans des réservoirs naturels, des aquifères salins par exemple, ou dans d'anciens sites miniers ou encore dans des gisements d'hydrocarbures soit actifs, soit éteints[2].

Les études empiriques visant à évaluer le potentiel de cette technologie sont relativement nombreuses et sont, le plus souvent, réalisées au moyen de modèles complexes d'évaluation intégrée (Edmonds et al., 2004, Hamilton et al., 2009, Kurosawa, 2004, Gerlagh and van der Zwaan, 2006, Grimaud et al., 2011). Cette complexité apparaît comme le prix à payer pour pouvoir disposer de modèles opérationnels et suffisamment précis pour pouvoir définir des politiques énergétiques

[1] Près de 85% de l'énergie commerciale provient aujourd'hui des trois principales sources d'énergie fossile carbonée : charbon, pétrole et gaz naturel (IPCC, 2007).

[2] Cependant, comme le fait remarquer Herzog (2011), les préoccupations à propos du changement climatique ne sont pas à l'origine de cette idée de séparer et de capturer le $CO_2$ des rejets provenant des centrales thermiques. Les premières unités de CSC construites aux Etats-Unis dans les années 70 avaient pour but d'améliorer le rendement de l'extraction des puits en cours d'exploitation, puits dont la pression peut être augmentée grâce à l'injection des émissions de $CO_2$ ainsi captées.

et d’abattement. Mais la multitude des rétroactions à l’œuvre dans de tels modèles tend le plus souvent à brouiller les lignes de force le long desquelles ces politiques devraient se déployer. De plus, du fait de cette complexité, ces modèles sont réduits à prendre comme données nombre de paramètres, pour certains essentiels dans l’explication de ces politiques. Pour marquer ces lignes de force et pour endogénéiser autant que faire se peut ces paramètres, un modèle théorique plus épuré se révèle plus approprié. Le développement d’un tel modèle constitue l’objet de la recherche présentée dans ce rapport.

Le modèle sur lequel notre recherche est fondée est issu des travaux de Lafforgue, Magné et Moreaux (2008-a et 2008-b), qui constituent eux-mêmes une extension du modèle séminal de Chakravorty, Magné et Moreaux (2006), et tiennent compte du fait que les capacités de stockage des rejets capturés ne sont pas illimitées. Les objectifs de ces travaux sont premièrement de déterminer quand et à quelle échelle la CSC doit être utilisée et, deuxièmement, de déterminer comment le recours à ce mode d’abattement du flux d’émission modifie le sentier optimal de consommation des ressources carbonées fossiles, et ce, lorsque la concentration atmosphérique en carbone ne doit pas dépasser un certain seuil jugé critique, ce qui est l’objectif déclaré de l’accord de Kyoto.

Les trois extensions que nous proposons ont pour objet d’identifier les facteurs économiques qui déterminent les coûts d’utilisation de la CSC, et qui régissent de ce fait la compétitivité relative des énergies non-renouvelables carbonées par rapport aux énergies renouvelables non émettrices de $CO_2$.

Les deux premières études supposent ce coût donné et caractérisent les sentiers d’exploitation optimale des deux types d’énergie ainsi que les politiques de séquestration qui en résultent. La première étude considère un coût moyen de séquestration constant, mais prend en compte le fait que les capacités de déploiement de la CSC dépendent des secteurs d’usages dans lesquels elles sont mises en œuvre. Il semble en effet évident que capter les rejets d’une centrale thermique au gaz sera moins coûteux que capter les rejets d’un parc de véhicules fonctionnant grâce à cette même source d’énergie.

Dans la deuxième étude, nous envisageons différentes configurations de structure du coût moyen de séquestration. Ce coût moyen peut dépendre soit du flux de rejets à traiter, soit du cumul de ces rejets, soit enfin des deux. Le coût moyen de capture peut être soit une fonction croissante du cumul des rejets séquestrés afin de rendre compte de la rareté des sites d’enfouissement les plus accessibles, et donc les moins coûteux, soit une fonction décroissante de ce même cumul grâce aux effets d’apprentissage dont bénéficie le secteur au fur et à mesure qu’il déploie la technologie de capture en question.

Enfin, la troisième étude est un essai de définition des politiques qu’il conviendrait de promouvoir pour obtenir les coûts raisonnables supposés déjà acquis dans les deux premiers essais. En effet, pouvoir disposer d’un dispositif de CSC à des coûts non prohibitifs n'est pas un don du Ciel, mais le fruit soit de l'expérience, soit d'efforts de recherche conséquents, soit d'une conjugaison des deux. Cet essai s'attache d'abord à préciser les politiques optimales permettant une percée technologique dans le secteur de la CSC, i.e. une réduction drastique du coût d'abattement, qui ne reposeraient que sur l'un ou l'autre des leviers permettant de la déclencher. Elle envisage ensuite la possibilité de combiner apprentissage à partir des technologies initialement existantes et effort de recherche pour réaliser la dite percée.

Le rapport est organisé comme suit. Les hypothèses communes aux différents modèles développés ainsi que les grandes lignes de leurs principes de fonctionnement sont exposés dans la section 2. Les résultats du modèle initial et ceux des trois extensions que nous proposons font l'objet de la section 3. En particulier on compare les différents scénarios de mise en place des politiques d'exploitation des différents types d'énergie et d'abattement obtenues dans les trois cas. Enfin dans la dernière section nous présentons la façon dont nous comptons valoriser les fruits de cette recherche.

# 2 STRUCTURE COMMUNE AUX DIFFERENTS MODELES DEVELOPPES

La structure commune aux trois études développées dans ce rapport, dont une version simplifiée est donnée par la Figure 1, est la suivante. La demande en services énergétiques des usagers finaux, qui sont supposés être d'une seul type, peut-être approvisionnée par deux ressources primaires, parfait substitut l'une de l'autre : une ressource carbonées fossile et émettrice de $CO_2$, le charbon, et une ressource renouvelable propre, le solaire. Le coût d'approvisionnement à partir de l'une ou l'autre de ces deux sources primaires comprend l'ensemble des coûts de transformation de l'énergie primaire en question en services énergétiques directement utilisables par lesdits usagers. Le coût de transformation de la ressource non-renouvelable en énergie utile est inférieur au coût de transformation de l'énergie renouvelable. Par ailleurs, ces deux coûts sont supposés constants.

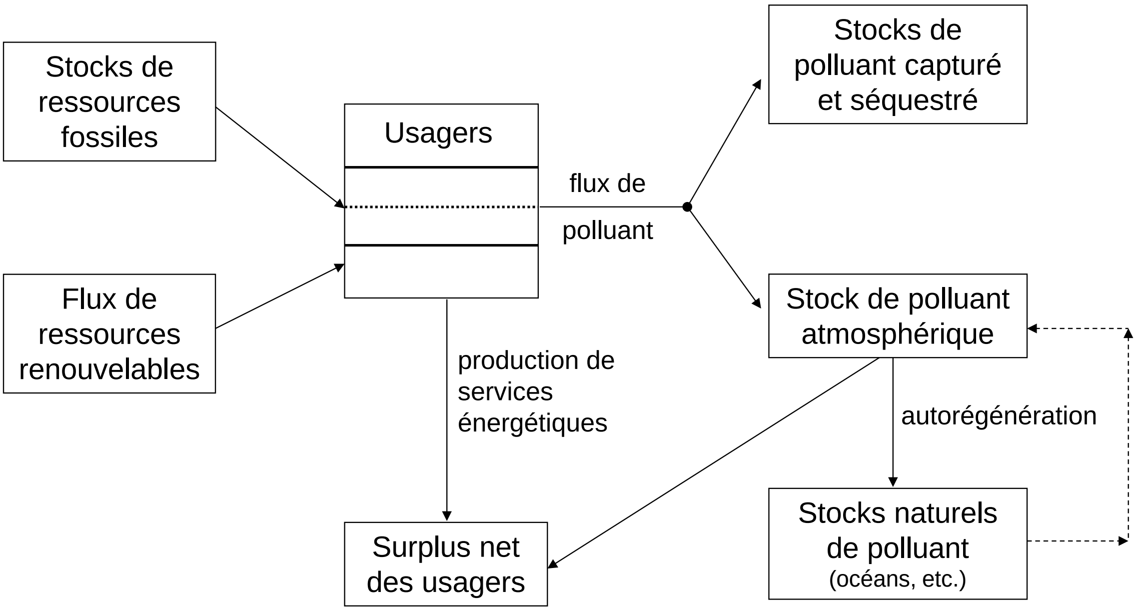


Figure 1 : La structure commune des modèles

L'exploitation de la ressource fossile carbonée génère des flux de rejets de $CO_2$ qui s'accumulent dans l'atmosphère. Une partie de ces gaz accumulés est progressivement éliminée par régénération naturelle[3], la partie restante est source de dommages pour les usagers. Cependant, possibilité est donnée à ces usagers de réduire leur empreinte carbone en capturant et en séquestrant tout ou partie de leurs émissions grâce à un dispositif de CSC. Partant de ce postulat, nous considérons alors deux types d'énergies fossiles aptes à approvisionner la demande finale en services énergétiques selon que leurs rejets polluants soient séquestrés ou non. Nous convenons d'appeler « charbon propre » la partie de la production de charbon dont les émissions sont capturées et « charbon sale », la partie dont les émissions sont directement relâchées dans l'atmosphère. La production de charbon propre implique donc, par rapport à la production de charbon sale, un surcoût correspondant au coût de capture et de séquestration.

Pour aller à l'essentiel, nous retenons comme modèle des dommages le modèle dit « à plafond », introduit par Chakravorty et al. (2006), qui consiste à poser que les dommages sont négligeables tant qu'un seuil critique de concentration atmosphérique n'est pas dépassé mais sont incommensurablement élevés dès que ce seuil est franchi[4]. Dans ce type de modèle, puisque les ressources carbonées fossiles sont disponibles en grande quantité et d'un coût de mobilisation relativement

[3] Il s'agit en réalité d'un processus de séquestration naturelle, donc gratuit, dans des puits de très grande capacité, essentiellement les océans (voir IPCC, 2007, pour plus de détails).
[4] La prise en compte de dommages commensurables pour des niveaux de concentration en-deçà du seuil en question ne modifie pas sensiblement les conclusions de l'analyse pour autant qu'on ne s'intéresse qu'aux propriétés qualitatives des sentiers optimaux, comme l'ont montré Amigues, Moreaux et Schubert (2011).

modeste, si le seuil de déclenchement d'évènements catastrophiques n'est pas excessivement élevé, la contrainte de non-franchissement du dit seuil sera nécessairement active le long du sentier optimal d'exploitation des ressources polluantes[5].

Deux points méritent alors d'être soulignés. Le premier est le fait que, le long d'un sentier optimal, la date à partir de laquelle la contrainte de plafond restreint la consommation de charbon sale est une date phare. Etant fonction du sentier de consommation de ce type de charbon suivi depuis l'instant initial, elle est de ce fait endogène. La contrainte de plafond doit donc faire sentir ses effets sur la totalité du sentier d'exploitation du charbon sale, mais aussi sur celui du charbon propre et celui de la ressource renouvelable puisque ces trois sources d'énergie sont de parfaits substituts les unes des autres. Le second point à souligner est que la société dispose de deux options pour relâcher cette contrainte de plafond, ces deux options pouvant être combinées. L'une consiste à recourir aux moyens de capture et de séquestration, et donc à substituer du charbon propre au charbon sale, l'autre à substituer la ressource renouvelable au charbon sale. De ce fait, le problème est double :

- Pour déverrouiller la contrainte, à supposer qu'il faille la déverrouiller, faut-il privilégier la capture et la séquestration des rejets émis par les ressources polluantes et retarder l'exploitation des ressources naturellement propres ? Ou faut-il au contraire privilégier d'abord l'exploitation des ressources naturelles dites propres ?
- Quelle que soit la réponse à la question précédente, faut-il attendre d'être contraint pour mobiliser l'une ou l'autre de ces deux sortes de ressources, naturellement propre ou rendue propre après traitement approprié, ou faut-il au contraire s'efforcer de les mettre en œuvre avant d'avoir à subir directement les effets de la contrainte comme le suggèrerait une acceptation large du principe de précaution ?

Clairement, les réponses à ces deux questions sont liées. L'argumentation est fondée sur la confrontation des coûts moyens totaux de chacune des trois options énergétiques : charbon propre, charbon sale et énergie renouvelable. La détermination du coût moyen de cette dernière option, à la fois non-polluante et abondante, est immédiate puisque ce coût ne comprend que le coût monétaire de transformation, supposé constant. En revanche, le coût des deux premières options implique trois composantes. Produire du charbon présente d'abord un coût de

[5] Si le seuil était suffisamment élevé et les stocks disponibles en ressource fossile suffisamment petits, la contrainte pourrait être négligée. Il faudrait alors mettre l'accent sur les dommages commensurables et mesurer au trébuchet ces dommages et les bienfaits des services procurés par la consommation d'énergie. Les seuils généralement admis sont compris entre 450 et 650 ppm (parties par million par volume). Le spectre peut sembler extrêmement large. Mais, compte tenu des stocks exploitables de ressources carbonées fossiles, tous les travaux de simulation montrent que la contrainte la moins prégnante, lorsque le plafond est fixé à 650 ppm, est active le long du sentier optimal (cf. par exemple Chakravorty, Magné et Moreaux, 2012).

transformation, également constant et supposé inférieur à celui de l'énergie renouvelable. Ensuite, à la consommation de charbon, qu'il soit propre ou sale, doit être associé un coût d'opportunité – une rente minière – comme pour toute ressource non-renouvelable disponible en quantité limitée. Ces deux premières composantes sont communes aux deux types de charbon exploité. Dans le cas de la production de charbon sale, il faut ajouter à ce coût commun un second coût d'opportunité correspondant au stock de carbone présent dans l'atmosphère. Ce coût marginal social de la pollution est équivalent au niveau de taxe, mesuré en termes de surplus marginal des utilisateurs, qu'il faudrait appliquer sur les flux d'émissions dans une économie décentralisée afin d'implémenter l'optimum de premier rang. Enfin, la production de charbon propre présente un coût additionnel direct correspondant à la séquestration des rejets ainsi qu'un coût d'opportunité spécifique correspondant au stock de carbone déjà séquestré. Les propriétés de ces coûts additionnels étant propres à chacune des trois études développées, elles seront explicitées plus loin.

# 3 RESULTATS DE LA RECHERCHE

## 3.1 Rappel des résultats du modèle initial

Les premières études théoriques (Chakravorty et al., 2006, Lafforgue et al., 2008-a et 2008-b) sur lesquelles s'appuient notre recherche considéraient un seul type d'utilisateur de services énergétiques pour lequel le coût moyen de capture et de séquestration est constant. Elles ont mis en évidence l'importance des capacités de stockage du carbone à différents coûts et des potentialités d'exploitation de leurs substituts renouvelables, plus ou moins abondants. La conclusion principale de ces études est que le recours à la CSC permet de prolonger un usage soutenu de la ressource fossile compatible avec le respect de la contrainte de plafond. Par ailleurs, il n'est pas optimal d'avoir recours à la CSC avant d'avoir atteint le dit plafond ni, lorsque cette option est exercée, de séquestrer la totalité des émissions polluantes. L'enchaînement type des différentes phases de consommation d'énergie et d'abattement est illustré à la Figure 2.

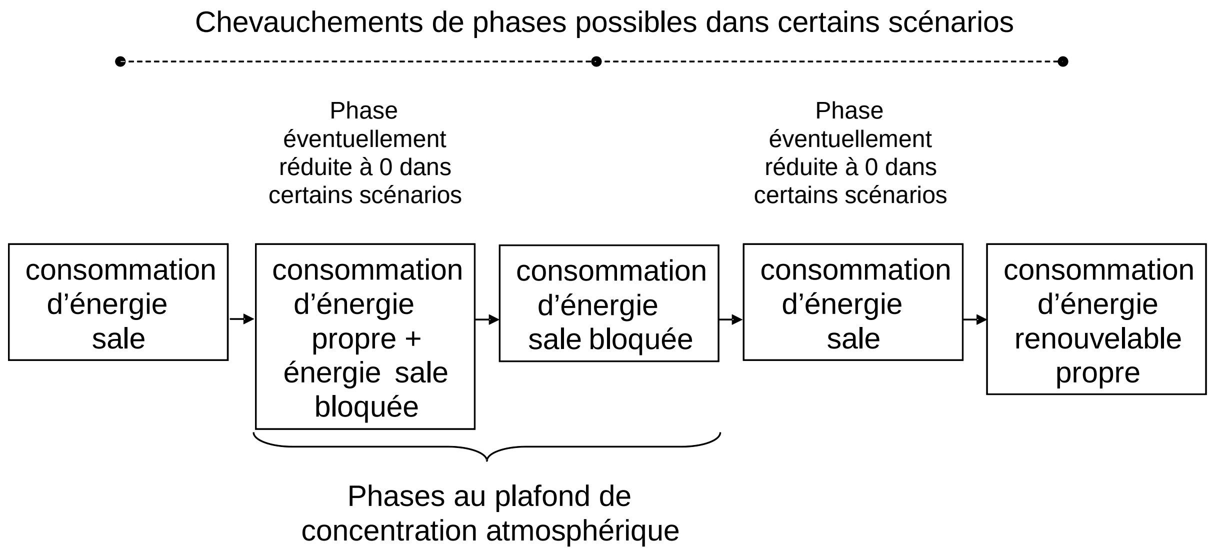


Figure 2 : Typologie des enchaînements de phases – Le modèle initial

Les extensions de ces modèles qui sont détaillées en annexe, mettent en évidence que la chronologie des phases de séquestration et de non-séquestration dépend de façon cruciale de la structure de la fonction de coût de capture dans les modèles à un seul type d'usager, et des différentiels de coûts de capture dans les modèles où le coût de capture est corrélé avec le type d'usager. Les trois sous-sections suivantes présentent ces résultats.

## 3.2 Hétérogénéité des usagers

Supposer que les possibilités de capture et donc leurs coûts ne dépendent pas des usages est une commodité pour l'analyse. Cependant, malgré son fort potentiel, la CSC présente l'inconvénient de ne pouvoir être mise en œuvre à des coûts raisonnables que pour les rejets qui émanent d'une partie des usagers : ceux qui sont à la source des émissions les plus importantes et les plus concentrées, typiquement les centrales électriques thermiques ou certaines industries lourdes (cimenteries, aciéries…). En effet, si capturer les gaz à effet de serre émis par une centrale à charbon est techniquement faisable à un coût qui n'est pas nécessairement exorbitant, capturer les gaz d'échappement des véhicules routiers ou des locomotives à moteur thermique est une mission presqu'impossible. Presque mais pas tout à fait. En effet, si la capture directe apparaît irréalisable, il reste la possibilité d'une capture indirecte en prélevant dans l'atmosphère les gaz qui y auraient été rejetés. L'extension du couvert végétal est un des procédés possibles, mais limité. D'autres voies industrielles semblent s'ouvrir à terme qui ne rencontreraient pas ces limites bien que probablement très coûteuses[6]. L'autre possibilité est d'imposer une norme de rejet sur les véhicules. Le différentiel de coût de production entre des véhicules qui satisfont la norme et ceux qui ne la satisfont pas peut être vu comme un coût d'abattement.

La première étude présentée en annexe a pour but d'étudier le cas de plusieurs secteurs d'utilisation de l'énergie qui se différencient par leur capacité de capture de leurs émissions à différents coûts. La modification du modèle initial qui en résulte est schématisée à la Figure 3. Pour simplifier, nous présentons le cas de deux types d'usagers. Les usagers de type 1 (U1) ont la possibilité de recourir à la CSC. Autrement dit, ils peuvent consommer de l'énergie sale, i. e. dont les rejets ne sont pas traités, et, moyennant un coût additionnel de traitement des rejets, également de l'énergie propre. Nous supposons que le coût moyen de capture et de séquestration est constant. On suppose en outre que la capacité de stockage des sites d'enfouissement permet d'y séquestrer tous les rejets qu'il conviendrait éventuellement de capturer. Aucune rente de rareté n'a donc à être retenue. Pour les utilisateurs de type 2 (U2), le coût de séquestration est prohibitif de sorte qu'ils ne consomment que de l'énergie sale. Enfin, nous supposons que la fonction de demande de services énergétiques est identique pour tous les usagers.

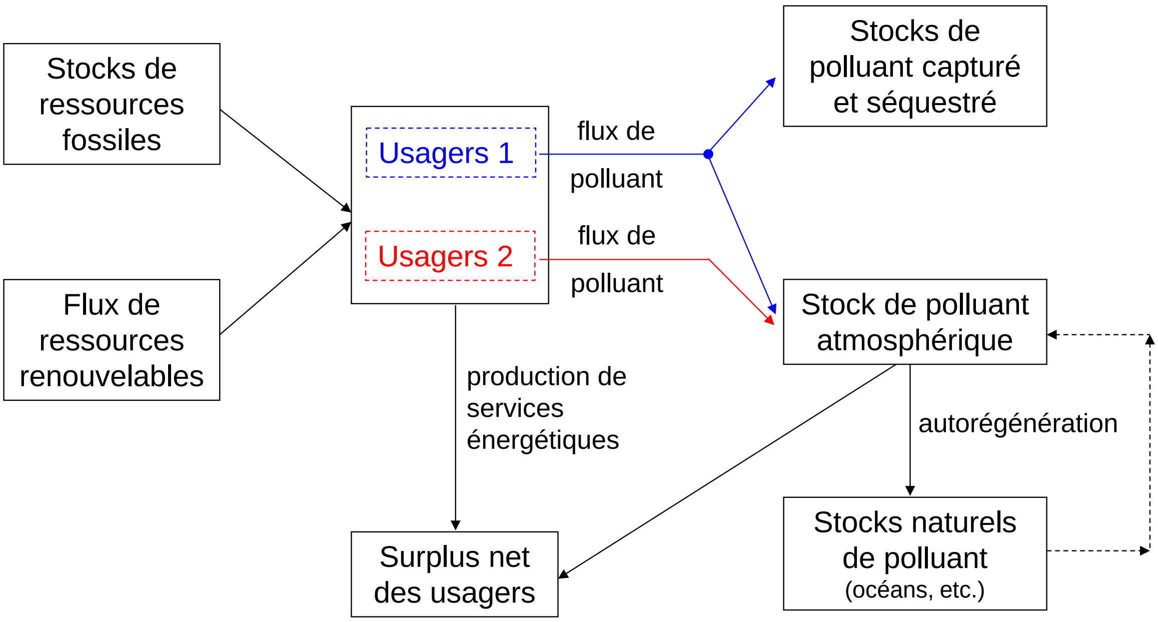


Figure 3 : Hétérogénéité des usagers

[6] Par souci d'homogénéité du présent rapport, nous ne développons pas les résultats issus de la prise en compte de cette seconde option d'abattement. Ceux-ci sont toutefois détaillés dans le premier article de recherche joint en Annexe.

Nous montrons que, s'il faut mettre en œuvre la technologie de CSC, il existe des sentiers optimaux le long desquels il faut commencer à capturer les émissions du secteur U1 avant de buter contre la contrainte de plafond. En outre, puisque cette technologie est inapte à traiter les rejets polluants des usagers de type U2, il est optimal, tout au moins au début, de séquestrer la totalité des émissions des usagers de type U1. Cette conclusion est généralisable à des situations moins extrêmes dans lesquelles les captures peuvent s'opérer directement dans plusieurs secteurs mais à des coûts moyens constants différents. La Figure 4 présente la typologie des enchaînements de phases de consommation des trois sources d'énergie primaire lorsque deux types d'utilisateurs sont considérés.

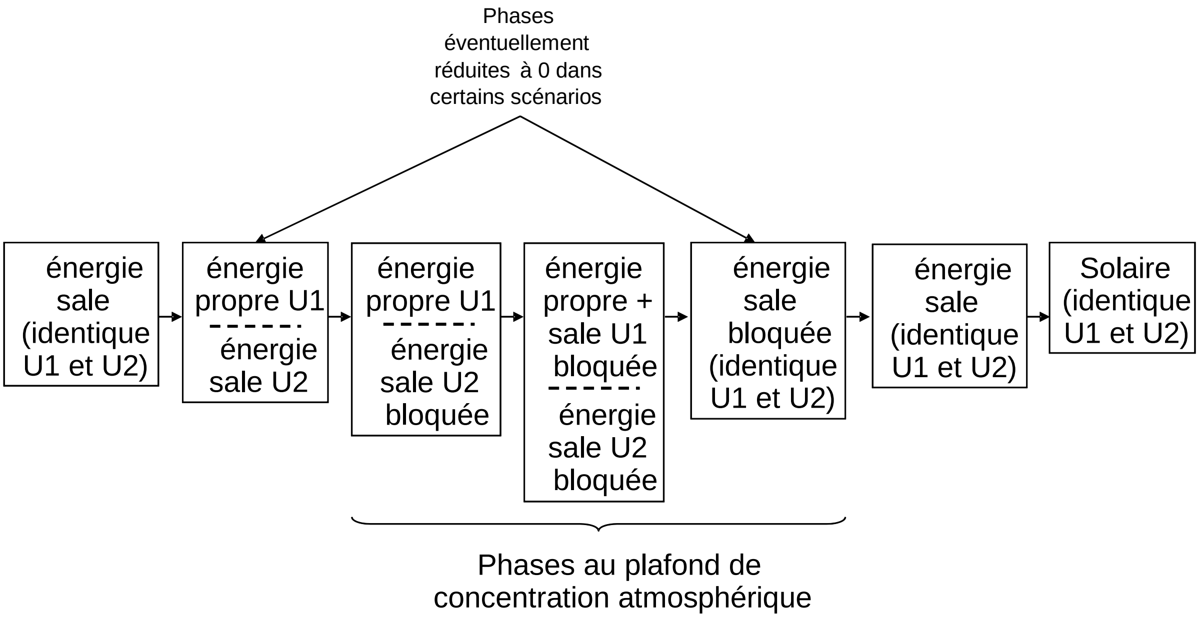


Figure 4 : Typologie des enchaînements de phase – Le cas de secteurs hétérogènes

Ayant la même fonction de demande, les deux secteurs d'utilisation ont un comportement identique tant que les usagers U1 n'ont pas recours à l'énergie propre. En revanche, la CSC permet aux usagers U1, lorsqu'ils exercent cette option, d'accroître leur consommation totale d'énergie en y intégrant une certaine proportion d'énergie propre et de soustraire ainsi une partie de leurs émissions au paiement de la taxe carbone. Au cours des phases où l'option est exercée, les quantités totales de charbon consommées par les deux types d'usagers diffèrent. Par conséquent, au cours de telles phases, chaque type d'usager fait face à un prix de l'énergie qui est différent de celui auquel fait face l'autre type d'usager. Le prix supporté par les usagers U2 est toujours supérieur à celui des usagers U1 du fait de leur impossibilité à se soustraire au paiement de la taxe carbone en substituant de l'énergie propre à de l'énergie sale, le coût de séquestration étant pour eux prohibitif. Nous montrons également que lorsque la contrainte de plafond est prégnante, seule la consommation d'énergie des usagers U2 est contrainte.

## 3.3 Structures alternatives des fonctions de coût de CSC

Dans ce deuxième essai, nous considérons un seul type d'usager final, mais supposons que les coûts spécifiques de séquestration ne sont pas constants. La structure générale du modèle reste la même que celle illustrée à la Figure 1. Deux types extrêmes de fonctions de coûts sont *a priori* envisageables. A chaque instant le coût moyen de capture peut être soit dépendant des quantités de rejets polluants à séquestrer, soit dépendant du cumul des émissions. Dans ce dernier cas, deux possibilités sont à considérer :

- Les sites de séquestration sont plus ou moins faciles d'accès et/ou requièrent des aménagements plus ou moins coûteux. L'actualisation commande de les mobiliser par ordre croissant de leur coût, les moins coûteux devant être mobilisés en priorité. Il en résulte qu'à chaque date, le coût moyen de capture et de séquestration est une fonction croissante du cumul des séquestrations effectuées jusqu'à cette date.
- Il est bien connu que toute activité est d'autant mieux organisée que l'expérience accumulée est grande. Si cet effet d'apprentissage devait être suffisamment puissant alors, à chaque instant, le coût moyen de capture et de séquestration devrait au contraire apparaître comme une fonction décroissante du cumul des séquestrations effectuées jusqu'à l'instant en question.

Ce que montre notre recherche c'est que, quel que soit celui des premiers effets qui est dominant, une politique active de capture et de séquestration ne doit jamais débuter avant que soit atteint le plafond de concentration en carbone lorsque les coûts moyens instantanés de capture sont indépendants des quantités à capturer au même instant.

Ce résultat était plus ou moins attendu lorsque le premier effet est dominant, effet qu'on appellera « effet de rareté » (sous-entendu des sites de séquestration facilement aménageables). En effet, le coût additionnel moyen induit par l'exploitation de l'énergie propre comprend à présent un coût monétaire direct de séquestration qui augmente à mesure que la capacité disponible d'enfouissement diminue, accru d'un coût d'opportunité associé à la limitation de cette capacité. La dominance de l'effet rareté pénalise donc la compétitivité de l'énergie propre par rapport à une situation où son coût additionnel serait constant. Il n'y a donc aucune raison de démarrer son exploitation avant d'être contraint par le plafond de concentration atmosphérique des gaz à effet de serre.

En outre, nous montrons que ce résultat ne dépend pas du coût du substitut renouvelable non carboné. Lorsque ce coût est élevé, l'enchaînement type des différentes phases est le même que celui de la Figure 2. Lorsque le coût de l'énergie solaire est faible, cet enchaînement est celui qui est illustré à la Figure 5.

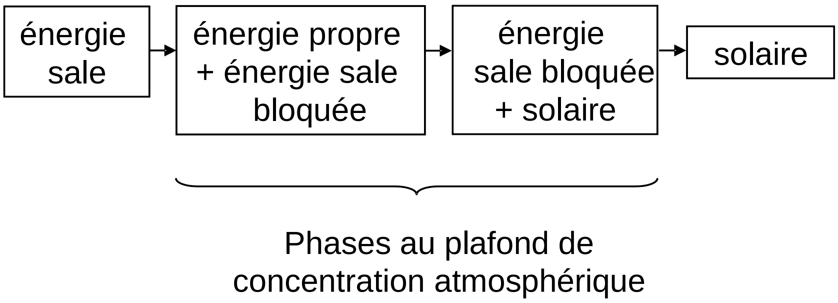


Figure 5 : Typologie des enchaînements de phase – Le cas d'un effet rareté dominant et d'une énergie solaire de faible coût

Le résultat est en revanche plus surprenant dans le cas d'un effet d'apprentissage dominant. On aurait pu croire que pour préparer un desserrement plus efficace de la contrainte dès que le plafond est atteint, il eut été opportun d'accumuler quelque expérience auparavant. En effet, le surcoût marginal de séquestration comprend à présent un coût monétaire direct qui décroît avec le stock des rejets déjà séquestrés, diminué d'un gain marginal lié à l'accumulation d'expérience acquise sur la technologie de CSC, gain qui justifie l'octroi d'une subvention à l'exploitation de l'énergie propre. Or, malgré ce renforcement de la compétitivité de l'énergie propre au cours du temps, il n'est jamais optimal d'en débuter l'exploitation avant d'avoir atteint le plafond.

Cependant, les profils temporels des prix de l'énergie et les sentiers de consommation des divers types d'énergies disponibles sont généralement très différents selon que domine soit l'effet de rareté soit l'effet d'apprentissage. Nous montrons qu'il peut être optimal dans certains cas de retarder encore davantage le recours à la CSC par rapport à l'instant où le stock de polluant bute sur la contrainte de plafond. Une illustration de ce résultat est donnée aux Figures 6 et 7, qui décrivent des exemples d'enchaînements de phases lorsque l'exploitation ne débute pas à l'instant où le plafond est atteint, et selon que l'énergie solaire est disponible à un coût élevé ou faible.

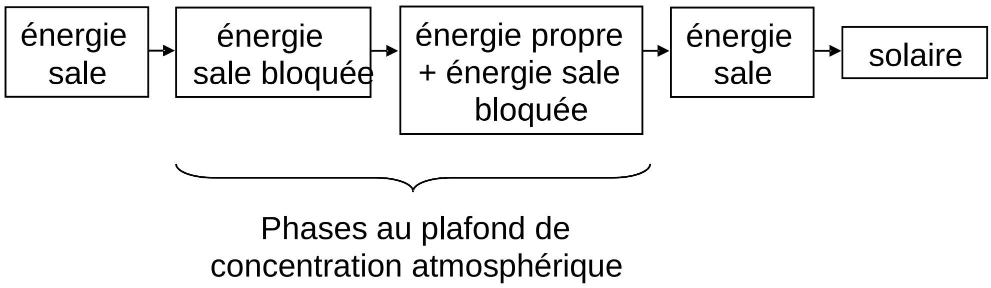


Figure 6 : Typologie des enchaînements de phase – Le cas d'un effet d'apprentissage dominant et d'une énergie solaire de coût élevé

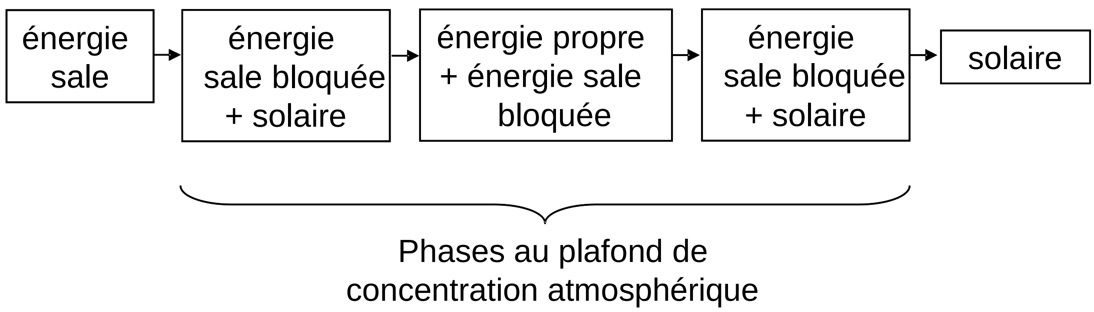


Figure 7 : Typologie des enchaînements de phase – Le cas d'un effet d'apprentissage dominant et d'une énergie solaire de faible coût

Lorsque le coût de l'énergie solaire est relativement bas, la période au plafond peut comprendre trois phases lorsque les disponibilités en ressource non renouvelable sont suffisamment élevées. La première phase combine exploitation de l'énergie solaire et production d'énergie sale en régime bloqué. Le coût additionnel de la séquestration compte tenu des possibilités d'apprentissage diminue au cours du temps du fait de l'actualisation. La seconde phase est celle au cours de laquelle l'effet de l'apprentissage est suffisamment fort pour que ce coût additionnel soit inférieur à celui de l'énergie solaire. Dès lors, il faut concentrer les efforts sur la séquestration. Les effets de l'apprentissage diminuant au cours du temps, le coût additionnel augmente et bientôt l'énergie solaire redevient compétitive. Lors de la troisième phase, il convient à nouveau d'exploiter conjointement l'énergie sale et l'énergie renouvelable.

L'autre hypothèse considérée dans cette étude est que le coût moyen de capture dépend à chaque instant des seuls volumes à capturer au même instant, indépendamment de tout effet de séquestration cumulée, et plus précisément que ce coût moyen est une fonction croissante des volumes à capturer. On montre alors que, contrairement au cas précédent, la politique optimale impose de commencer la séquestration avant que le plafond soit atteint, comme illustré à la Figure 8.

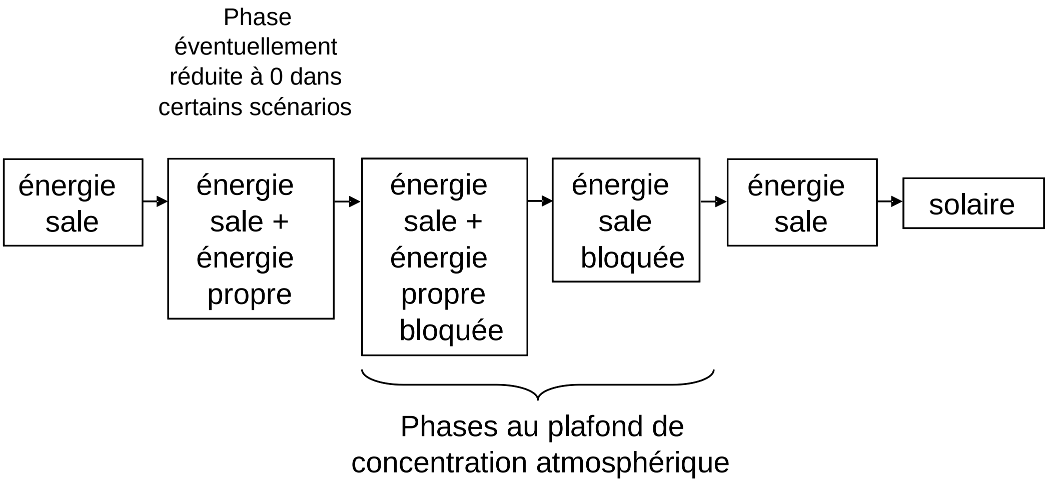


Figure 8 : Typologie des enchaînements de phase – Le cas d'une fonction de coût de séquestration ne dépendant que du flux des rejets

## 3.4 Progrès technique drastique, apprentissage et R&D

Pouvoir disposer de techniques de capture et de séquestration susceptibles d'être mises en œuvre à des coûts raisonnables n'est pas un don du Ciel. C'est soit le fruit de l'expérience, soit le produit d'efforts soutenus de recherche et de développement, soit d'une conjugaison des deux.

La troisième contribution du présent mémoire s'attache à préciser d'abord les politiques optimales permettant une percée technologique, une réduction drastique des coûts d'abattement, qui ne reposeraient que sur l'un ou l'autre de ces leviers en supposant que chacun seul soit assez puissant pour conduire à un tel bouleversement des coûts ; ensuite, de montrer comment il faut les déployer au cours du temps lorsqu'on s'applique à tirer profit de leur complémentarité.

La recherche et développement présente deux avantages par rapport à l'apprentissage. Elle évite de mettre en œuvre trop tôt une technologie de coût initial par hypothèse excessivement élevé tant qu'on n'a pas acquis suffisamment d'expérience pour provoquer la percée technologique voulue.

Ces différents aspects ont nourri une vive controverse parmi les économistes. Pour les uns, le potentiel d'apprentissage justifie un soutien public significatif aux technologies d'abattement, à même de contrebalancer leur surcoût initial et de favoriser leur adoption par le secteur de production d'énergie. Pour d'autres, il est préférable de laisser le temps à la recherche d'identifier les options les plus prometteuses techniquement et économiquement  et d'en assurer le développement. Un soutien trop précoce à des technologies immatures peut s'avérer contre-productif et devrait donc être évité.

Les premières réflexions sur cette question ont confirmé ce diagnostic : la prise en compte des possibilités d'apprentissage induit une action optimale plus précoce tandis que la prise en compte des potentialités de la recherche conduit à retarder l'action. La faiblesse de ces analyses vient de ce qu'elles ne considèrent que des cas extrêmes où l'avancement technologique ne résulterait soit que de l'apprentissage, soit que de la recherche. Mais une politique de soutien public à l'abattement va conduire les entreprises du secteur énergétique vers des stratégies de réponse variées, combinant dans des proportions diverses des efforts de déploiement précoce de la séquestration pour bénéficier d'effets d'apprentissage avec des efforts d'innovation vers des options techniques nouvelles et potentiellement moins coûteuses à mettre en œuvre. L'objet de la troisième étude présentée en annexe est de construire une analyse endogène du choix entre apprentissage et recherche dans un modèle où les deux effets se combinent pour provoquer une percée technologique dans le secteur de l'abattement.

Dans un premier temps, on explore les cas extrêmes où le progrès technique ne résulterait que de l'apprentissage ou bien de la seule recherche. Les trajectoires technologiques combinant recherche et apprentissage sont étudiées dans un second temps.

Une politique fondée sur la seule recherche-développement ne devrait déboucher sur une révolution des coûts qu'à la date à partir de laquelle la contrainte de plafond commence à restreindre directement la consommation d'énergie polluante et pas avant, dans la mesure où les sommes à engager sont d'autant plus élevées qu'il faut réussir la percée plus tôt. Il ne sert à rien de disposer dès aujourd'hui d'une technologie que l'on aura à mettre en œuvre que demain[7]. Mais il faut noter que la date à laquelle les effets directs de cette contrainte doivent être pris en compte est elle-même endogène.

Une mobilisation des possibilités d'apprentissage nécessite la combinaison de deux moyens de pilotage, une taxe sur les dégagements de gaz dans l'atmosphère et une subvention pour utilisation des technologies de dépollution. Tel n'est pas le cas lorsque la percée est obtenue par la recherche-développement seule. La taxation des émissions suffit alors pour inciter à produire les efforts de recherche optimaux.

La possibilité de combiner apprentissage à partir de technologies initialement existantes et parfois balbutiantes, et efforts de recherche, permet d'élargir

[7] L'argument est similaire à celui mis en avant par Henriet (2012) qui étudie les politiques de recherche optimales visant à mettre au point des techniques d'exploitation à coûts modérés des ressources renouvelables propres.

considérablement la perspective. L'accumulation de carbone dans l'atmosphère et son élimination progressive, et le développement de technologies d'atténuation des émissions apparaissent alors comme deux processus dynamiques en interaction mais avec leurs propres logiques et leurs propres contraintes. On montre ainsi qu'il est possible qu'il faille introduire la capture et la séquestration avant d'atteindre le plafond. On montre aussi que les politiques optimales de recours aux deux types de moyens peuvent s'appuyer initialement soit uniquement sur de l'apprentissage, soit uniquement sur des efforts de recherche.

Dans des scénarios où il convient de recourir simultanément à la recherche et à l'apprentissage pour provoquer une rupture technologique, on montre enfin que l'effort d'apprentissage doit croître à un rythme plus soutenu que celui des efforts de recherche. Ces derniers en effet ne produisent de résultats qu'au moment de la percée. Les efforts conjugués de l'apprentissage ne produisent eux aussi de résultats qu'au même moment pour autant qu'on ne considère que la seule percée. Mais avant que cette percée ait lieu, ils réduisent les rejets dans l'atmosphère et contribuent à réduire le poids de la contrainte.

# 4 CONCLUSION

Les recherches sur l'économie de l'abattement des émissions de gaz à effet de serre sont actuellement en plein essor. La stratégie de valorisation des résultats de nos travaux que nous comptons mettre en œuvre est la suivante.

La première étude, mise en forme comme article de recherche, est actuellement en révision pour la revue *Environmental and Resources Economics,* revue phare en Europe dans le domaine de l'économie de l'environnement. Les deux études suivantes sont beaucoup plus récentes et pas encore soumises à des revues internationales. Un travail de réécriture préalable est nécessaire pour les réduire au format usuel des supports de publication en économie. Ce travail accompli, nous comptons soumettre ces recherches à l'automne à des revues cibles. La seconde étude pourrait être soumise au *Journal of Environmental Economics and Management* ou à *Resource and Energy Economics*. Ces deux revues sont les supports majeurs de publication internationale en économie des ressources naturelles et de l'environnement. L'intérêt suscité aujourd'hui par le sujet laisse espérer des chances raisonnables de succès dans l'un ou l'autre support. La troisième étude soulève des questions d'ordre plus général, portées à l'attention des économistes par Scott Barrett dans l'*Américan Economic Review* en 2006. Nous prévoyons de toucher un lectorat plus large pour cette contribution en visant une revue de théorie économique générale.

Cet effort principal de valorisation sera complété par d'autres initiatives. Nous sommes sollicités pour une participation à un numéro spécial *d'Economie et Prévision* sur le thème de l'économie des ressources naturelles. Une synthèse destinée à un lectorat francophone plus large pourrait être réalisée pour la *Revue Française de l'Energie.* Enfin différentes opportunités de présentation de nos travaux dans des colloques ou séminaires internationaux nous sont offertes. Citons les prochaines journées du CREE (*Canadian Resource and Environmental Economics Study Group Annual Conference 2012*, University of British Columbia, Vancouver, 28-30 septembre, 2012) et la vingtième édition de la conférence annuelle de l'Association Européenne des économistes de l'environnement à Toulouse en juin 2013.

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# ANNEXES

Article 1: Optimal carbon capture and sequestration from heterogeneous consuming sectors.

Article 2: Optimal timing of carbon capture policies under alternative CCS cost function.

Article 3: Triggering the technological revolution in carbon capture and sequestration costs.

# Optimal Carbon Capture and Sequestration From Heterogeneous Energy Consuming Sectors

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# Optimal Carbon Capture and Sequestration From Heterogeneous Energy Consuming Sectors

**Abstract**

We characterize the optimal exploitation paths of two primary energy resources, a non-renewable polluting resource and a carbon-free renewable one. Both resources can supply the energy needs of two sectors. Sector 1 is able to reduce its carbon footprint at a reasonable cost owing to a CCS device. Sector 2 has only access to the air capture technology, but at a significantly higher cost. We assume that the atmospheric carbon stock cannot exceed some given ceiling. We show that there may exist paths along which it is optimal to begin by fully capturing the sector 1's emissions before the ceiling has been reached. Also there may exist optimal paths along which both capture devices have to be activated, in which case the sector 1's emissions are first fully abated and next sector 2 partially abates.

**Keywords:** Air capture; Carbon stabilization cap; CCS; Fossil resource; Heterogeneity.

**JEL classifications:** Q32, Q42, Q54, Q58.

# Contents

# 1 Introduction

Among all the alternative abatement technologies aiming at reducing the anthropogenic carbon dioxide emissions, a particular interest should be given to the carbon capture and sequestration (CCS) according to the IPCC (2005, 2007). Even if the efficiency of this technology is still under assessment[1], current engineering estimates suggest that CCS could be a credible cost-effective approach for eliminating most of the emissions from coal and natural gas power plants (MIT, 2007). Along this line of arguments, Islegen and Reichelstein (2009) point out that CCS has considerable potential to reduce $CO_2$ emissions at a "reasonable" social cost, given the social costs of carbon emissions predicted for a business-as-usual scenario. CCS is also intended to have a major role in limiting the effective carbon tax, or the market price for $CO_2$ emission permits under a cap-and-trade system. The crucial point is then to estimate how far would the $CO_2$ price have to rise before the operator of power plants would find it advantageous to install CCS technology rather then buy emission permits or pay the carbon tax. The International Energy Agency (2006) estimates such a break-even price in the range of \$30-90/t$CO_2$ (depending on technology) but, assuming reasonable technology advances, projected CCS cost by 2030 is around \$25/t$CO_2$.

However, geologic CCS presents the disadvantage to apply to the sole large point sources of pollution such as power plants or huge manufacturings. This technology is prohibitively costly to filter for instance the $CO_2$ emissions from transportation as far as the energy input is gasoline or kerosene, small residence heating or scattered agricultural activities. Hence, the ultimate device to abate carbon dioxide fluxes from any concentrated as well as diffuse sources would consist in capturing them directly from the atmosphere.

According to Keith et al. (2006), atmospheric carbon capture – or air capture – differs from conventional mitigation in three key aspects. First, it removes emissions from any part of the economy with equal ease of difficulty, so its cost provides an absolute cap on the mitigation cost. Second, it permits reduction in concentrations faster than the natural carbon cycle. Third, because it is weakly coupled with existing energy infrastructure, air capture may offer stronger economies of scale and smaller adjustment costs than the more

[1] CCS technology consists in filtering $CO_2$ fluxes at the source of emission, that is, in fossil energy-fueled power plants, by use of scrubbers installed near the top of chimney stacks. The carbon would be sequestered in reservoirs, such as depleted oil and gas fields or deep saline aquifers.

conventional mitigation technologies. As underlined by Keith (2009), though this abatement technology costs more than CCS, it allows one to treat small and mobile emission sources, advantage that may compensate for the intrinsic difficulty of capturing carbon from the air. Finally, deliberately expressing a double meaning, McKay (2009) claims about this alternative that "*capturing carbon dioxide from thin air is the last thing we should talk about*" (p.240). On the one hand, the energy requirements for atmospheric carbon capture are so enormous that, according to McKay, it seems actually almost absurd to talk about it. But on the other hand, "*we should talk about it because capturing carbon from thin air may turn out to be our last line of defense if humanity fails to take the cheaper and more sensible options that may still be available today*" (p.240).

Technically speaking, sucking carbon from thin air can be achieved in different ways.[2] The probably most credible one is to use a chemical process. This involves a technology that brings air into contact with a chemical "sorbent" (an alkaline liquid). The sorbent absorbs $CO_2$ in the air, and the chemical process then separates out the $CO_2$ and recycles the sorbent. The captured $CO_2$ is stored in geologic deposits, just like the CCS from power plant. However, chemical air capture is expensive. Estimates of marginal cost range from \$100-200/t$CO_2$, which is larger than the cost of alternatives for reducing emissions such as CCS. They are also larger than current estimates of the social cost of carbon, which range from about \$7-85/t$CO_2$. But, as concluded by Barrett (2009), bearing the cost of chemical air capture can become profitable in the future under constraining cap-and-trade scenarios. Furthermore, we may hope that the cost will decrease, thanks to R&D and learning by doing.

In the present study, we address the question of the heterogeneity of energy users regarding the way their carbon footprint can be reduced. We then consider two abatement technologies and two sectors. The first technology is a conventional emission abatement device (CCS) which is available at a marginal cost assumed to be socially acceptable.

[2]The most obvious approach consists in exploiting the process of photosynthesis by increasing the forestlands or changing the agricultural processes, but this is not the type of device we consider in the present paper. A close idea can be transposed to the oceans. To make them able to capture carbon faster then normal, phytoplankton blooms can be stimulated by fertilizing some oceanic iron-limited regions. A third way is to enhance weathering of rocks, that is to pulverize rocks that are capable of absorbing $CO_2$, and leave them in the open air. This idea can be pitched as the acceleration of a natural process. Unfortunately, as claimed by Barrett (2009), the effects of all these devices are difficult to verify, their potential is limited in any event, and there are concerns about some unknown ecological consequences.

However, this abatement technology cannot apply to carbon emissions from any type of activity, but only from large point sources of emissions. The second technology directly captures carbon in the atmosphere. Its marginal cost is much higher than the emission capture technology, but it allows to reduce carbon from any sources since the capture process and the generation of emissions are now disconnected. The first sector, in which pollution sources are spatially concentrated, can abate its carbon emissions, but not the second one since energy users are too small and too scattered. The ultimate way for abating pollution is to directly capture carbon in the atmosphere. But since the atmosphere is a public good, this kind of pollution reduction will also benefit to sector 1. Whatever the capture process, we assume that carbon is stockpiled into reservoirs whose size is very large. Then, as in Chakravorty et al. (2006), this suggests a generic abatement scheme of unlimited capacity. Finally, energy in each sector can be supplied either by a carbon-based fossil fuel, contributing to climate change (oil, coal, gas), or by a carbon-free renewable and non biological resource such, as solar energy.

Using a standard Hotelling model for the non-renewable resource and assuming that the atmospheric carbon stock should not exceed some critical threshold – as in Chakravorty et al. (2006) – we characterize the optimal time path of sectoral energy prices, sectoral energy consumptions, emission and atmospheric abatements. The key results of the paper are: i) Irrespective of the availability of the air capture technology, it may happen that it is optimal for the first sector to abate its carbon emissions before the atmospheric carbon concentration cap is attained.[3] ii) Since this type of carbon capture is unable to filter the emissions from the second sector, it is also optimal for the first sector to abate the totality of its own emissions, at least at the beginning. These two first results are at variance with Chakravorty et al. (2006), Lafforgue et al. (2008-a) and (2008-b) who consider a single sector using energy and a single abatement technology. iii) The atmospheric carbon capture is only used when the atmospheric carbon stock reaches the ceiling, maintaining the stock at its critical level. Hence the flow of carbon captured in the atmosphere is lower than the emission flow of the second sector and the whole carbon emissions coming from

[3]This result is in accordance with Coulomb and Henriet (2010) who show that, in a model with a single abatement technology, when technical constraints make it impossible to capture emissions from all energy consumers, CCS should be used before the ceiling is reached if non capturable emissions are large enough.

the two sectors are only partially abated.

The paper is organized as follows. Section 2 presents the model. In section 3, we lay down the social planner program and we derive the optimality conditions. In section 4, we examine the restricted problem in which only the emission carbon capture device is available. In section 5, we examine how the model reacts when the atmospheric carbon capture technology is introduced. We also investigate the time profile of the optimal carbon tax as well as, for each sector, the total burden induced by the mitigation of their emissions. Finally, we briefly conclude in section 6.

## 2 Model and notations

Let us consider a stationary economy with two sectors, indexed by $i = 1, 2$, in which the instantaneous gross surplus derived from energy consumption are the same.[4] For an identical energy consumption in the two sectors, $q_1 = q_2 = q$, the sectoral gross surplus $u_1(q)$ and $u_2(q)$ are such that: $u_1(q) = u_2(q) = u(q)$. We assume that this common function $u$ satisfies the following standard assumptions. $u : \mathbb{R}_+ \to \mathbb{R}_+$ is a function of class $\mathcal{C}^2$, strictly increasing, strictly concave and verifying the Inada conditions: $\lim_{q \downarrow 0} u'(q) = +\infty$ and $\lim_{q \uparrow +\infty} u'(q) = 0$. We denote by $p(q)$ the sectoral marginal gross surplus function and by $q^d(p) = p^{-1}(q)$, the sectoral direct demand function.

In each sector, energy can be supplied by two primary natural resources: a dirty non-renewable resource (let say oil for instance) and a carbon-free renewable resource (let say solar energy). Let us denote by $X^0$ the initial oil endowment of the economy, by $X(t)$ the remaining part of this initial endowment at time $t$, and by $x_i(t)$, $i = 1, 2$, the instantaneous consumption flow of oil in sector $i$ at time $t$, so that:

$$\dot{X}(t) = -[x_1(t) + x_2(t)], \quad \text{with } X(0) \equiv X^0 \text{ and } X(t) \geq 0 \tag{2.1}$$

$$x_i(t) \geq 0, \ i = 1, 2. \tag{2.2}$$

The delivery cost of oil is the same for both sectors. We denote by $c_x$ the corresponding

[4]Since the focus of the paper is on the effect of the heterogeneity of the energy consumers regarding the type of abatement technologies they can use, we consider the simple case of two sectors with the same gross surplus function and the same cost structure, excepted the abatement cost. Introducing different demand functions and/or different delivery costs for these sectors would imply a more complex analysis without altering the key message of the paper.

average cost, assumed to be constant and hence equal to the marginal cost. The delivery cost includes the extraction cost of the resource, the cost of industrial processing (refining of the crude oil) and the transportation cost, so that the resource is ready for use by the consumer in the concerned sector. To keep matter as simple as possible, we assume that no oil is lost during the delivery process. Equivalently, the oil stock $X(t)$ may be understood as measured in ready for use units.

Let $Z(t)$ be the stock of carbon within the atmosphere at time $t$, and $Z^0$ be the initial stock, $Z^0 \equiv Z(0)$. We assume that a carbon cap policy is prescribed to prevent catastrophic damages which would be infinitely costly. This policy consists in forcing the atmospheric stock to stay under some critical level $\bar{Z}$, with $\bar{Z} > Z^0$.

The atmospheric carbon stock is fed by carbon emission flows resulting from the use of oil. Let $\zeta$ be the quantity of carbon which would be potentially released per unit of oil consumption whatever the sector in which the oil is used. Thus, the gross pollutant flow amounts to $\zeta[x_1(t) + x_2(t)]$. However, this gross emission flow can be abated before being released into the atmosphere. We assume that emissions from sector 1 can be abated, but not emissions from sector 2 (or at a prohibitive cost). Emission abatement by carbon capture and sequestration (CCS) can be achieved when burning oil is spatially concentrated, as it is the case for instance in the electricity or cement industries, which are good examples of sector 1's activities. At the other extreme of the spectrum, i.e. in sector 2, there exists some activities with prohibitively costing emission captures since users are too small or too scattered. Transportation by cars, trucks and diesel train are good examples of sector 2's industry.[5]

Let $s_e(t)$ be this part of carbon emissions from sector 1 which is captured and sequestered at some average cost $c_e$, assumed to be constant. Then the net pollution flow issued from sector 1 amounts to:

$$\zeta x_1(t) - s_e(t) \geq 0, \quad s_e(t) \geq 0. \tag{2.3}$$

In sector 2, the net pollution flow amounts to $\zeta x_2(t)$.

[5]Note that electric traction trains could be good examples of sector 1 users, as well as electric cars (cf. Chakravorty et al., 2010).

Carbon emission capture is not the unique way to reduce the atmospheric carbon concentration. The other process consists in capturing the carbon present in the atmosphere itself. We denote by $s_a(t)$ the instantaneous carbon flow which is abated owing to this second device, and by $c_a$ the corresponding average cost, also assumed to be constant. Although atmospheric carbon capture seems technically feasible, it is proved to be more costly than emission capture: $c_a > c_e$. The only constraint on this capture flow is:

$$s_a(t) \geq 0. \tag{2.4}$$

Whatever the capture process, from emissions or from the atmosphere, we assume that carbon is stockpiled into reservoirs whose capacities are unlimited.[6]

Last, there is also some natural self regeneration effect of the atmospheric carbon stock. We assume that the natural proportional rate of decay, denoted by $\alpha > 0$, is constant. Taking into account all the components of the dynamics of $Z(t)$ results into:

$$\dot{Z}(t) = \zeta[x_1(t) + x_2(t)] - [s_a(t) + s_e(t)] - \alpha Z(t), \quad Z(0) \equiv Z^0 < \bar{Z} \tag{2.5}$$

$$\bar{Z} - Z(t) \geq 0. \tag{2.6}$$

When the atmospheric carbon stock reaches its critical level, i.e. when $Z(t) = \bar{Z}$, and absent any active capture policy, i.e. $s_a(t) = s_e(t) = 0$, then the total oil consumption $x(t) \equiv x_1(t)+x_2(t)$ is constrained to be at most equal to $\bar{x}$, where $\bar{x}$ is solution of $\zeta x - \alpha\bar{Z} = 0$, that is $\bar{x} = \alpha\bar{Z}/\zeta$. Then, since the two sectors have the same weight, each one consumes the quantity $\bar{x}/2$ of oil at the ceiling when neither CCS nor air capture are activated.

We assume that it may be optimal to abate the pollution for delaying the date of arrival at the critical threshold and for relaxing the constraint on the oil consumption flow, that is: $c_x + c_e < c_x + c_a < u'(\bar{x})$.

The alternative energy source is supplied by the carbon-free renewable resource, the solar energy. We denote by $y_i(t)$ the solar energy consumption in sector $i$, $i = 1, 2$, and by $c_y$ the average delivery cost of this alternative energy. Because $c_x$ and $c_y$ both include all

[6] In order to focus on the abatement options for each sector and their respective costs, we dispense from considering reservoirs of limited capacity. The question of the size of carbon sinks and of the time profile of their filling up is addressed by Lafforgue et al. (2008-a) and (2008-b).

the costs necessary to deliver a ready for use energy unit to the potential users, then both resources may be seen as perfect substitutes for the consumers, so that we may define the aggregate energy consumption of sector $i$ as $q_i = x_i + y_i$, $i = 1, 2$, as far as the costs $c_x$ and $c_y$ are incurred.

The average cost $c_y$ is assumed to be constant, the same for both sectors, and higher than $u'(\bar{x}/2)$. This last condition implies that the optimal energy consumption paths can be split into two periods: a first one during which only oil is consumed and a second one during which only solar energy is used.[7] We also have to assume that the natural flow of available solar energy, denoted by $y^n$, is large enough to supply the energy needs in both sectors during the second period described above.[8] Let $\tilde{y}$ be the sectoral energy consumption that it would be optimal to consume at the marginal cost $c_y$, that is $\tilde{y} = q^d(c_y)$ for which $u'(\tilde{y}) = c_y$. Then we assume that $y^n > 2\tilde{y}$. Under this assumption, no rent has ever to be imputed for using the solar energy. Thus the only constraint on $y_i(t)$ having to be taken into account along any optimal path is a non-negativity constraint:

$$y_i(t) \geq 0, \quad i = 1, 2. \tag{2.7}$$

Finally, the instantaneous social rate of discount, denoted by $\rho$, $\rho > 0$, is assumed to be constant over time.

# 3 Social planner problem and optimality conditions

The problem of the social planner consists in maximizing the sum of the discounted net current surplus. Let $(P)$ be this program:

$$\begin{aligned}(P) \qquad \max_{s_a, s_e, x_i, y_i, i=1,2} \int_0^\infty & \{u\left[x_1(t) + y_1(t)\right] + u\left[x_2(t) + y_2(t)\right] - c_x\left[x_1(t) + x_2(t)\right] \\ & -c_y\left[y_1(t) + y_2(t)\right] - c_a s_a(t) - c_e s_e(t)\} e^{-\rho t} dt\end{aligned}$$

subject to (2.1)-(2.7).

[7]Since both $c_x$ and $c_y$ are set constant, oil and solar cannot be used simultaneously. Using a stock-dependent marginal extraction cost, but a constant marginal cost of the backstop, together with a damage function increasing with the atmospheric carbon stock, Hoel and Kverndokk (1996) and Tahvonen (1997) have shown that there may be a period of simultaneous use of the nonrenewable and the renewable resource. Furthermore, as underlined by Tahvonen (1997), the conjunction of these assumptions gives rise to a multiplicity of possible scenarios.

[8]The case of a rare renewable substitute is analyzed in Lafforgue et al. (2008-b).

Let us denote by $\lambda_X$ the costate variable of the state variable $X$, by $\lambda_Z$ minus the costate variable of the state variable $Z$, by $\gamma$'s the Lagrange multipliers associated with the non-negativity constraints on the command variables, and by $\nu$ the Lagrange multiplier associated with the ceiling constraint on $Z$. As usually done in this kind of problem, we do not take explicitly into account the non-negativity constraint on $X$. Thus, droping out the time index for notational convenience, we may write the current value Lagrangian $\mathcal{L}$ of problem $(P)$ as follows:

$$\begin{aligned}
\mathcal{L} &= u(x_1+y_1)+u(x_2+y_2)-c_x(x_1+x_2)-c_y(y_1+y_2)-c_a s_a - c_e s_e \\
&\quad -\lambda_X(x_1+x_2)-\lambda_Z[\zeta(x_1+x_2)-(s_a+s_e)-\alpha Z]+\nu(\bar{Z}-Z) \\
&\quad +\sum_i \gamma_{x_i}x_i+\sum_i \gamma_{y_i}y_i+\gamma_{s_a}s_a+\gamma_{s_e}s_e+\bar{\gamma}_{s_e}(\zeta x_1 - s_e)
\end{aligned}$$

The static and dynamic first-order conditions are:

$$u'[x_1(t)+y_1(t)] = c_x+\lambda_X(t)+\zeta[\lambda_Z(t)-\bar{\gamma}_{s_e}(t)]-\gamma_{x_1}(t) \tag{3.8}$$

$$u'[x_2(t)+y_2(t)] = c_x+\lambda_X(t)+\zeta\lambda_Z(t)-\gamma_{x_2}(t) \tag{3.9}$$

$$u'[x_i(t)+y_i(t)] = c_y-\gamma_{y_i}(t), \quad i=1,2 \tag{3.10}$$

$$c_a = \lambda_Z(t)+\gamma_{s_a}(t) \tag{3.11}$$

$$c_e = \lambda_Z(t)-\bar{\gamma}_{s_e}(t)+\gamma_{s_e}(t) \tag{3.12}$$

$$\dot{\lambda}_X(t) = \rho\lambda_X(t) \tag{3.13}$$

$$\dot{\lambda}_Z(t) = (\rho+\alpha)\lambda_Z(t)-\nu(t) \tag{3.14}$$

together with the associated complementary slackness conditions. Last, the transversality conditions take the following forms:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \tag{3.15}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{3.16}$$

## Remarks:

1. As expected with a constant marginal delivery cost, the shadow marginal value of the stock of oil, or mining rent, $\lambda_X(t)$, must grow at the social rate of discount $\rho$. From (3.13), we get: $\lambda_X(t)=\lambda_{X_0}e^{\rho t}$, with $\lambda_{X_0}\equiv\lambda_X(0)$. Thus the transversality condition

(3.15) reduces to $\lambda_{X_0} \lim_{t\uparrow\infty} X(t) = 0$. If oil is to have some value, $\lambda_{X_0} > 0$, then it must be exhausted along the optimal path.

2. Concerning the shadow marginal cost of the atmospheric carbon stock, $\lambda_Z(t)$, note that before the date $\underline{t}_Z$ at which the ceiling constraint is beginning to be active, we must have $\nu(t) = 0$ since $\bar{Z} - Z(t) > 0$. Then (3.14) reduces to $\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z$ so that: $t < \underline{t}_Z \;\Rightarrow\; \lambda_Z(t) = \lambda_{Z_0} e^{(\rho+\alpha)t}$, with $\lambda_{Z_0} \equiv \lambda_Z(0)$. Once the ceiling constraint is no more active and forever, $\lambda_Z(t) = 0$. Thus, denoting by $\bar{t}_Z$ the latest date at which $Z(t) = \bar{Z}$, we get: $t > \bar{t}_Z \;\Rightarrow\; \lambda_Z(t) = 0$.[9]

3. In order to simplify the notations in the next sections, it is useful to define the following prices or full marginal costs and the corresponding sectoral consumption levels for which the F.O.C's (3.8) and (3.9) relative to $x_1(t)$ and to $x_2(t)$, respectively, are satisfied:[10]

   - Price or full marginal cost of oil and sectoral oil consumption before the ceiling and absent any abatement, whatever the sector under consideration: $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta \lambda_{Z_0} e^{(\rho+\alpha)t}$ and $\tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) \equiv q^d\left(p^1(t, \lambda_{X_0}, \lambda_{Z_0})\right)$.

   - Price or full marginal cost of oil for consumption in sector 1 given that emissions from this sector are fully or partially abated, i.e. $s_e(t) > 0$, and corresponding oil consumption of sector 1: $p_e^2(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta c_e$ and $\tilde{q}_e^2(t, \lambda_{X_0}) \equiv q^d\left(p_e^2(t, \lambda_{X_0})\right)$.

   - Price or full marginal cost of oil for consumption in sector 2 given that some part of the atmospheric carbon stock is captured, $s_a(t) > 0$, and corresponding consumption in this sector: $p_a^2(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta c_a$ and $\tilde{q}_a^2(t, \lambda_{X_0}) \equiv q^d\left(p_a^2(t, \lambda_{X_0})\right)$.

   - Price or full marginal cost of oil once the ceiling constraint $\bar{Z} - Z(t) \geq 0$ is no more active and forever, and corresponding sectoral consumptions, whatever the sector: $p^3(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t}$ and $\tilde{q}^3(t, \lambda_{X_0}) \equiv q^d\left(p^3(t, \lambda_{X_0})\right)$. This last case corresponds to a pure Hotelling regime.

[9] This characteristics is standard under the assumption of a linear natural regeneration function of the atmospheric carbon stock. For non linear decay functions, see Toman and Withagen (2000) for instance.

[10] The upper indexes $n = 1, 2, 3$ correspond to the order in which the price $p^n$ and the quantity $\tilde{q}^n$ are appearing along the optimal path. If both $p^n(t, ...)$ and $p^{n+m}(t', ...)$ are appearing along the same path, then it implies that $t < t'$.

**Solving strategy of the social planner:**

In order to solve her problem $(P)$, the social planner can proceed as follows. First, she checks whether the most costly device to capture the carbon has ever to be used. The test consists in solving her problem assuming that the atmospheric carbon capture device is not available. This is inducing some path of atmospheric carbon shadow cost $\lambda_Z(t)$. Next, according to the outcome of the first step:

- either this shadow cost is permanently lower than the marginal cost of atmospheric carbon capture, that is $\lambda_Z(t) < c_a$ for any $t \geq 0$, and then the atmospheric carbon capture device has never to be used because too costly;

- or there exists some time interval during which $\lambda_Z(t)$ is higher than $c_a$ so that, in this case, the atmospheric carbon capture device must be activated since the loss in the marginal net surplus induced by not using it is higher than its marginal cost of use.

This test is performed in Section 4. Section 5 deals with the case in which it is optimal to activate the air capture device.

# 4 Optimal policy without atmospheric carbon capture device

This kind of policies have been investigated and characterized in Chakravorty et al. (2006), and in Lafforgue et al. (2008-a) and (2008-b), but for economies in which any potential emissions can be captured and sequestered irrespective of the oil consumption sector. Thus, in their models, there is a single consumption sector, similar to the sector 1 of the present model. Two important conclusions of these studies are that: i) it is never optimal to abate the potential flow of emissions before attaining the critical level $\bar{Z}$ of atmospheric carbon concentration; ii) along the phase at the ceiling during which it is optimal to abate, only some part of the potential emission flow must be abated. Because abating is never optimal excepted during this phase, then it is never optimal to fully abate the potential flow of emissions along the optimal path.

As we shall show, it may happen in the present context that: i) abating the potential emissions of the sector 1 has to begin before the ceiling level $\bar{Z}$ is attained; ii) when it is optimal to begin to capture the sector 1 potential emissions, before the ceiling is attained, then it is optimal to capture its whole potential emission flow.

## 4.1 Restricted social planner problem

Assuming that the atmospheric carbon capture technology is not available, the social planner problem reduces to the following restricted problem $(R.P)$:

$$(R.P) \qquad \max_{s_e, x_i, y_i, i=1,2} \int_0^{\infty} \{u\left[x_1(t)+y_1(t)\right]+u\left[x_2(t)+y_2(t)\right]-c_x\left[x_1(t)+x_2(t)\right] \\ -c_y\left[y_1(t)+y_2(t)\right]-c_e s_e(t)\} e^{-\rho t} dt$$

subject to (2.1), (2.2), (2.3), (2.6), (2.7) and:

$$\dot{Z}(t)=\zeta[x_1(t)+x_2(t)]-s_e(t)-\alpha Z(t), \quad Z(0)=Z^0<\bar{Z} \tag{4.17}$$

The new F.O.C's relative to the command variables, except $s_a$, and to the state variables are the same then the ones of the unrestricted problem $(P)$, namely (3.8)-(3.14). Also the associated complementary slackness condition and the transversality conditions (3.15) and (3.16) must hold. We can conclude that remarks 1 and 2 of the previous section 3 also hold in the present restricted context.

The opportunity for sector 1 to fully or partially abate its emissions strongly depends upon the level of $c_e$. Hence, we have to distinguish the cases of a full abatement phase or a partial abatement phase, before or after being at the ceiling. The next subsections describe these different possibilities.

## 4.2 Optimal paths along which it is optimal to capture and sequester before being at the ceiling

Let us assume that the initial oil endowment is large enough to justify some period at the ceiling during which $Z(t)=\bar{Z}$, and that there exists some period during which the emissions of sector 1 are abated, $s_e(t)>0$. Figure 1 below illustrates the optimal price path which is obtained in this case.

The optimal price path is a seven phases path. Denoting by $p_i(t)$, for $i=1,2$, the price – or full marginal cost – of oil for sector $i$, these phases are the following:

**- Phase 1, before the ceiling and without abatement:** $[0, \underline{t}_e)$

During this phase, the oil price is the same for each sector and it is given by $p_1(t)=p_2(t)=p^1(t, \lambda_{X_0}, \lambda_{Z_0})$. The existence of such a phase requires that $\lambda_{Z_0}<c_e$, so $p^1(t, \lambda_{X_0}, \lambda_{Z_0})<$

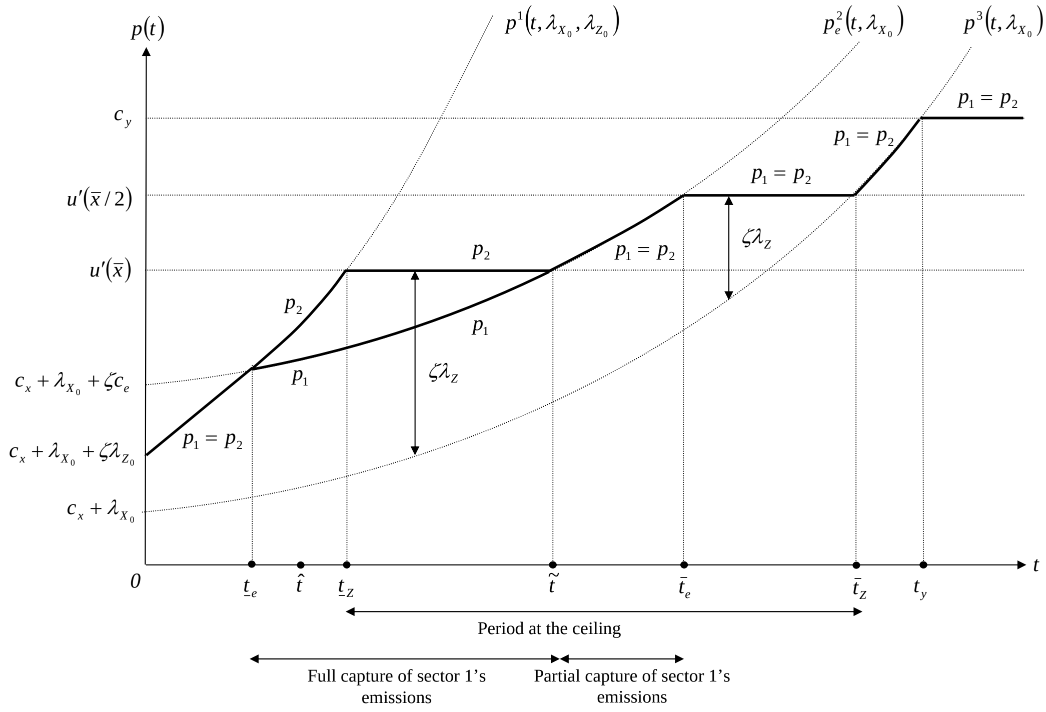


Figure 1: Optimal path along which it is optimal to abate before the ceiling

$p^2_e(t, \lambda_{X_0})$, that is capturing sector 1's emissions would be too costly. $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) - p^2_e(t, \lambda_{X_0}) = \zeta \left[\lambda_{Z_0} e^{(\rho+\alpha)t} - c_e\right] < 0$, is increasing so that supporting the marginal shadow cost of the atmospheric carbon stock, $\lambda_Z(t) = \lambda_{Z_0} e^{(\rho+\alpha)t}$, is less costly than abating, that is supporting the marginal cost of abating the sector 1's emissions, $c_e$.

The oil consumption of each sector is given by $x_1(t) = x_2(t) = \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})$.

The common oil price $p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ is increasing at an instantaneous rate which is higher than the rate of growth of $p^2_e(t, \lambda_{X_0})$. At the end of the phase, denoted by $\underline{t}_e$, both prices are equated $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p^2_e(t, \lambda_{X_0})$.

Note that, since $p_1(t) = p_2(t) < u'(\bar{x})$ and $Z^0 < \bar{Z}$, then during this phase both $x_1(t)$ and $x_2(t)$ are higher than $\bar{x}$ so that $Z(t)$ is increasing. However, the existence of this phase requires that, at its end, $Z(t)$ is lower than the critical level $\bar{Z}$: $Z(\underline{t}_e) < \bar{Z}$.

**- Phase 2, before the ceiling with full abatement of sector 1's emissions:** $[\underline{t}_e, \underline{t}_Z)$

From $\underline{t}_e$ onwards, we have $p^2_e(t, \lambda_{X_0}) < p^1(t, \lambda_{X_0}, \lambda_{Z_0})$. Thus it is now strictly less costly for sector 1 to abate than not to abate, hence $p_1(t) = p^2_e(t, \lambda_{X_0})$, implying that $x_1(t) =$

$\tilde{q}_e^2(t, \lambda_{X_0})$.[11] Moreover, since the inequality is strict then the potential sector 1's emissions are fully abated: $s_e(t) = \zeta x_1(t)$.

Sector 2 is not able to abate its emissions and it must support the carbon shadow cost $\zeta \lambda_{Z_0} e^{(\rho+\alpha)t}$ per unit of burned oil, so that $p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ and $x_2(t) = \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})$.

Note that, during this phase, since $Z(\underline{t}_e) < \bar{Z}$ and $p_2(t) < u'(\bar{x})$, then $x_2(t) > \bar{x}$ and the atmospheric carbon stock increases. Finally, since $p_2(t) > p_1(t)$, the first of these two prices reaching $u'(\bar{x})$ is $p_2(t)$. However, in order that sector 2's consumption begins to be blockaded at $t = \underline{t}_Z$, we must have simultaneously $p_2(t) = u'(\bar{x})$ and $Z(t) = \bar{Z}$ at the end of the phase.

**- Phase 3, at the ceiling with sector 2's oil consumption blockaded and sector 1's emissions fully abated:** $[\underline{t}_Z, \tilde{t})$

During this phase, the oil price in sector 2 is given by $p_2(t) = u'(\bar{x})$ and the oil consumption of this sector is set to the maximum consumption level allowed by the ceiling constraint, i.e. $x_2(t) = \bar{x}$. Note that this implies that $\lambda_Z(t) = [u'(\bar{x}) - p^3(t, \lambda_{X_0})]/\zeta$ is decreasing over time during the phase.[12]

Since $p_e^2(\underline{t}_Z, \lambda_{X_0}) < u'(\bar{x})$, then $c_e < \lambda_Z(t)$ at the beginning of the phase. Then, once again, abating emissions is proved to be less costly for sector 1 than supporting the shadow cost of the atmospheric carbon stock. Consequently, the sector 1's emissions are fully captured: $s_e(t) = \zeta x_1(t)$. Since $p_1(t) = p_e^2(\underline{t}_Z, \lambda_{X_0})$, we still have $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.

Given that sector 2's emissions are $\zeta x_2(t) = \zeta \bar{x}$, full abatement in sector 1 implies that, during this phase at the ceiling, the atmospheric carbon stock stays at its critical level: $\dot{Z}(t) = 0$ and $Z(t) = \bar{Z}$. Finally, $p_1(t) = p_e^2(t, \lambda_{X_0})$ is increasing during the phase. At the end of the phase, $p_e^2(t, \lambda_{X_0}) = u'(\bar{x})$ or, equivalently, $\lambda_Z(t) = c_e$.

[11] Note that during such a phase, because $s_e(t) > 0$ then $\gamma_{s_e}(t) = 0$, so that from (3.12) we obtain: $\lambda_Z(t) = c_e + \bar{\gamma}_{s_e}(t)$. Substituting for $\lambda_Z(t)$ in (3.8) and taking into account that $x_1(t) > 0$, hence $\gamma_{x_1}(t) = 0$, and $y_1(t) = 0$, we get: $u'(x_1(t)) = c_x + \lambda_{X_0} e^{\rho t} + \zeta c_e$, from which we conclude that $p_1(t) = p_e^2(t, \lambda_{X_0})$ and $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.

[12] Since the ceiling constraint is active, then $\nu(t)$ is strictly positive and sufficiently high so that $\dot{\lambda}_Z(t) = (\rho + \alpha)\lambda_Z(t) - \nu(t) < 0$.

**- Phase 4, at the ceiling with partial abatement of sector 1's emissions:** $[\tilde{t}, \bar{t}_e)$

From time $\tilde{t}$ onwards, $p_e^2(t, \lambda_{X_0})$ becomes higher than $u'(\bar{x})$. Thus, the only way to satisfy simultaneously the F.O.C's (3.8) and (3.9) on the $x_i$'s is to set $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0})$, which implies $x_1(t) = x_2(t) = \tilde{q}_e^2(t, \lambda_{X_0})$ together with a partial abatement of sector 1's emissions. As far as $p_e^2(t, \lambda_{X_0})$ is staying under $u'(\bar{x}/2)$, then the potential emissions amount to $2\zeta\tilde{q}_e^2(t, \lambda_{X_0}) > \zeta\bar{x} = \alpha\bar{Z}$. As far as $p_e^2(t, \lambda_{X_0})$ is now higher than $u'(\bar{x})$, then the potential emissions $2\zeta\tilde{q}_e^2(t, \lambda_{X_0})$ stays at a lower level than $2\zeta\bar{x}$, so that:

$$\bar{x} < 2\tilde{q}_e^2(t, \lambda_{X_0}) < 2\bar{x}. \tag{4.18}$$

In order to satisfy the atmospheric carbon constraint $Z(t) = \bar{Z}$, it is sufficient to abate this part $s_e(t)$ of the sector 1's emissions for which $\dot{Z}(t) = 0$. Thus we may have:

$$2\zeta\tilde{q}_e^2(t, \lambda_{X_0}) - s_e(t) = \zeta\bar{x}. \tag{4.19}$$

Conditions (4.18) and (4.19) imply that:

$$s_e(t) = \zeta\left[2\tilde{q}_e^2(t, \lambda_{X_0}) - \bar{x}\right] < \zeta\tilde{q}_e^2(t, \lambda_{X_0}) = \zeta x_1(t). \tag{4.20}$$

Hence, during this phase, emissions from sector 1 are only partially abated and, since $\tilde{q}_e^2(t, \lambda_{X_0})$ is decreasing through time then the instantaneous rate of capture $s_e(t)$ is also decreasing. This solution may be optimal if and only if abating and supporting the shadow marginal cost of the atmospheric carbon stock are resulting into the same full marginal cost, that is if and only if $\lambda_Z(t)$ is constant and equal to $c_e$. Since sector 2 cannot abate its emissions, it is supporting the marginal shadow cost of atmospheric carbon and the condition $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0}) = c_x + \lambda_{X_0}e^{\rho t} + \zeta\lambda_Z(t)$ guarantees that $\lambda_Z(t) = c_e$ is satisfied.[13]

Since $p_e^2(t, \lambda_{X_0})$ is increasing over time, there exists some date $\bar{t}_e$ at which $p_e^2(t, \lambda_{X_0}) = u'(\bar{x}/2)$. At this date, $x_1(t) = x_2(t) = \bar{x}/2$ and sector 1 ceases to capture its emissions, $s_e(t) = 0$. From $\bar{t}_e$ onwards, we have $p_e^2(t, \lambda_{X_0}) > u'(\bar{x}/2)$ so that the cost of capture of sector 1's emissions becomes prohibitive.

[13] Again, because the ceiling constraint is effective then $\nu(t) > 0$ and, in order that $\dot{\lambda}_Z(t) = 0$, we have: $\nu(t) = (\rho + \alpha)\lambda_Z(t) = (\rho + \alpha)c_e$.

**- Phase 5, at the ceiling and without abatement of sector 1's emissions:** $[\bar{t}_e, \bar{t}_Z)$

Since abating the sector 1's emissions is now too costly, there is no more abatement and, in order to not overshoot the critical atmospheric carbon level, we must have $p_1(t) = p_2(t) = u'(\bar{x}/2)$ and $x_1(t) = x_2(t) = \bar{x}/2$, so that $\dot{Z}(t) = 0$.

During such a phase, $\lambda_Z(t) = [u'(\bar{x}) - p^3(t, \lambda_{X_0})]/\zeta$ is decreasing. The phase is ending at time $t = \bar{t}_Z$ when $\lambda_Z(t) = 0$, which implies that $p^3(t, \lambda_{X_0}) > u'(\bar{x}/2)$ for $t > \bar{t}_Z$.

**- Phase 6, pure Hotelling phase:** $[\bar{t}_Z, t_y)$

This phase is the last one during which energy needs are supplied by oil. This is a pure Hotelling phase. The energy price is the same for the two sectors: $p_1(t) = p_2(t) = p^3(t, \lambda_{X_0}) > u'(\bar{x}/2)$, also generating an identical oil consumption in the two sectors: $x_1(t) = x_2(t) < \bar{x}/2 \Rightarrow x(t) < \bar{x}$.

Since $x(t) < \bar{x}$ and $Z(t) = \bar{Z}$ at the beginning of the phase, then $Z(t) < \bar{Z}$ for $t > \bar{t}_Z$ justifying the fact that now $\lambda_Z(t) = 0$ from $\bar{t}_Z$ onwards.[14] Then $\lambda_Z(t)Z(t) = 0$ and the transversality condition (3.16) is satisfied.

During the phase, the price is ever increasing and there must exist some time $t = t_y$ at which $p^3(t, \lambda_{X_0}) = c_y$. At this time, this level of oil price makes the renewable resource competitive. To be optimal, the switch from the pure Hotelling regime to a pure renewable regime requires that, at time $t = t_y$, $X(t) = 0$ so that from $t_y$ onwards $\lambda_X(t)X(t) = 0$ warranting that the transversality condition (3.15) relative to $X$ is satisfied.

**- Phase 7, carbon-free renewable energy permanent regime:** $[t_y, +\infty)$

From $t_y$ onwards, the economy follows a pure renewable energy regime which is free of carbon emissions: $p_1(t) = p_2(t) = c_y$, $x_1(t) = x_2(t) = 0$ and $y_1(t) = y_2(t) = \tilde{y}$. Since $x_i(t) = 0$, $i = 1, 2$, then $\dot{Z}(t) = -\alpha Z(t)$ so that $Z(t)$ is permanently decreasing down to 0 at infinity: $Z(t) = Z(t_y)e^{-\alpha(t-t_y)}$.

[14]However, note that $Z(t)$ is not necessarily monotonically decreasing during this phase. What is sure is that there exists some critical time interval $(\bar{t}_Z, \bar{t}_Z + \epsilon)$, with $\epsilon$ positive and small enough, during which $\dot{Z}(t) < 0$. For $t > \bar{t}_Z + \epsilon$, it may happen that $\dot{Z}(t) > 0$. But, because $x(t) < \bar{x}$, even if $\dot{Z}(t)$ were temporally increasing, it would not be able to go back to $\bar{Z}$.

**Determination of the characteristics of the optimal path:**

The optimal path described above is parametrized by eight variables whose values have to be determined: $\lambda_{X_0}$, $\lambda_{Z_0}$, $\underline{t}_e$, $\underline{t}_Z$, $\tilde{t}$, $\bar{t}_e$, $\bar{t}_Z$ and $t_y$. They are given as the solutions of the following eight equations system.

- Balance equation of non-renewable resource consumption and supply:

$$\begin{aligned}
2\int_0^{\underline{t}_e} \tilde{q}^1(t,\lambda_{X_0},\lambda_{Z_0})dt + \int_{\underline{t}_e}^{\underline{t}_Z} \left[\tilde{q}_1(t,\lambda_{X_0},\lambda_{Z_0}) + \tilde{q}_e^2(t,\lambda_{X_0})\right] dt & \\
+ \int_{\underline{t}_Z}^{\tilde{t}} \left[\tilde{q}_e^2(t,\lambda_{X_0}) + \bar{x}\right] dt + 2\int_{\tilde{t}}^{\bar{t}_e} \tilde{q}_e^2(t,\lambda_{X_0})dt & \\
+ \left[\bar{t}_Z - \bar{t}_e\right]\bar{x} + 2\int_{\bar{t}_Z}^{t_y} \tilde{q}^3(t,\lambda_{X_0})dt &= X^0. \qquad (4.21)
\end{aligned}$$

- Continuity of the carbon stock at time $\underline{t}_Z$:

$$\begin{aligned}
Z^0 e^{-\alpha \underline{t}_Z} + 2\zeta \int_0^{\underline{t}_e} \tilde{q}^1(t,\lambda_{X_0},\lambda_{Z_0}) e^{-\alpha(\underline{t}_Z - t)} dt & \\
+\zeta \int_{\underline{t}_e}^{\underline{t}_Z} \tilde{q}^1(t,\lambda_{X_0},\lambda_{Z_0}) e^{-\alpha(\underline{t}_Z - t)} dt &= \bar{Z}. \qquad (4.22)
\end{aligned}$$

- Price continuity equations:

$$p^1\left(\underline{t}_e,\lambda_{X_0},\lambda_{Z_0}\right) = p_e^2\left(\underline{t}_e,\lambda_{X_0}\right) \qquad (4.23)$$

$$p^1\left(\underline{t}_Z,\lambda_{X_0},\lambda_{Z_0}\right) = u'(\bar{x}) \qquad (4.24)$$

$$p_e^2\left(\tilde{t},\lambda_{X_0}\right) = u'(\bar{x}) \qquad (4.25)$$

$$p_e^2\left(\bar{t}_e,\lambda_{X_0}\right) = u'(\bar{x}/2) \qquad (4.26)$$

$$p^3\left(\bar{t}_Z,\lambda_{X_0}\right) = u'(\bar{x}/2) \qquad (4.27)$$

$$p^3(t_y,\lambda_{X_0}) = c_y. \qquad (4.28)$$

Assuming a positive solution of system (4.21)-(4.28), then it is easy to check that all the optimality conditions of the restricted problem $(R.P)$ are satisfied. Reciprocally, it is clear that there exists values of the parameters of the system $c_x$, $c_y$, $c_e$, $\zeta$, $\alpha$ and $\rho$ together with values of initial endowments of oil $X^0$ and of atmospheric carbon stock $Z^0$ such that the path described above is the solution of the restricted problem $(R.P)$. However, other solutions may exist, such as the one in which sector 1's emissions have to be captured from the beginning of the planning horizon.

## 4.3 Paths along which the oil price is the same for the two sectors

### 4.3.1 Paths along which it is optimal to abate sector 1's emissions

Example of such a path, solution of the restricted problem $(R.P)$, is illustrated in Figure 2 below.

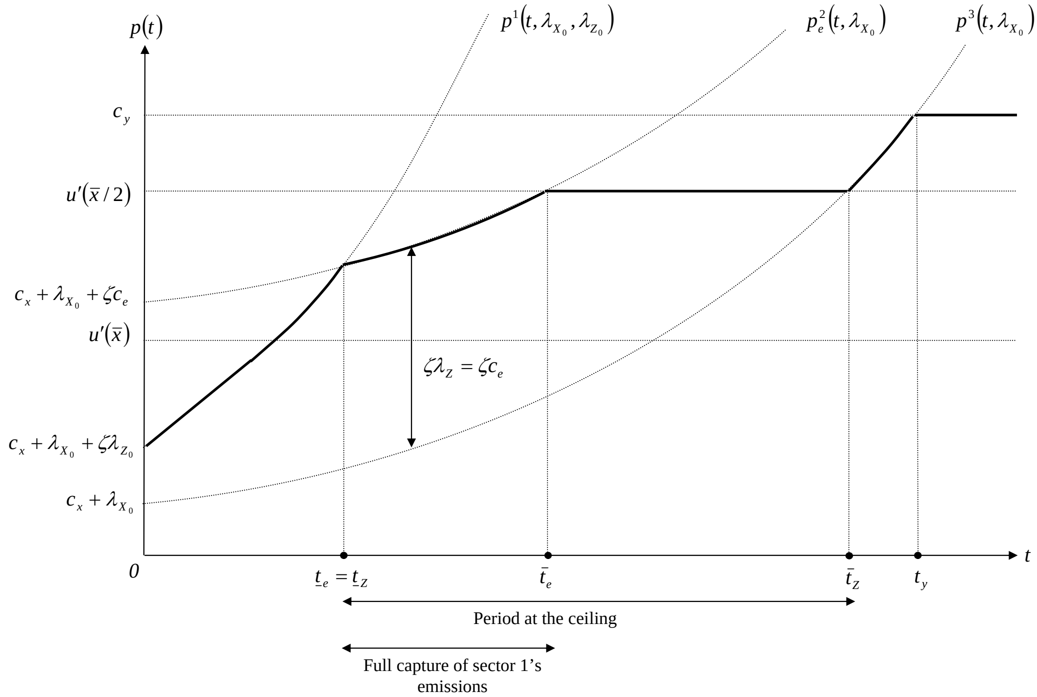


Figure 2: Optimal path along which the energy price is the same for each sector and it is optimal to abate sector 1's emissions

This kind of paths is characterized by the fact that, at time $t = \underline{t}_e$ at which $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$, then the common value of these two prices is larger than $u'(\bar{x})$ while $Z(\underline{t}_e) = \bar{Z}$ simultaneously.

Because $Z^0 < \bar{Z}$ there must exist a first phase $[0, \underline{t}_e)$ during which the ceiling $\bar{Z}$ is not yet attained and $p_1(t) = p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0}) < p_e^2(t, \lambda_{X_0})$, hence it is not optimal to abate sector 1's emissions. At the end of this first phase, both $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$ and $Z(\underline{t}_e) = \bar{Z}$ so that $\underline{t}_e$ coincides with $\underline{t}_Z$.

The next phase $[\underline{t}_e, \bar{t}_e)$ is a phase at the ceiling during which $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0})$. As in the phase 4 of the previous case – $[\tilde{t}, \bar{t}_e)$ of the path illustrated in Figure 1 – because sector 2 cannot abate its emissions, we must have $\lambda_Z(t) = c_e$ during the second phase of the present path. Also because $u'(\bar{x}) < p_e^2(t, \lambda_{X_0}) < u'(\bar{x}/2)$, then only some part of the sector 1's emissions have to be captured (cf. the above equation (4.20)), $s_e(t) < \zeta \tilde{q}_e^2(t, \lambda_{X_0}) = \zeta x_1(t)$, and the capture intensity $s_e(t)$ diminishes. At the end of this phase, $p_e^2(t, \lambda_{X_0}) = u'(\bar{x}/2)$, $x_1(t) = x_2(t) = \bar{x}/2$ and $s_e(t) = 0$.

The third phase $[\bar{t}_e, \bar{t}_Z)$ is still a phase at the ceiling but without capture of sector 1's emissions: $p_1(t) = p_2(t) = u'(\bar{x}/2)$ and $x_1(t) = x_2(t) = \bar{x}/2$. The phase is ending when $p^3(t, \lambda_{X_0}) = u'(\bar{x}/2)$, that is when $\lambda_Z(t) = 0$. The fourth and fifth phases are respectively the standard pure Hotelling phase $[\bar{t}_Z, t_y)$ and the pure renewable energy phase $[t_y, \infty)$.

#### 4.3.2 Paths along which it is never optimal to capture sector 1's emissions

When the abatement cost $c_e$ is very high, capturing is proved to never be an optimal strategy. In this case, we get a four phases optimal price path as illustrated in Figure 3.

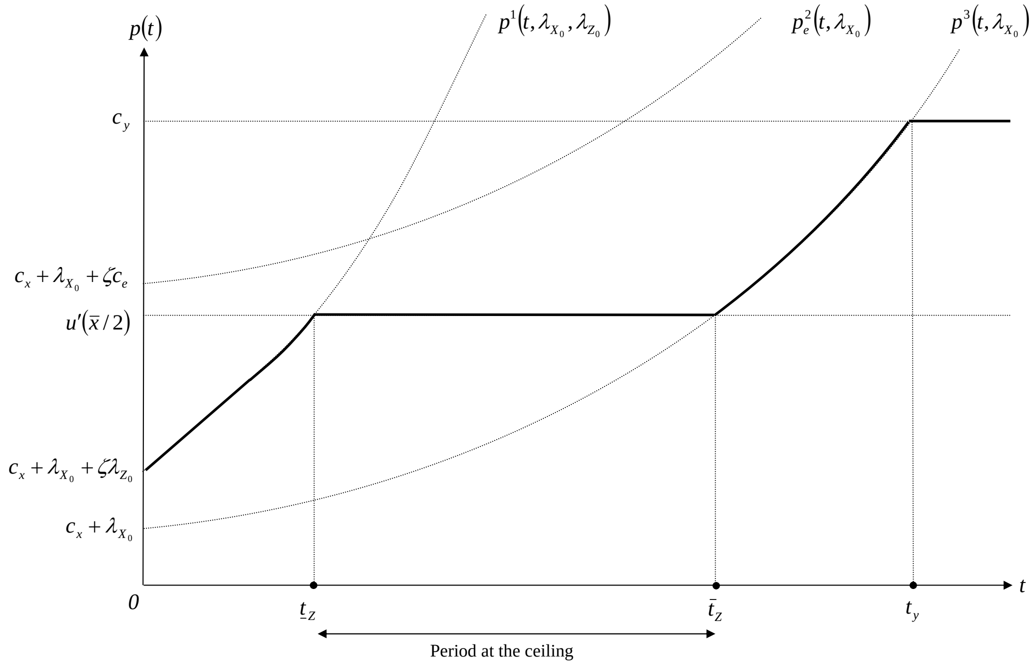


Figure 3: Optimal path along which the energy price is the same for each sector and it is not optimal to abate sector 1's emissions

In Figure 3, $p_e^2(t, \lambda_{X_0})$ is higher than $p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ along the whole time interval $[0, \underline{t}_Z)$

before the ceiling. Hence, it is never optimal to capture sector 1's emissions. Such optimal paths have been characterized in Chakravorty et al. (2006).

# 5 Optimal policies requiring to activate both capture devices

In this section, we first determine the conditions under which it is optimal to activate the atmospheric carbon capture device. Next we characterize the optimal paths along which both carbon capture technologies must be used. Last, we discuss about the time profile of the optimal carbon marginal shadow cost, that is the optimal unitary carbon tax, as well as the total burden induced by climate change mitigation policies in each sector, including the tax burden and the abatement cost.

## 5.1 Checking whether the atmospheric carbon capture device must be used along the optimal path

Let us consider the three kinds of optimal price paths which may solve the planner restricted problem $(R.P)$ and which have been discussed in the previous section. Clearly, since $p_a^2(t,\lambda_{X_0}) > p_e^2(t,\lambda_{X_0})$, then for the two last kinds of optimal paths illustrated in Figures 2 and 3 in subsection 4.3, the price trajectory $p_a^2(t,\lambda_{X_0})$ (not depicted in these figures) is always located above the optimal price path. Hence, it is never optimal to use the atmospheric carbon capture device.

For the optimal path illustrated in Figure 1 in subsection 4.2, it may happen that using the atmospheric carbon capture technology reveals optimal. To check whether this technology is optimal or not, the test runs as follows. Consider the price path $p_a^2(t,\lambda_{X_0})$ (not depicted in Figure 1). Then at time $t = \underline{t}_Z$, either $p_a^2(t,\lambda_{X_0}) < u'(\bar{x})$ or $p_a^2(t,\lambda_{X_0}) \geq u'(\bar{x})$. In the first case, there must exist a time interval around $t = \underline{t}_Z$ such that $p_2(t) > p_a^2(t,\lambda_{X_0})$ and it would be less costly for sector 2 to bear the cost of the atmospheric capture $c_a$ than the burden of the shadow cost of the atmospheric carbon stock $\lambda_Z(t)$. In the second case, using the atmospheric carbon capture technology could not allow to improve the welfare.

## 5.2 Optimal paths

Let us assume now that the atmospheric carbon capture technology has to be used. Then we may obtain two kinds of optimal paths depending on whether the least costly emission capture technology has to be activated from the beginning or not. The typical optimal path along which it is not optimal to capture the sector 1's emission flows from the start is illustrated in Figure 4 below.

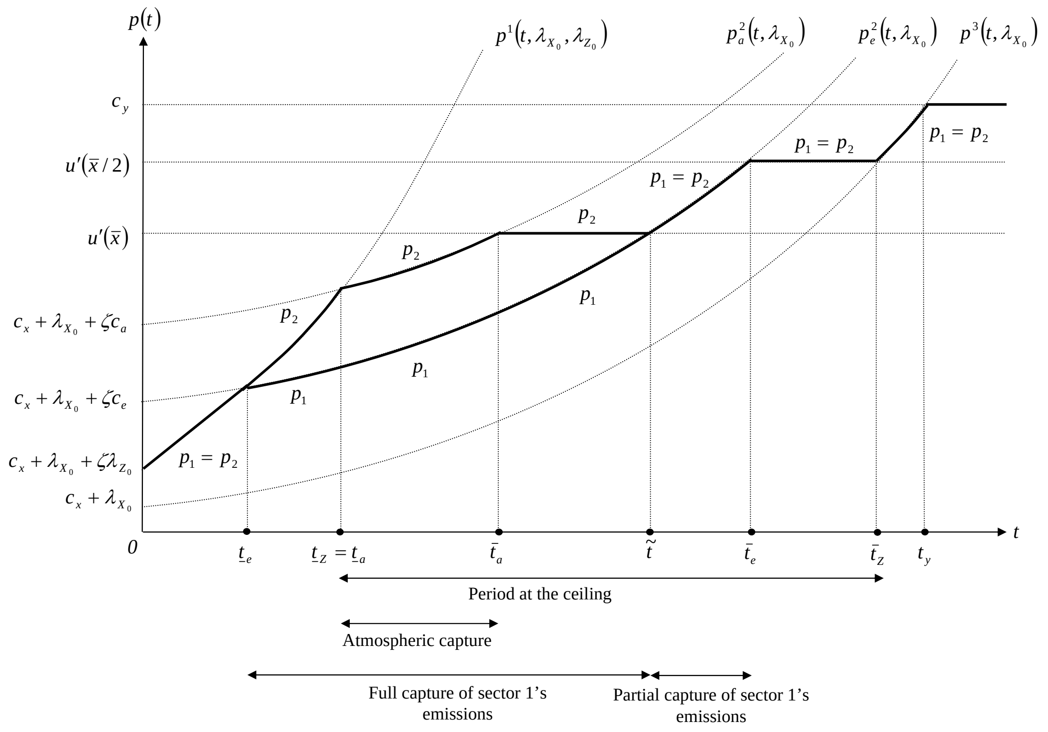


Figure 4: Optimal path requiring to activate the both carbon capture devices

The path is an eight phases path and the difference with the trajectory depicted in Figure 1 is that a new phase $[\underline{t}_a, \bar{t}_a)$ – the third one in the present case – appears now during which some of the atmospheric carbon is captured. The seven other phases are similar to the ones which have been described in section 4.2. This new phase begins at $t = \underline{t}_a$ when $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_a^2(t, \lambda_{X_0})$, that is when $\lambda_Z(t) = c_a$. Then for $t > \underline{t}_a$, it becomes less costly for sector 2 to undertake atmospheric carbon capture rather than to pay the social cost of the carbon accumulation within the atmosphere. At the time sector 2's abatement begins, the ceiling is reached, so that $\underline{t}_a$ coincides with $\underline{t}_Z$.

During this phase $[\underline{t}_a, \bar{t}_a)$, each sector uses simultaneously its own abatement technology. We have $p_1(t) = p_e^2(t, \lambda_{X_0})$ and $p_2(t) = p_a^2(t, \lambda_{X_0})$, which implies $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$ and $x_2(t) = \tilde{q}_a^2(t, \lambda_{X_0})$. Since $c_e < c_a$, we also have $p_1(t) < p_2(t)$ and then $x_1(t) > x_2(t)$.

Remember that, during this phase, as in the phase 3 of subsection 4.2, sector 1's emissions are fully captured: $s_e(t) = \zeta x_1(t)$. Because this is a phase at the ceiling, sector 2 has just to capture in the atmosphere the necessary amount of carbon in order to maintain the atmospheric carbon stock at its critical level. It is thus optimal for sector 2 to abate at a level which is smaller than its own carbon emissions: $s_a(t) = \zeta x_2(t) - \alpha\bar{Z} < \zeta x_2(t)$. Moreover, since $s_a(t) > 0$, we have $\zeta x_2(t) > \alpha\bar{Z}$, or equivalently, $x_2(t) > \bar{x}$, implying in turns $p_2(t) < u'(\bar{x})$. The price path $p_2(t) = p_a^2(t, \lambda_{X_0})$ being increasing through time, first the amount of abated carbon by the atmospheric device $s_a(t)$ is decreasing, second there must exist a date at which $p_2(t) = u'(\bar{x})$, that is at which $x_2(t) = \bar{x}$ and $s_a(t) = 0$. At that time, denoted by $\bar{t}_a$, since sector 1 still fully abates all its emissions, it is no more optimal for sector 2 to pursue the atmospheric carbon capture. All the efforts to maintain the carbon stabilization cap are now supported by the sole sector 1 and the economy behaves as in section 4.2 from phase 3, that if from the date $\underline{t}_Z$ as depicted in Figure 1.

To the eight variables parameterizing the optimal path in the case without atmospheric capture technology (cf. subsection 4.2), we must here determine the values of two additional variables: $\underline{t}_a$ and $\bar{t}_a$. But because $\underline{t}_a = \underline{t}_Z$, then only one more variable has to be determined. Hence we are left with nine variables that must solve the following nine equations system:

- Balance equation of non-renewable resource consumption and supply:

$$\begin{aligned} 2\int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})dt + \int_{\underline{t}_e}^{\underline{t}_a = \underline{t}_Z} \left[\tilde{q}_1(t, \lambda_{X_0}, \lambda_{Z_0}) + \tilde{q}_e^2(t, \lambda_{X_0})\right] dt & \\ + \int_{\underline{t}_a = \underline{t}_Z}^{\bar{t}_a} \left[\tilde{q}_e^2(t, \lambda_{X_0}) + \tilde{q}_a^2(t, \lambda_{X_0})\right] dt + \int_{\bar{t}_a}^{\tilde{t}} \left[\tilde{q}_e^2(t, \lambda_{X_0}) + \bar{x}\right] dt & \\ +2\int_{\tilde{t}}^{\bar{t}_e} \tilde{q}_e^2(t, \lambda_{X_0})dt + \left[\bar{t}_Z - \bar{t}_e\right]\bar{x} + 2\int_{\bar{t}_Z}^{t_y} \tilde{q}^3(t, \lambda_{X_0})dt & = X^0. \end{aligned} \quad (5.29)$$

- Continuity of the carbon stock at time $\underline{t}_Z$: identical to (4.22).

- Price continuity equations: identical to (4.23)-(4.28) except that (4.24) is now replaced by the two following equations:

$$p^1\left(\underline{t}_a, \lambda_{X_0}, \lambda_{Z_0}\right) = p_a^2(\underline{t}_a, \lambda_{X_0}) \quad (5.30)$$

$$p_a^2\left(\bar{t}_a, \lambda_{X_0}\right) = u'(\bar{x}) \quad (5.31)$$

## 5.3 Time profile of the optimal carbon tax

The trajectory of the carbon marginal shadow cost corresponding to the optimal path illustrated in Figure 4 is characterized by:

$$\lambda_Z(t) = \begin{cases} \lambda_{Z_0} e^{(\rho+\alpha)t} & , t \in [0, \underline{t}_Z) \\ c_a & , t \in [\underline{t}_Z, \bar{t}_a) \\ \left[u'(\bar{x}) - p^3(t, \lambda_{X_0})\right]/\zeta & , t \in [\bar{t}_a, \tilde{t}) \\ c_e & , t \in [\tilde{t}, \bar{t}_e) \\ \left[u'(\bar{x}/2) - p^3(t, \lambda_{X_0})\right]/\zeta & , t \in [\bar{t}_e, \bar{t}_Z) \\ 0 & , t \in [\bar{t}_Z, \infty) \end{cases} \tag{5.32}$$

This shadow cost can be interpreted as the optimal unitary tax to be levied on the net carbon emissions. Its time profile is illustrated in Figure 5 below.

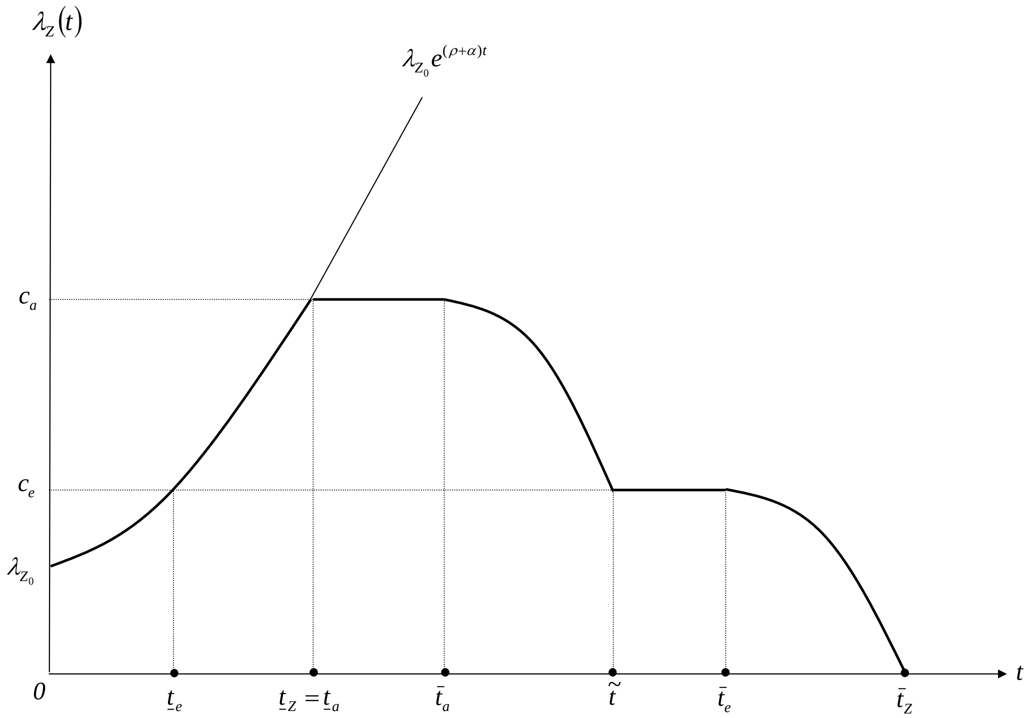


Figure 5: Time profile of the optimal unitary carbon tax

The unitary tax rate is first increasing but is bounded from above by the highest marginal abatement cost $c_a$ which is attained when it becomes optimal to use this abatement device and, simultaneously, when the atmospheric carbon stock constraint begins to be active, that is at time $t = \underline{t}_a = \underline{t}_Z$. Given that it is always possible to choose to abate rather than release the carbon in the atmosphere, the maximal tax rate of carbon emissions is necessarily determined by the highest marginal cost permitting to avoid polluting carbon releases.

During the ceiling phases, from $\underline{t}_Z$ up to $\bar{t}_Z$, the carbon tax is either constant or decreasing. First, as long as sector 2 abates, that is between $\underline{t}_a$ and $\bar{t}_a$, it is sufficient to set the tax rate equal to $c_a$ to induce an optimal atmospheric capture by sector 2, given that sector 1 fully abates its own emissions. The same applies between $\tilde{t}$ and $\bar{t}_e$ for sector 1 by setting the tax rate equal to $c_e$, given that sector 2 no more abates. Between these two phases, that is between $\bar{t}_a$ and $\tilde{t}$, and during the last phase at the ceiling, that is between $\bar{t}_e$ and $\bar{t}_Z$, the tax rate strictly decreases. This is due to the oil price increase and to the fact that the emission level is constrained by $\bar{x}$ for sector 2 during $[\bar{t}_a, \tilde{t})$, and by $\bar{x}/2$ for each sector during $[\bar{t}_e, \bar{t}_Z)$.

### 5.4 Time profile of the tax burdens and the sequestration costs

Assume now that the above tax optimal rate is implemented. Such a tax is inducing a fiscal income $\Gamma_1(t) \equiv [\zeta x_1(t) - s_e(t)]\lambda_Z(t)$ for sector 1 and $\Gamma_2(t) \equiv [\zeta x_2(t) - s_a(t)]\lambda_Z(t)$ for sector 2. The sequestration cost in each sector simply writes as the sequestered carbon flow times the respective marginal cost of sequestration: $S_1(t) \equiv s_e(t)c_e$ and $S_2(t) \equiv s_a(t)c_a$. Then, the total burden of carbon for each sector is the sum of the fiscal burden and the sequestration cost. Denoting by $B_i(t)$ $i = 1,2$ this total burden, the two following tables detail its components for each sector.

| $\Gamma_1(t)$ | $S_1(t)$ | $B_1(t)$ | Phases |
|---|---|---|---|
| $\zeta \tilde{q}^1(t)\lambda_{Z_0} e^{(\rho+\alpha)t}$ | $0$ | $\zeta \tilde{q}^1(t)\lambda_{Z_0} e^{(\rho+\alpha)t}$ | $[0, \underline{t}_e)$ |
| $0$ | $\zeta \tilde{q}_e^2(t) c_e$ | $\zeta \tilde{q}_e^2(t) c_e$ | $[\underline{t}_e, \tilde{t})$ |
| $\zeta \left[\bar{x} - \tilde{q}_e^2(t)\right] c_e$ | $\zeta \left[2\tilde{q}_e^2(t) - \bar{x}\right] c_e$ | $\zeta \tilde{q}_e^2(t) c_e$ | $[\tilde{t}, \bar{t}_e)$ |
| $(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$ | $0$ | $(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$ | $[\bar{t}_e, \bar{t}_Z)$ |
| $0$ | $0$ | $0$ | $[\bar{t}_Z, \infty)$ |

Table 1. Decomposition of the total carbon burden for sector 1

| $\Gamma_2(t)$ | $S_2(t)$ | $B_2(t)$ | Phases |
|---|---|---|---|
| $\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$ | $0$ | $\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$ | $[0, \underline{t}_a)$ |
| $\zeta\bar{x}c_a$ | $\zeta[\tilde{q}_a^2(t) - \bar{x}]c_a$ | $\zeta\tilde{q}_a^2(t)c_a$ | $[\underline{t}_a, \bar{t}_a)$ |
| $\bar{x}\left[u'(\bar{x}) - p^3(t)\right]$ | $0$ | $\bar{x}\left[u'(\bar{x}) - p^3(t)\right]$ | $[\bar{t}_a, \tilde{t})$ |
| $\zeta\tilde{q}_e^2(t)c_e$ | $0$ | $\zeta\tilde{q}_e^2(t)c_e$ | $[\tilde{t}, \bar{t}_e)$ |
| $(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$ | $0$ | $(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$ | $[\bar{t}_e, \bar{t}_Z)$ |
| $0$ | $0$ | $0$ | $[\bar{t}_Z, \infty)$ |

Table 2. Decomposition of the total carbon burden for sector 2

Their time profile are depicted upon Figure 6 below.

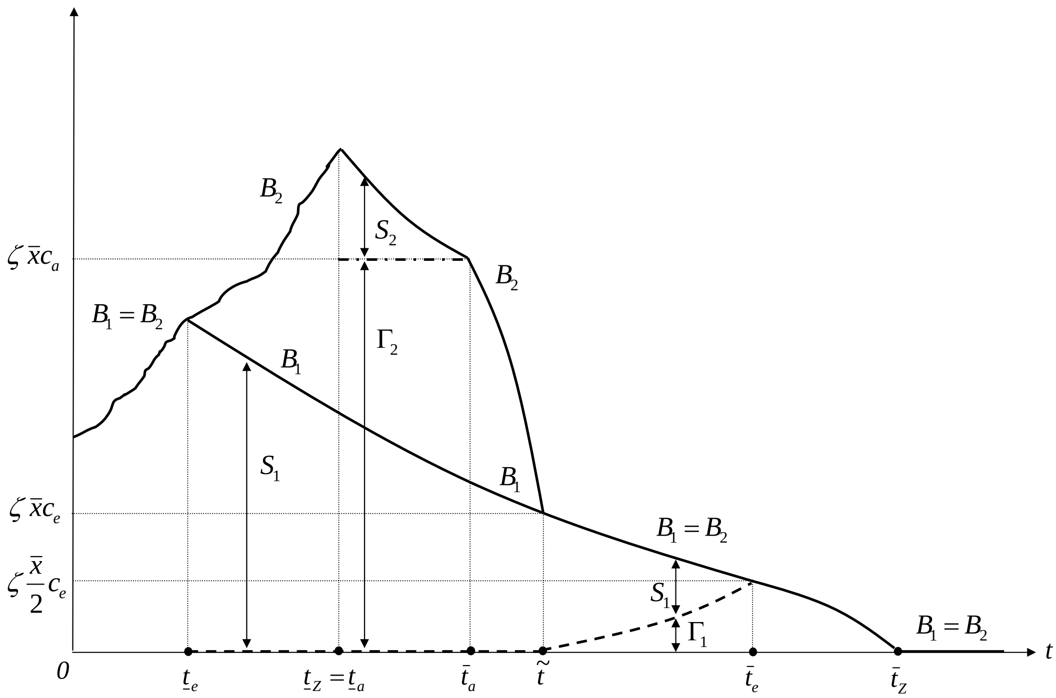


Figure 6: Total burden of carbon for each sector

Before the ceiling phases, the shapes of the total burden trajectories may be either increasing or decreasing depending upon oil demand elasticity. Once the ceiling is reached, the total burden gradually declines down to zero at the end of the ceiling phase.

For sector 1, the total burden identifies to the sole tax burden as long as abatement is not activated, that is before $\underline{t}_e$. Between $\underline{t}_e$ and $\tilde{t}$, sector 1, fully abating its emissions, does not bear the carbon tax burden ($\Gamma_1(t) = 0$), but bears the sequestration cost $S_1(t)$. During this phase, since sector 1's emissions decrease, so does its sequestration cost and then its total burden. During the next phase, between $\tilde{t}$ and $\bar{t}_e$, it is no more optimal for sector 1 to fully abate its emissions and then, this sector bears a mix of tax burden and abatement cost. Its gross carbon emissions decrease, but its sequestration flow decreases at an even higher rate resulting in an increase in the net emission flow. The cost of sequestration thus decreases. Since the tax rate is constant and equal to the sequestration marginal cost $c_e$, the fiscal burden rises. The combined effect of these two evolutions results in a declining total carbon burden for sector 1. Over the last ceiling phase, between $\bar{t}_e$ and $\bar{t}_Z$, sector 1 no more abates and bears only the fiscal burden. Then its total burden is declining down to zero when the ceiling constraint becomes no more active, that is at time $\bar{t}_Z$.

During the atmospheric capture phase, that is between $\underline{t}_a$ and $\bar{t}_a$, sector 2 is indifferent between paying the tax and abating from the atmosphere. Since it does not fully abate, it bears both the tax on this part of its emissions which are not captured, and the sequestration cost burden. During this phase, its carbon burden is constant because i) the tax rate is constant and equal to $c_a$ and ii) sector 1 fully abates its emissions and sector 2's net emissions are constrained by $\bar{x}$. Its sequestration effort decreases since gross emissions decline. After $\bar{t}_a$ and during all next phases at the ceiling, the total burden of sector 2 reduces to the sole fiscal burden and it is thus decreasing over time as discussed above.

We conclude by two remarks. First, the total fiscal income, that is $\Gamma_1(t) + \Gamma_2(t)$, jumps down twice at each time when either sector 1 or sector 2 begins to abate. Hence, any environmental policy should take into account the ability of polluters to undertake abatement activities and thus to escape from the tax. Second, since sector 2 is constrained by the higher cost of its abatement technology, its fiscal contribution as well as its total burden are larger or equal than the total burden of sector 1 when pollutive potential intensities and demand functions are the same for both sectors.

## 6 Conclusion

In a Hotelling model, we have determined the optimal CCS and air capture policies for an economy composed of two kinds of energy users differing by the degree of concentration of their carbon emissions. The concentrated emissions sector has access to geological carbon capture in addition to air capture while the diffuse emissions sector can only abate its emissions through air capture. Both sectors face a global maximal atmospheric carbon concentration constraint.

In this framework, we have shown that carbon sequestration by the first sector must begin strictly before the atmospheric carbon stock reaches its critical threshold and that sector 1's emissions have to be fully abated during a first time phase with constant marginal costs of abatement and a stationary demand schedule. This result stands in contrast with the findings of Chakravorty et al. (2006) that abatement should begin only whence the atmospheric ceiling has been attained in a model with one energy using sector and a single abatement technology.

This difference appears as a consequence of the emission concentration heterogeneity of energy users, CCS being only available for concentrated emissions sectors like thermic electricity plants, steel mills or cement factories and not for the diffuse emissions by transport of house heating. This heterogeneity constrains the potential of CCS to be at most equal to the sole emissions of sector 1 and thus to be always smaller than the total carbon emissions of fossil energy consumers. In a constant CCS cost setting there is no limitation over the amount of abated emissions below the gross emission level and in a case where diffuse emissions alone would drive atmospheric concentration up to its maximum threshold, full abatement by sector 1 of its emissions appears as the only optimal choice for the economy. Furthermore, with or without air capture possibilities, delaying CCS after the atmospheric carbon stock reaches is maximum level is dominated by an earlier development of CCS by sector 1 because of the inability of sector 2 to use carbon sequestration. However, even with air capture availability, the total carbon emission flow from the two sectors remains only partially abated resulting in a time phase during which the atmospheric carbon constraint binds over the fossil fuel consumption possibilities of the two sectors.

Note also that atmospheric capture is undertaken only after the beginning of the atmospheric carbon ceiling phase and that sector 2's abatement effort is always smaller than its gross contribution to carbon emissions, a result which stands now in accordance with Chakravorty et al. (2006). It is interesting to observe that the economy may experience a rather complex dynamic pattern of energy price while being constrained by the atmospheric carbon ceiling. With constant abatement unit costs, the energy price at the consumer stage is composed of a sequence of constant price phases separated by increasing price phases. This complex shape translates to the time profile of the carbon tax implemented to meet the atmospheric concentration objective.

The carbon tax must increase over time before the ceiling but note that sector 1 escapes the tax when fully abating its emissions and bears a comparatively lower sequestration cost, the fiscal burden being transferred over sector 2. Such a discrepancy between sectors is justified by the fact that sector 2 benefits from the carbon sequestration efforts of sector 1, a sort of positive "external" effect of sector 1 upon sector 2. Of course this is not a true external effect since it comes through the carbon price. But this opens interesting policy questions regarding the use of carbon regulation to develop non polluting transportation devices, like the electric car, electricity being provided by plants making use of CCS technologies. During the ceiling phase, the carbon tax has an overall decreasing shape down to zero at the end of the phase. But this general shape is actually composed of a complex sequence of decreasing rates phases separated by constant rates phases, these last phases corresponding respectively to the air capture phase and to the partial carbon sequestration phase by sector 1 which should follow the full carbon abatement phase by this sector. Thus inducing through the carbon tax the optimal sequence of abatement efforts by the two sectors appears as a rather complicated exercise in fiscal policy, the policy maker having to adjust over time the carbon tax rate according to the optimal sequence of abatement phases.

A second source of heterogeneity between sectors comes from the differing availability of the two carbon abatement technologies. As stated before, CCS is only available for sector 1 while air capture may apply to emissions coming from any source. Alternatively we could have assumed that sector 2 abates its emissions at a unit cost $c_a$ through some dedicated technology while sector 1 abates through CCS at a unit cost $c_e$, $c_e < c_a$, without altering the results of our analysis. To reinforce the heterogeneity argument, it can be shown (Amigues et al., 2011) that, when energy users have a access to a single carbon abatement technology, then even learning or R&D over this technology do not justify to abate before being at the atmospheric ceiling. However, because the time at which the ceiling is attained is endogenous, learning by doing will affect the time profile of the ceiling phase. An interesting extension of the work would be to analyze the effects of learning by doing or dedicated R&D over CCS and air capture in an heterogeneous use framework.

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# Optimal Timing of Carbon Capture Policies Under Alternative CCS Cost Functions

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# Optimal Timing of Carbon Capture Policies Under Alternative CCS Cost Functions


**Abstract**

We determine the optimal exploitation time-paths of three types of perfect substitute energy resources: The first one is depletable and carbon-emitting (dirty coal), the second one is also depletable but carbon-free thanks to a carbon capture and storage (CCS) process (clean coal) and the last one is renewable and clean (solar energy). We assume that the atmospheric carbon stock cannot exceed some given ceiling. These optimal paths are considered along with alternative structures of the CCS cost function depending on whether the marginal sequestration cost depends on the flow of clean coal consumption or on its cumulated stock. In the later case, the marginal cost function can be either increasing in the stock thus revealing a scarcity effect on the storage capacity of carbon emissions, or decreasing in order to take into account some learning process. We show among others the following results: Under a stock-dependent CCS cost function, the clean coal exploitation must begin at the earliest when the carbon cap is reached while it must begin before under a flow-dependent cost function. Under stock-dependent cost function with a dominant learning effect, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation. Last, the scarcity effect implies a carbon tax trajectory which is also unusual in this kind of ceiling models, its increasing part been extended for some time during the period at the ceiling.



**Keywords:** Carbon capture and storage; Energy substitution; Learning effect; Scarcity effect; Carbon stabilization cap.

**JEL classifications:** Q32, Q42, Q54, Q55, Q58.

# Contents

# 1 Introduction

Carbon dioxide capture and storage (CCS) is a process consisting of the separation of $CO_2$ from the emissions stream from fossil fuel combustion, transporting it to storage location, and storing it in a manner that ensures its long-run isolation from the atmosphere (IPCC, 2005). Currently, the major CCS effort focus on the removal of $CO_2$ directly from industrial or utility plants and storing it in secure geological reservoirs. Given that fossil fuels supply over 85% of all primary energy demands, CCS appears as the only technology that can substantially reduce $CO_2$ emissions while allowing fossil fuels to meet the world's pressing needs (Herzog, 2011). Moreover, CCS technology may have considerable potential to reduce $CO_2$ at a "reasonable" social cost, given the social costs of carbon emissions predicted for a business-as-usual scenario (Islegen and Reichelstein, 2009). According to Hamilton et al. (2009), the mitigation cost for capture and compression of the emissions from power plants running with gas is about \$52 per metric ton $CO_2$. Adding the transport and storage costs[1] in a range of \$5-15 per metric ton $CO_2$, a carbon price of about \$60-65 per metric ton $CO_2$ is needed to make these plants competitive.

The CCS technology has motivated a large number of empirical studies, mainly through complex integrated assessment models (see for instance McFarland et al. (2003), Kurosawa, 2004, Edenhofer et al., Gerlagh, 2006, Gerlagh and van der Zwaan, 2006, Grimaud et al., 2011). In these models, the only reason to use CCS technologies is to reduce $CO_2$ emissions[2] and then, climate policies are essential to create a significant market for these technologies. These empirical models generally conclude that an early introduction of sequestration can lead to a substantial decrease in the social cost of climate change. However a high level of complexity for such models, aimed at defining some specific climate policies and energy scenarios, may be required so as to take into account the various interactions at the hand.

The theoretical economic literature on CCS is more succinct. Grimaud and Rouge (2009) study the implications of the CCS technology availability on the optimal use of polluting exhaustible resources and on optimal climate policies within an endogenous growth model. Ayong Le kama et al. (2010) develop a growth model aiming at exhibiting the main driving forces that should determine the optimal CCS policy when the command variable

[1] As explained in Hamilton et al. (2009), the transport and storage costs are very site specific.

[2] As mentioned by Herzog (2009), the idea of separating and capturing $CO_2$ from the flue gas of power plants did not originate out of concern about climate change. The first commercial CCS plants that have been built in the late 1970s in the United States aimed at achieving enhanced oil recovery (EOR) operations, where $CO_2$ is injected into oil reservoirs to increase the pressure and thus the output of the reservoir.

of such a policy is the sequestration rate instead of the sequestration flow. Lafforgue et al. (2008-a) characterize the optimal timing of the CCS policy in a model of energy substitution when carbon emissions can be stockpiled into several reservoirs of finite size. However, the outcomes of these models cannot be easily compared since they strongly vary according to a crucial feature: the structure of the CCS cost function.

In the present study, we address the question of the qualitative impacts of such cost function properties on the optimal use of carbon capture and storage. Using a standard Hotelling model for the fossil resource and assuming, as in Chakravorty et al. (2006), that the atmospheric carbon stock should not exceed some critical threshold, we characterize the optimal time paths of energy price, energy consumption, carbon emissions and atmospheric abatement for various types of CCS cost functions. In that sense, we generalize the model of Lafforgue et al. (2008) in which the marginal sequestration cost is assumed to be constant.

The sketch of the model is the following. The energy needs can be supplied by three types of energy resources that are perfect substitutes: The first one is depletable and carbon-emitting (dirty coal), the second one is also depletable but carbon-free thanks to a CCS device (clean coal) and the last one is renewable and clean (solar energy). Hence, we consider two alternative mitigation options allowing to relax the carbon cap constraint: the exploitation of the solar energy and of the clean coal. The design of the optimal energy consumption path thus results from the comparison of the respective marginal costs of these three energy sources. Both the marginal extraction cost of coal and the marginal production cost of the solar energy are assumed to be constant, the former been lower than the later. However, producing clean coal requires an additional CCS cost whose characteristics can vary. We consider alternative structures of the CCS cost function depending on whether the marginal sequestration cost depends on the flow of clean coal consumption or on its cumulated stock. In the later case, the marginal cost function can first be increasing in the stock thus revealing a scarcity effect on the storage capacity of carbon emissions[3]. Second, since as pointed out by Gerlagh (2006) or by Mannea and Richelsb (2004), the cumulated experience in carbon capture generates in most cases some beneficial learning tending to reduce the involved costs, the average cost function can be decreasing in the cumulated clean coal consumption.

We show among others the following results: Under a stock-dependent CCS cost func-

[3]This effect is taken into account in Lafforgue et al. (2008) through the definition of a physical limit of sequestration. In the present study, such a limit in capacity is also tackled in an economical way by assuming that the marginal sequestration cost increases as the carbon reservoir is filled up.

tion, the clean coal exploitation must begin at the earliest when the carbon cap is reached while it must begin before under a flow-dependent cost function. Under stock-dependent cost function with a dominant learning effect, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation. Last, the scarcity effect implies a carbon tax trajectory which is unusual in this kind of ceiling models, its increasing part been extended for some time during the period at the ceiling.

The paper is organized as follows. Section 2 presents the model and characterizes the various structures of CCS cost function that are under study. Section 3 describes the optimal path in the case of flow-dependent CCS cost functions by distinguishing different possibilities for the solar energy to be more or less expensive as compared with the clean coal exploitation. Section 4 studies the optimal paths under cost-dependent CCS cost functions according to whether the scarcity effect or the learning effect dominates and according to whether the solar energy cost is high or low. Section 5 investigates the main qualitative dynamical properties of the carbon tax required to enforce the carbon cap constraint that are obtained in the various cases described above, and it compares them. Last Section 6 briefly concludes.

# 2 The model

Let us consider an economy in which the energy services can be produced from two primary resources, a polluting non-renewable one, say coal, and a clean renewable one, say solar.

## 2.1 The polluting non-renewable primary resource

Let $X(t)$ be the available stock of coal at time $t$, $X^0$ be its initial endowment, $X(0) = X^0 > 0$, and $x(t)$ its instantaneous extraction rate so that:

$$\dot{X}(t) = -x(t),\ X(t) \geq 0,\ t \geq 0 \ \text{ and } X(0) = X^0 > 0 \tag{2.1}$$

$$x(t) \geq 0,\ t \geq 0 \tag{2.2}$$

The average cost of coal exploitation, denoted by $c_x$, is assumed to be constant, hence equal to its marginal cost. This cost includes all the different costs having to be borne to produce ready-for-use energy services to the final users, that is the extraction cost, the processing cost and the transportation and distribution costs.

Let $\zeta$ be the unitary pollutant content of coal so that, absent any abatement policy, the pollution flow which would be released into the atmosphere would amount to $\zeta x(t)$.

### 2.2 Atmospheric pollution stock

Denote by $Z(t)$ the current level of the atmospheric carbon concentration at time $t$ and by $Z^0$ the initial concentration inherited from the past: $Z(0) = Z^0 \geq 0$. This atmospheric pollution stock is assumed to be self-regenerating at some constant proportional rate $\alpha$, $\alpha > 0$.

To get the dynamics of $Z(t)$, we must take into account that its supplying flow can be lower than the potential pollution flow $\zeta x(t)$ generated by coal burning thanks to some carbon capture and sequestration option. Let $s(t)$ be this share of the potential emission flow which is captured and sequestered:

$$s(t) \geq 0 \text{ and } \zeta x(t) - s(t) \geq 0 \tag{2.3}$$

The dynamics of the atmospheric pollution stock is driven by both the coal consumption policy and the capture and sequestration policy, that is:

$$\dot{Z} = \zeta x(t) - s(t) - \alpha Z(t), \; Z(0) = Z^0 \geq 0 \tag{2.4}$$

Having adopted this formalization, the next step consists in introducing the CCS average cost as some function of either the current emission captured flow $s(t)$, or of the cumulated captures $S(t)$, $S(t) = S^0 + \int_0^t s(\tau)d\tau$, where $S^0 \equiv S(0)$, in order to take into account the scarcity of accessible sequestering sites and/or the learning effects resulting from the experience in the capture and sequestration activity.

### 2.3 Clean versus dirty energy services

Instead of expressing the CCS cost as some function of the sequestration flow $s(t)$ and/or of the cumulated sequestration $S(t)$, we proceed formally otherwise by considering two types of fossil energies allowing to produce final energy services together with the clean renewable substitute. We define the clean coal as this part of coal consumption whose emissions are captured and the dirty coal as this part whose emissions are directly released into the atmosphere. Let us denote respectively by $x_c(t)$ and $x_d(t)$ the instantaneous consumption rates of clean and dirty coals. Since $x_c(t) + x_d(t) = x(t)$, then (2.1) and (2.2) have to be rewritten as:

$$\dot{X}(t) = -[x_c(t) + x_d(t)],\ X(t) \geq 0\ t \geq 0 \ \text{ and } X(0) = X^0 > 0 \tag{2.5}$$

$$x_c(t) \geq 0 \ \text{ and } \ x_d(t) \geq 0 \tag{2.6}$$

We denote by $S_c(t)$ be the cumulated clean coal consumption from time 0 up to time $t$. For the sake of simplicity, we assume that $S_c(0) = 0$, so that:

$$S_c(t) = \int_0^t x_c(\tau)d\tau \Rightarrow \dot{S}_c(t) = x_c(t) \tag{2.7}$$

equivalently:

$$S_c(t) = \frac{1}{\zeta}S(t) \tag{2.8}$$

Since only the dirty coal is supplying the atmospheric carbon stock, its dynamics (2.4) may be simply rewritten as:

$$\dot{Z}(t) = \zeta x_d(t) - \alpha Z(t),\ t \geq 0 \ \text{ and } Z(0) = Z^0 \geq 0 \tag{2.9}$$

## 2.4 Sequestration costs

Producing energy services from clean coal is more costly than from dirty coal since some additional capture and sequestration costs must be incurred. Let $c_s$ be the additional cost per unit of clean coal. Clearly, the implications of such a way to relax the pollution constraint should depend upon the characteristics of this additional cost.

The CCS average cost $c_s$ may first depend upon the current quantity of clean coal which is consumed, and only upon this flow.

- **CCS.1 Flow-dependent capture cost function:**
  $c_s : \mathbb{R}_+ \to \mathbb{R}_+^*$ is a $\mathcal{C}^2$ function, strictly increasing and strictly convex, $c_s'(x_c) > 0$ and $c_s''(x_c) > 0$ for any $x_c > 0$, with $\lim_{x_c \downarrow 0} c_s(x_c) = \underline{c}_s > 0$.

Under CCS.1, the total additional cost required for consuming clean coal rather than dirty coal thus amounts to $c_s(x_c)x_c$. The associated marginal cost of clean coal, denoted by $c_{ms}(x_c)$, amounts to: $c_{ms}(x_c) = c_s(x_c) + c_s'(x_c)x_c > 0$, and is increasing: $c_{ms}'(x_c) = 2c_s'(x_c) + c_s''(x_c)x_c > 0$.

Second, the CCS cost function may depend upon the cumulated clean coal consumption, which may give rise to two different effects working in quite opposite directions. On the one hand, due to the scarcity of the most accessible sites into which the carbon can be sequestered[4], the average CCS cost may increase with $S_c$ up to some upper bound $\bar{S}_c$ corresponding to the global capacity of such reservoir sites, hence the following constraint:

$$\bar{S}_c - S_c(t) \geq 0 \tag{2.10}$$

Although not sufficient, a necessary condition for such a condition to be effective is that $\bar{S}_c$ be lower than the maximal cumulated emissions of coal, that is: $\bar{S}_c < X^0$.

On the other hand, the higher $S_c$, the larger the cumulated experience in carbon capture generating in most cases some beneficial learning tending to reduce the involved costs, in which case the CCS cost function decreases with $S_c$.

We define stock-dependent capture costs as average capture cost functions depending upon the cumulated clean coal consumption $S_c$ and only the cumulated clean coal consumption, so that at any time $t$ the total additional cost having to be incurred for using the friendly environmental coal instead of the carbon emitting one, amounts to $c_s(S_c(t))x_c(t)$. A stock-dependent capture cost with a dominant effect is a cost function for which the marginal balance sheet between the scarcity and the learning effects does not depend upon the cumulated clean coal consumption. In brief, it is the polar case in which the sign of the derivative of $c_s(S_c)$ does not depend upon $S_c$ and thus, cannot alternate.

In the case of a dominant scarcity effect, $c_s$ must be defined in the range $[0, \bar{S}_c]$.

- **CCS.2 Stock-dependent capture cost with dominant scarcity effect:** $c_s : \left[0, \bar{S}_c\right] \to \mathbb{R}_+^*$ is a $\mathcal{C}^2$ function, strictly increasing and strictly convex, $c_s'(S_c) > 0$ and $c_s''(S_c) > 0$ for any $S_c \in \left(0, \bar{S}_c\right)$, with $\lim_{S_c \downarrow 0} c_s(S_c) = \underline{c}_s > 0$.

In the case of a pure dominant learning effect, no restriction has to be put on the global capacity of the reservoirs. Such a constraint would introduce in some sense a scarcity effect blurring the learning effect. The objective of the paper being to isolate the pure learning effect, we neglect an eventual locking of this process that would be involved by a constrained capacity of the reservoirs, even if such a constraint is empirically relevant.

- **CCS.3 Stock-dependent capture cost with dominant learning effect:** $c_s : \left[0, X^0\right] \to \mathbb{R}_+^*$ is a $\mathcal{C}^2$ function, strictly decreasing and strictly convex, $c_s'(S_c) < 0$

[4]Lafforgue *et al.* (2008-a) show that the different reservoirs should be completely filled by increasing order of their respective sequestration costs. The present setting assumes that there is no correlation between the extraction and consumption costs and the sequestration costs.

and $c_s''(S_c) > 0$ for any $S_c \in \left(0, X^0\right)$, with $\lim_{S_c \downarrow 0} c_s(S_c) = \bar{c}_s < \infty$ and $c_s(X^0) = \underline{c}_s > 0$.

## 2.5 The clean renewable primary resource

The other primary resource can be processed at some constant average cost $c_y$. As for the non-renewable resource this cost includes all the costs having to be supported to supply ready-for-use energy services to the final users. Thus once $c_x$, possibly $c_s$, and $c_y$ are supported, the both types of the main primary energy resources are perfect substitutes as far as consuming energy services generates some surplus. Denoting by $y(t)$ the renewable energy consumption, we may define the aggregate energy consumption $q(t)$ as $q(t) = x(t) + y(t) = x_c(t) + x_d(t) + y(t)$, with the usual non-negativity constraint:

$$y(t) \geq 0 \tag{2.11}$$

The natural flow of solar energy $y^n$ is assumed to be sufficiently large to provide all the energy needs of the society at the marginal cost $c_y$ so that no rent has ever to be charged for an efficient exploitation of the resource. Last, we assume that $c_y$ is larger than $c_x$ to justify the use of coal during some time period. Since relaxing the ceiling constraint can be achieved by using either clean coal or solar energy, the relative competitiveness of these two options may depend upon their respective costs. That is why we will distinguish the cases of a "high" or a "low" solar energy costs in the following analysis. What we mean by "high" or "low" will be made more precise in the next sections.

## 2.6 Gross surplus generated by energy service consumption

The energy service consumption $q(t)$ is generating an instantaneous gross surplus $u(q(t))$. Function $u(.)$ is assumed to satisfy the following standard assumptions: $u : \mathbb{R}_+ \to \mathbb{R}$ is a $\mathcal{C}^2$ function, strictly increasing and strictly concave verifying the Inada condition: $\lim_{q \downarrow 0} u'(q) = +\infty$.

We denote by $p(q)$ the marginal gross surplus function $u'(q)$, and by $q(p)$ its inverse, i.e. the energy demand function. When the solar energy is the unique energy source, then its optimal consumption would amount to $\tilde{y}$ solution of $u'(q) = c_y$, provided that $y^n$ is not smaller than $\tilde{y}$, what we mean by assuming that $y^n$ is sufficiently large.

### 2.7 Pollution damages

Turning now to the main focus of the paper, we assume that, as far as the atmospheric pollution stock does not overshoot some critical level $\bar{Z}$, the damages due to the atmospheric carbon accumulation are negligible[5]. However, for pollution stocks that are larger than $\bar{Z}$, the damages would be immeasurably larger than the sum of the discounted gross surplus generated along any path triggering this overshoot. By doing that, we assume a lexicographic structure of the preferences over the set of the time paths of energy consumption and pollution stock. Technically, this lexicographic structure translates into two constraints, the first one on the state variable $Z$ and the second one on the control variable $x_d$.

Since the overshoot of this critical cap would destroy all that could be gained otherwise, then we must impose:

$$\bar{Z} - Z(t) \geq 0 \; t \geq 0 \tag{2.12}$$

The other constraint states that, when the ceiling is reached, the maximum quantity of dirty coal which can be consumed is this quantity whose emissions are balanced by the natural regeneration of the atmosphere. Denoting by $\bar{x}_d$ this maximum consumption rate of dirty coal, (2.9) implies that $\bar{x}_d = \alpha\bar{Z}/\zeta$.

### 2.8 The social rate of discount and the social planner program

We denote by $\rho$ the instantaneous rate of discount, which is assumed to be constant over time and strictly positive. The social planner program thus consists in determining the paths of $x_c$, $x_d$ and $y$ that maximize the sum of the discounted net surplus.

## 3 Flow-dependent CCS cost functions

### 3.1 Problem formulation and preliminary remarks

Under CCS.1, the social planner program takes the following form:

$$(P) \quad \max_{x_c, x_d, y} \int_0^\infty \{u(x_c(t) + x_d(t) + y(t)) - c_x[x_c(t) + x_d(t)] - c_s(x_c(t))x_c(t) - c_y y(t)\} \, e^{-\rho t} dt$$

subject to constraints (2.5), (2.9) and to the inequality constraints (2.6), (2.11) and (2.12).

[5]See Amigues, Moreaux and Schubert (2011) for a model in which the both types of effects are explicitly taken into account.

Let $\mathcal{H}$ be the Hamiltonian in current value of problem $(P)$ (we drop the time argument for notational convenience):

$$\mathcal{H} = u(x_c + x_d + y) - c_x[x_c + x_d] - c_s(x_c)x_c - c_y y - \lambda_X[x_c + x_d] - \lambda_Z[\zeta x_d - \alpha Z]$$

where $\lambda_X$ and $-\lambda_Z$ are the costate variables of $X$ and $Z$ respectively[6]. Denoting by $\nu$'s the Lagrange multipliers associated with the inequality constraints on the state variables and by $\gamma$'s the multipliers corresponding to the inequality constraints on the control variables, the Lagrangian in current value writes:

$$\mathcal{L} = \mathcal{H} + \nu_X X + \nu_Z[\bar{Z} - Z] + \gamma_{x_c}x_c + \gamma_{x_d}x_d + \gamma_y y$$

The first order optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial x_c} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + c_{ms}(x_c) - \gamma_{x_c} \tag{3.13}$$

$$\frac{\partial \mathcal{L}}{\partial x_d} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + \zeta\lambda_Z - \gamma_{x_d} \tag{3.14}$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_y - \gamma_y \tag{3.15}$$

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{\partial X} \quad \Rightarrow \quad \dot{\lambda}_X = \rho\lambda_X - \nu_X \tag{3.16}$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \quad \Rightarrow \quad \dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \tag{3.17}$$

together with the usual complementary slackness conditions.

The transversality conditions are:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \tag{3.18}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{3.19}$$

As it is well known, with a constant marginal extraction cost $c_x$, the mining rent $\lambda_X$ must grow at the social rate of discount as long as the stock of coal is not exhausted. From (3.16), we have:

$$X(t) > 0 \Rightarrow \lambda_X(t) = \lambda_{X0}e^{\rho t},\ \lambda_{X0} = \lambda_X(0) \tag{3.20}$$

so that $e^{-\rho t}\lambda_X(t)X(t) = \lambda_{X0}X(t)$. Hence from the transversality condition (3.18), if coal have some positive initial value, i.e. if $\lambda_{X0} > 0$, then its stock must be exhausted in the long run along the optimal path.

[6]Using $-\lambda_Z$ as the costate variable of $Z$ makes it possible to directly interpret $\lambda_Z \geq 0$ as the unitary tax having to be charged for the pollution emissions generated by dirty coal consumption.

Initially, we have $\nu_Z = 0$ as long as the ceiling constraint is not binding. Denoting by $\underline{t}_Z$ the time at which the atmospheric carbon cap $\bar{Z}$ is reached, (3.17) implies:

$$t \leq \underline{t}_Z \Rightarrow \lambda_Z(t) = \lambda_{Z0} e^{(\rho+\alpha)t}, \quad \text{where } \lambda_{Z0} = \lambda_Z(0) \tag{3.21}$$

Once the ceiling constraint is no more active and forever, $\lambda_Z$ must be nil. Denoting by $\bar{t}_Z$ the last time at which the constraint is active, it comes[7]:

$$t \geq \bar{t}_Z \Rightarrow \lambda_Z(t) = 0 \tag{3.22}$$

### 3.2 The optimal paths

The dynamics of consumption of the two types of coal is driven by the dynamics of their respective full marginal costs. A common component of these costs is the processing cost $c_x$ augmented by the mining rent $\lambda_X(t)$. We denote by $p^F(t)$ ($F$ for free of tax and free of cleaning cost) this common component:

$$p^F(t) = c_x + \lambda_{X0} e^{\rho t} \Rightarrow \dot{p}^F(t) = \rho \lambda_{X0} e^{\rho t} > 0 \tag{3.23}$$

In addition to this common component, the full marginal cost of the dirty coal, which is denoted by $c_m^d(x_d)$, must also include the imputed marginal cost of the carbon emissions generated by its consumption:

$$c_m^d(x_d(t)) = p^F(t) + \zeta \lambda_Z(t) \tag{3.24}$$

The full marginal cost of the clean coal must include the marginal cleaning cost. Thus denoting by $c_m^c(x_c)$ this full marginal cost, we get:

$$c_m^c(x_c(t)) = p^F(t) + c_{ms}(x_c(t)) \tag{3.25}$$

where $c_{ms}(x_c(t)) = c_s(x_c) + c_s'(x_c)x_c > 0$.

The day-to-day dynamics of exploitation of the two types of coal and solar energy are driven by the dynamics of their instantaneous full marginal costs. Given that we assume a constant marginal cost of the solar energy, free of pollution tax since clean, we may organize the discussion depending on whether this marginal cost of the clean renewable substitute

[7] Solving the ordinary differential equations (2.9) and (3.17) respectively results in $Z(t) = \left[Z^0 + \int_0^t \zeta x_d(\tau) e^{\alpha\tau} d\tau\right] e^{-\alpha t}$ and $\lambda_Z(t) = \left[\lambda_{Z0} - \int_0^t \nu_Z(\tau) e^{-(\rho+\alpha)\tau} d\tau\right] e^{(\rho+\alpha)t}$. The transversality condition (3.19) can thus be written as: $\lim_{t\to\infty} \left[\lambda_{Z0} - \int_0^t \nu_Z(\tau) e^{-(\rho+\alpha)\tau} d\tau\right] \left[Z^0 + \int_0^t \zeta x_d(\tau) e^{\alpha\tau} d\tau\right] = 0$, which implies $\lambda_{Z0} = \int_0^\infty \nu_Z(\tau) e^{-(\rho+\alpha)\tau} d\tau$. Then, $\lambda_Z(t) = \int_t^\infty \nu_Z(\tau) e^{-(\rho+\alpha)(\tau - t)} d\tau$ and, as a consequence, $\lambda_Z(t) = 0$ for any $t \geq \underline{t}_Z$.

is "high" or "low", meaning that either $c_y > u'(\bar{x}_d)$ or $c_y < u'(\bar{x}_d)$ and assuming that the initial coal endowment $X^0$ is large enough for having the ceiling constraint $\bar{Z} - Z(t) \geq 0$ binding along the optimal path.

### 3.2.1 The high solar cost case: $c_y > u'(\bar{x}_d)$

Let us assume that solar cost is high. In this case, we show that the optimal path is a five or six phases path when the ceiling constraint is active.

**Types of phases**

For sufficiently low $\lambda_Z(t)$, that is for $\zeta\lambda_Z(t) < \underline{c}_s$, dirty coal is more competitive than dirty coal and than solar energy, and it thus must be the only source of supplied energy.

Consider now a phase of simultaneous exploitation of the both types of coal and the composition of the resulting energy supply. Denote by $\underline{t}_c$ the time at which clean coal begins to be exploited. If a simultaneous use of both types of coal is possible before the ceiling is attained, $\underline{t}_c < \underline{t}_Z$, then the full marginal costs of the both types of coal must be equal, that is $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_{ms}(x_c(t))$. Differentiating this expression with respect to time and solving for $\dot{x}_c$, we get:

$$\dot{x}_c(t) = \frac{\zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t}}{c'_{ms}(x_c(t))} > 0 \tag{3.26}$$

where $c'_{ms}(x_c(t)) = 2c'_s(x_c(t)) + c''_s(x_c(t))x_c(t) > 0$. The consumption of clean coal must increase over time during such a phase. Since the energy price $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ is increasing, then the consumption of energy services decreases hence the consumption of the dirty coal must simultaneously decrease.

During a phase along which the ceiling constraint is binding and both types of coal are used, assuming again that it is possible, minimizing the energy production cost implies that the dirty coal must be used as far as possible: $x_d(t) = \bar{x}_d$. The clean coal consumption is thus determined by the condition (3.13): $u'(x_c(t)+\bar{x}_d) = c_x + \lambda_{X0}e^{\rho t} + c_{ms}(x_c(t))$. Time differentiating this expression and solving for $\dot{x}_c$, we obtain:

$$\dot{x}_c(t) = \frac{\rho\lambda_{X0}e^{\rho t}}{u''(x_c(t)+\bar{x}_d) - c'_{ms}(x_c(t))} < 0 \tag{3.27}$$

Since the energy consumption $q(t) = x_c(t)+\bar{x}_d$ decreases during such a phase at the ceiling, the energy price must increase.

A crucial problem for characterizing the optimal path is to identify the timing of the different types of phases and their sequencing. The following Proposition 1 states that if the clean coal has to be ever exploited because the ceiling constraint is effective during some phase of the optimal path, then its exploitation must begin before the ceiling constraint is attained. Thus the clean coal use must be seen as some costly device allowing to delay the time at which the ceiling constraint will become effective. Another possibility would be to use the solar energy, but it is assumed to be too costly here, too costly meaning that $c_y > u'(\bar{x}_d)$.

**Proposition 1** *Under flow-dependent CCS cost functions CCS.1, assuming that the solar energy cost is high, that clean coal is exploited and that the ceiling constraint is effective along the optimal path, then the clean coal exploitation must begin before the ceiling constraint is active:* $\underline{t}_c < \underline{t}_Z$.

**Proof:** We first show that $\zeta\lambda_Z(t)$ is always decreasing for $t \in [\underline{t}_Z, \bar{t}_Z)$. During this interval of time, either $x_c(t) = 0$ so that $\zeta\lambda_Z(t) = u'(\bar{x}_d) - p^F(t)$ and $\zeta\dot{\lambda}_Z(t) = -\dot{p}^F(t) < 0$, or $x_c(t) > 0$ so that $\zeta\lambda_Z(t) = c_{ms}(x_c(t))$ and $\zeta\dot{\lambda}_Z(t) = c'_{ms}(x_c(t))\dot{x}_c(t)$, which is also negative from (3.27). Hence, since we know that $\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}$ for $t \in [0, \underline{t}_Z)$, the maximal value of $\zeta\lambda_Z(t)$ is attained at time $\underline{t}_Z$: $\underline{t}_Z = \text{argmax}\,\{\lambda_Z(t)\}$.

At this point of time, assume that sequestration has not begun yet: $\underline{t}_c > \underline{t}_Z$ so that $x_c(\underline{t}_Z) = 0$. It means that $\zeta\lambda_Z(\underline{t}_Z) < \underline{c}_s$ and then, since $\zeta\lambda_Z(t)$ is decreasing for $t \geq \underline{t}_Z$, we must have $x_c(t) = 0$ for any $t \geq \underline{t}_Z$. If sequestration has not begun yet at time $\underline{t}_Z$, it will never be used thereafter. In order to have any interest, the problem must be such that $\zeta\lambda_Z(\underline{t}_Z) = c_{ms}(x_c(\underline{t}_Z)) > \underline{c}_s$. Consequently, any clean coal consumption phase must begin at some date $\underline{t}_c < \underline{t}_Z$. ∎

Proposition 2 below characterizes the behavior the economy during any phase at the ceiling.

**Proposition 2** *Under a flow-dependent cleaning cost function, assuming that the cost of solar energy is high, if clean coal has to be used, then there must exist two phases at the ceiling, the first one during which the both types of coal are exploited and the next one during which only dirty coal must be exploited.*

**Proof:** According to Proposition 1 and (3.26), the clean coal production is strictly positive when the ceiling is attained. This is possible if and only if $\zeta\lambda_Z(\underline{t}_Z) > \underline{c}_s$. Since the price path must be continuous then there must exist some time interval $(\underline{t}_Z, \underline{t}_Z + \delta)$, $\delta > 0$, during which the clean coal production is still positive and decreasing from (3.27).

Assume now that clean coal is produced during the entire period at the ceiling. At the end of the period, at time $t = \bar{t}_Z$, we must have $\lambda_Z(\bar{t}_Z) = 0$ as pointed out by (3.22). Hence, by the price continuity argument, there would exist some time interval $(\bar{t}_Z - \delta, \bar{t}_Z)$ during which $\zeta\lambda_Z(t) < \underline{c}_s$. During such a time interval, the full marginal cost of clean coal would be higher than the energy price, a contradiction. ■

As a consequence, clean coal exploitation allows not only to delay the date at which the ceiling constraint begins to be effective, but also to relax this constraint once it begins to be effective.

The last phase of coal exploitation is the phase of exclusive dirty coal use that follows the phase at the ceiling. Since $\lambda_Z(t) = 0$ from (3.22), the dirty coal is necessarily less costly than the clean one and the production rate of the later must be nil, implying $u'(x_d(t)) = c_x + \lambda_{X0}e^{\rho t}$. Time differentiating this last expression and solving for $\dot{x}_d$, we get:

$$\dot{x}_d(t) = \frac{\rho\lambda_{X0}e^{\rho t}}{u''(x_d(t))} < 0 \tag{3.28}$$

Note that, since $c_x + \lambda_{X0}e^{\rho t} > u'(\bar{x}_d)$ along such a phase, then $x_d(t) < \bar{x}_d$ so that $Z(t) < \bar{Z}$.

We denote by $\bar{t}_c$ and $t_y$, respectively, the time at which the clean coal consumption ends and the time at which the solar energy becomes competitive. A typical optimal path of energy prices and full marginal costs is illustrated in Figure 1 when the coal endowment is sufficiently large to trigger the binding of the ceiling constraint.[8]

Initially, we have $\zeta\lambda_{Z0} < \underline{c}_s$ implying that only dirty coal is used. Since the marginal cost of emissions $\zeta\lambda_Z(t)$ grows at rate $(\rho+\alpha)$, there exists some time $\underline{t}_c$ at which $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = \underline{c}_s$. Then $\underline{t}_c$ corresponds to the beginning of a phase of simultaneous use of both types of coal although the ceiling is not reached yet. During this phase the consumption of clean coal increases while the consumption of dirty coal decreases. This phase is ending at time $\underline{t}_Z$ when the ceiling is attained and the consumption of dirty coal is precisely equal to $\bar{x}_d$. At this time, a new phase begins, which is still characterized by a simultaneous exploitation of the both types of coal, but now at the ceiling. During this phase, the consumption of

[8] A full analytical characterization of the optimal paths under CCS.1 is given in appendix A.1 for the cases of high and low solar costs.

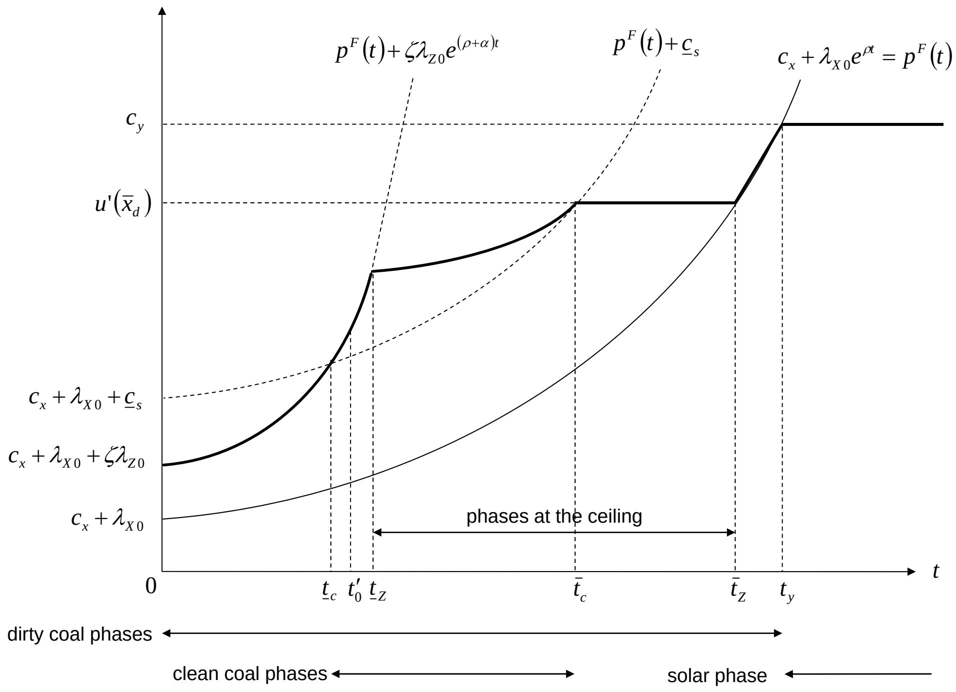


Figure 1: Optimal price path. Flow-dependent CCS average cost and high solar cost: $c_y > u'(\bar{x}_d)$

clean coal decreases while the consumption of dirty coal stays constant and equal to $\bar{x}_d$. The phase stops at time $\bar{t}_c$, when the consumption of clean coal falls to zero.

Note that during the two first phases, the price path is given by the same function $p^F(t)+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. The reason is that before the ceiling is attained, the unitary pollution tax must grow at the same proportional rate $\rho+\alpha$. But during the third phase, at the ceiling, $p(t) = u'(x_c(t)+x_d(t)) = p^F(t)+c_{ms}(x_c(t))$. We can write:

$$\lim_{t\uparrow\underline{t}_Z} \dot{p}(t) = \dot{p}^F(\underline{t}_Z) + \zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} > \dot{p}^F(\underline{t}_Z)$$

and, since from (3.27) $\dot{x}_c(t) < 0$ for any $t \in (\underline{t}_Z, \bar{t}_c)$, we also have:

$$\lim_{t\downarrow\underline{t}_Z} \dot{p}(t) = \dot{p}^F(\underline{t}_Z) + \lim_{t\downarrow\underline{t}_Z}\left[c'_{ms}(x_c(t))\dot{x}_c(t)\right] \leq \dot{p}^F(\underline{t}_Z)$$

Hence, as illustrated in Figure 1, the time derivative of the energy price, while increasing both before and after $\underline{t}_Z$, is discontinuous at $t=\underline{t}_Z$, its speed of growth being abruptly decelerated at this time.

The next phase is still a phase at the ceiling during which only the dirty coal is used at rate $\bar{x}_d$. The energy price is constant and equal to $u'(\bar{x}_d)$ and, from (3.14), $\lambda_Z(t) =$

$[u'(\bar{x}_d) - (c_x + \lambda_{X0} e^{\rho t})]/\zeta$ goes on to decrease as in the preceding phase. The phase ends at time $\bar{t}_Z$ when $\lambda_Z$ is nil.

During the following phase, $\lambda_Z = 0$ and the full marginal cost of the dirty coal is $p^F(t)$. The energy price increases up to that time $t_y$ at which the solar energy is becoming competitive: $p^F(t_y) = c_y$. At this time, the stock of coal must be exhausted. Then the solar energy time begins, forever.

The optimal consumption paths of the clean and dirty coals corresponding the price path described above, are illustrated in Figure 2. Although the total coal consumption is always either decreasing or constant, the clean coal consumption first increases, reaches an upper bound and next decreases down to zero. Moreover, clean coal use must begin before attaining the ceiling and must end before leaving it. This result is strongly linked with the increasing CCS marginal cost assumption and, as we shall see in the next section, it is no more valid for stock-dependent structures of marginal costs.

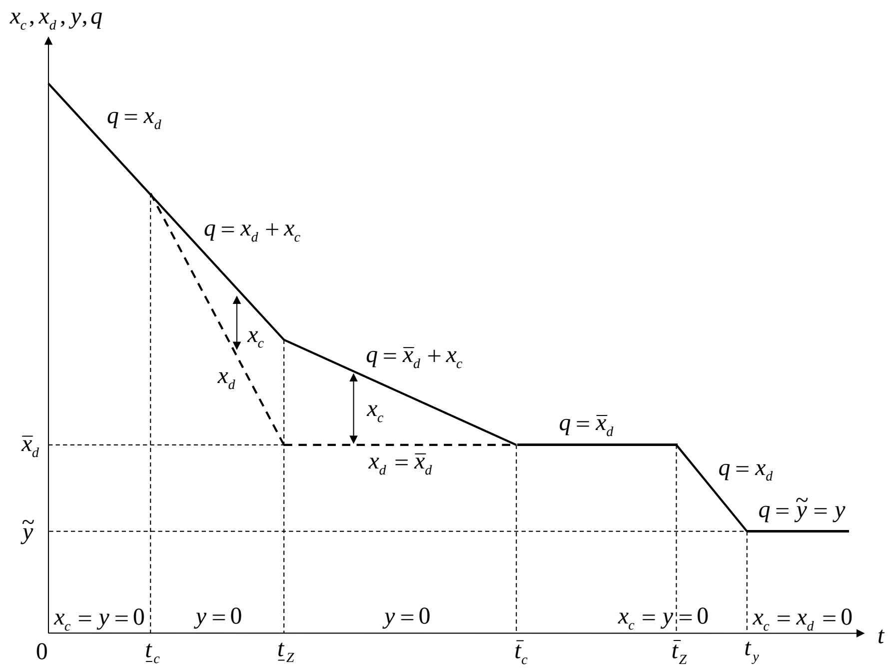


Figure 2: Optimal energy consumption paths. Flow-dependent CCS average cost and high solar cost: $c_y > u'(\bar{x}_d)$

Designing such an optimal path requires some evident necessary conditions. We must impose $c_x < u'(\bar{x}_d) < c_y$, a large enough coal initial endowment and a not too high initial average CCS cost $\underline{c}_s$. This last condition about the $\underline{c}_s$'s value is endogenous but can be more precisely explained by the following test. Assume that the clean coal option is not available and that initial coal endowments are large enough so that the ceiling constraint have to be active. Then the optimal price path is a path as the one illustrated in Figure 3, whose the main characteristics are similar to those underlined in Chakravorty et al. (2006).

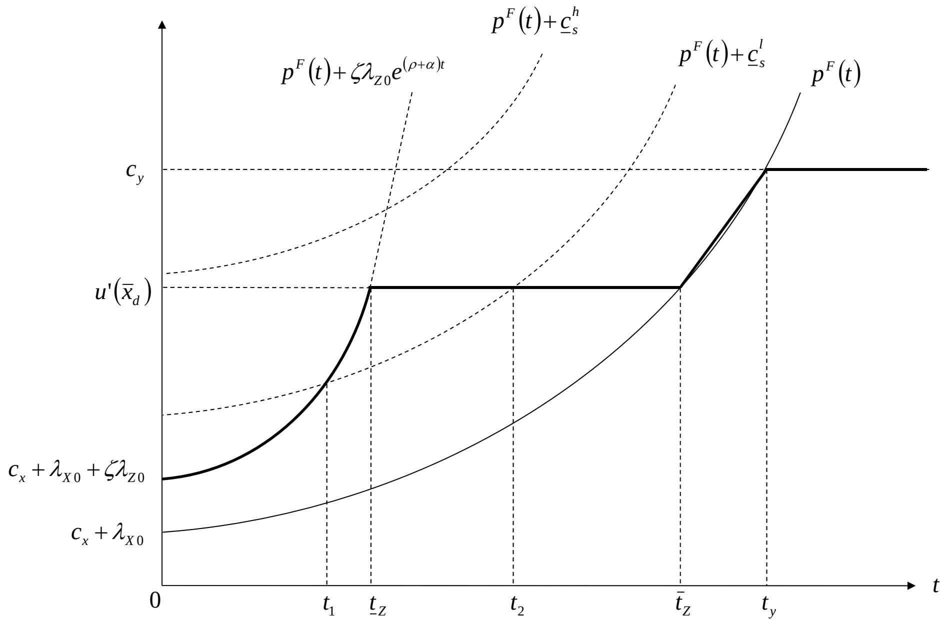


Figure 3: Optimal price path absent the clean coal option

Assume that $\underline{c}_s$ is very high so that the trajectory of $p^F(t)+\underline{c}_s^h$ (superscript $h$ for high) lies above the optimal price path which would be obtained in the absence of the clean coal option, as depicted in Figure 2. It is then clearly never optimal to use the clean coal since its full marginal cost is always higher than the full marginal cost of the dirty coal. On the contrary, if the additional sequestration cost is low enough, $\underline{c}_s^l$ ($l$ for low), then the full marginal cost of the clean coal would be lower than the full marginal cost of the dirty one over the time interval $(t_1, t_2)$ so that the policy consisting in producing energy without clean technology would reveal never optimal.

In the case where the initial atmospheric carbon concentration $Z^0$ is close to the critical level $\bar{Z}$, CCS appears to be an urgent action in the policy agenda and should be started immediately at time $t = 0$. However, there always exists an initial phase during which the pollution stock increases from its initial level to its critical level since $Z^0 < \bar{Z}$. Thus the optimal scenario is a five phases scenario in which the initial phase $[0, \underline{t}_c)$, as illustrated in Figure 1, disappears. The optimal path looks like the truncated path starting from $t_0'$, $\underline{t}_c < t_0' < \underline{t}_Z$, in Figure 1.

The optimal path as illustrated in Figure 1 is entirely characterized once the seven variables $\lambda_{X0}$, $\lambda_{Z0}$, $\underline{t}_c$, $\underline{t}_Z$, $\bar{t}_c$, $\bar{t}_Z$ and $t_y$ are determined. We detail in Appendix A.1.1 the seven-equation system these variables are solving, resulting in $\zeta\lambda_{Z0} < \underline{c}_s$. When the initial pollution stock is very large, only six parameters have to be determined since $\underline{t}_c$ vanishes, resulting in $\underline{c}_s < \zeta\lambda_{Z0}$.

### 3.2.2 The low solar cost case: $c_y < u'(\bar{x}_d)$

In the case of a low solar cost, $c_y < u'(\bar{x}_d)$, there may not exist any phase at the ceiling with the energy consumption provided by the dirty coal and the dirty coal only since the solar average cost is undercutting the price $u'(\bar{x}_d)$, which would have to prevail during such a phase. As compared with the high solar cost case, this rises the possibility to have two new types of phases at the ceiling during which solar energy is simultaneously used with either the two types of coal or only the dirty one.

Consider first the possibility of a simultaneous exploitation of the three primary energy sources during a phase at the ceiling. This implies that $p(t) = c_y = p^F(t) + c_{ms}(x_c(t))$, whose time differentiation leads to:

$$\dot{x}_c = -\frac{\dot{p}^F(t)}{c'_{ms}(x_c(t))} < 0 \tag{3.29}$$

where $\dot{p}^F(t) = \rho\lambda_{X0}e^{\rho t}$.

During such a phase, the clean coal consumption must decrease, the dirty coal consumption is constant and equal to $\bar{x}_d$ since this is a phase at the ceiling, and the total energy consumption is also constant since $p(t) = c_y$. Hence, during such a phase, the solar energy consumption must increase in such a way that it always balances the decrease in clean coal consumption: $\dot{y}(t) = -\dot{x}_c(t)$.

Next, consider a phase at the ceiling during which only dirty coal and solar energy are simultaneously used. Since this is a phase at the ceiling, then $x_d(t) = \bar{x}_d$. Since solar energy is used, then $p(t) = c_y$, hence $q(t) = \tilde{y}$ and $y(t) = \tilde{y} - \bar{x}_d$. The consumption paths of dirty coal and solar energy are both constant during such a phase.

A typical optimal price path is a six phases path as illustrated in Figure 4. The corresponding energy consumption paths are illustrated in Figure 5.

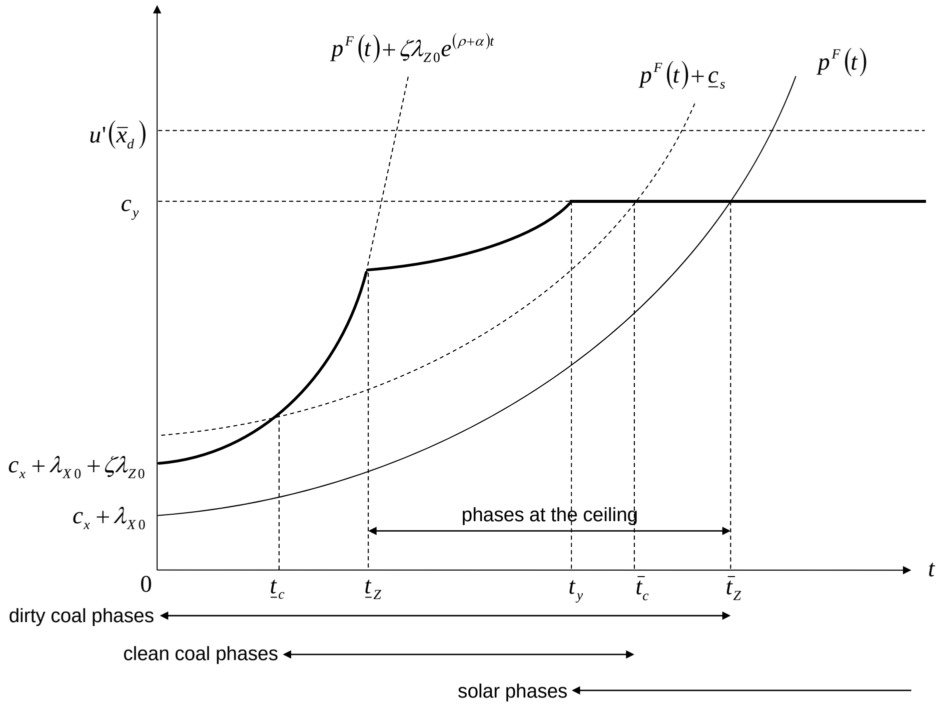


Figure 4: Optimal price path. Flow-dependent CCS average cost and low solar cost: $c_y < u'(\bar{x}_d)$

The three first phases of this optimal path are qualitatively the same as in the high solar cost case: First use dirty coal and only dirty coal, next exploit the both types of coal, that is begin the clean coal exploitation before attaining the ceiling, and third continue with this simultaneous use at the ceiling. From this step, the optimal path differs. Here, the third phase ends when the energy price reaches the marginal cost of solar energy $c_y$. Then begins phase $(t_y, \bar{t}_c)$ of simultaneous exploitation of the three types of energies – solar, clean and dirty coals – at the ceiling. The phase ends when $p^F(t) + \underline{c}_s = c_y$ so that clean coal is not competitive anymore as compared with solar energy. Since $\underline{c}_s > 0$, dirty coal remains competitive provided that its exploitation rate be maintained at $x_d(t) = \bar{x}_d$ in order to respect the ceiling constraint. Hence the next phase is a phase of simultaneous use of dirty coal and solar energy. This phase must end at $t = \bar{t}_Z$ when $p^F(t) = c_y$ or, equivalently, when $\lambda_Z(t) = 0$. At this time the coal stock must be exhausted. From $\bar{t}_Z$ onwards, solar energy is used alone and forever. Since there is no more pollution flow, the pollution stock $Z(t)$ starts to decrease and the ceiling constraint is no more active and forever.

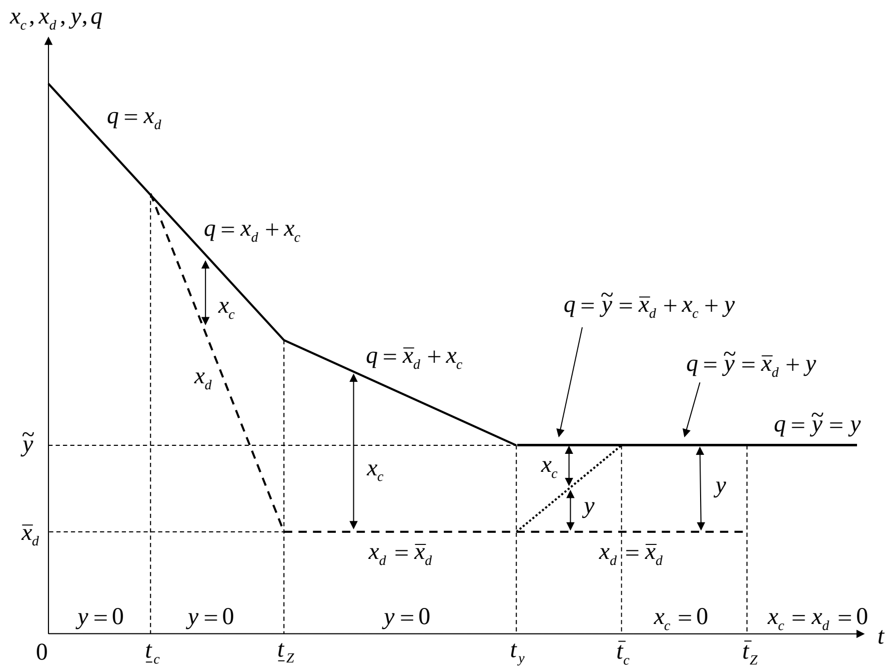


Figure 5: Optimal energy consumption paths. Flow-dependent CCS average cost and low solar cost: $c_y < u'(\bar{x}_d)$

The system of equations allowing to determine the endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\underline{t}_c$, $\underline{t}_Z$, $t_y$ and $\bar{t}_Z$ in the case of a low solar cost is detailed in Appendix A.1.2.

The main conclusion of this section is that, whatever the marginal cost of the solar clean substitute, either high or low provided that it is constant, assuming that the average abatement cost of the potential pollution flow is an increasing and convex function of the flow of abatement implies that abatement must be activated before the pollution stock constraint begins to bind. Moreover, in the case of low solar costs, the three types of resources – clean coal, dirty coal and solar energy – are simultaneously exploited during the second and the third phases of the period at the ceiling (the third and fourth phases of the scenarios).

As we shall see in the next section, such characteristics of the optimal paths can never be obtained with stock-dependent CCS average cost functions.

# 4 Stock-dependent CCS cost functions

Although giving rise to contrasted optimal paths according to whether the scarcity effect or the learning one dominates, the optimal paths generated by CCS stock-dependent cost functions have some strongly similar formal features. We first point out these similarities before focusing on the specificities induced by the dominance of each effect.

## 4.1 Problem formulation and preliminary remarks

Whatever the effect of clean coal cumulative production which is dominant, either the scarcity effect or the learning effect, the social planner problem has the same following general structure:

$$\max_{x_c,x_d,y} \int_0^\infty \{u(x_c(t)+x_d(t)+y(t)) - c_x[x_c(t)+x_d(t)] - c_s(S_c(t))x_c(t) - c_y y(t)\}\, e^{-\rho t} dt$$

subject to constraints (2.5), (2.7), (2.9), to the inequality constraints (2.6), (2.11) and (2.12), all common to the both cases, and to the constraint (2.10) for the case of a dominant scarcity effect. This last condition is the only one which is differentiating the two dominant effect sub-cases.

Let us denote by $\lambda_S$ the costate variable of $S_c$ and keep the notations of the previous section for the other costate variables, that is $\lambda_X$ for $X$ and $-\lambda_Z$ for $Z$. Then the current valued Hamiltonian of the program reads:

$$\mathcal{H} = u(x_c+x_d+y) - c_x(x_c+x_d) - c_s(S_c)x_c - c_y y - \lambda_X[x_c+x_d] - \lambda_Z[\zeta x_d - \alpha Z] + \lambda_S x_c$$

Also adopting the same notations for the Lagrange multipliers and denoting by $\nu_S$ the multiplier associated with constraint (2.10), the current valued Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \nu_X X + \nu_Z[\bar{Z} - Z] + \nu_S[\bar{S}_c - S_c] + \gamma_{x_c}x_c + \gamma_{x_d}x_d + \gamma_y y$$

with $\nu_S = 0$ for all $S_c \in [0, X^0]$ in the dominant learning effect case, a formal device to include the both CCS.2 and CCS.3 cases in a generic expression of the Lagrangian.

Among the first-order conditions (3.13)-(3.17) of the flow-dependent case, the condition (3.13) relative to the optimal use of $x_c$ must be replaced by:

$$u'(x_c+x_d+y) = c_x + \lambda_X + c_s(S_c) - \lambda_S - \gamma_{x_c} \tag{4.30}$$

A new condition relative to the dynamics of $\lambda_S$ must be introduced:

$$\dot{\lambda}_S = \rho\lambda_S + c_s'(S_c)x_c + \nu_S \tag{4.31}$$

together with the usual complementary slackness condition on $\nu_S$ . The associated transversality condition is:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_S(t)S_c(t) = 0 \tag{4.32}$$

The other first-order conditions (3.14)-(3.17) relative to the use of the other primary energies, $x_d$ and $y$, and to the dynamics of $\lambda_X$ and $\lambda_Z$ remain unchanged, as well as the transversality conditions (3.18) and (3.19) relative to the long run values of $X$ and $Z$.

Finally, note that from (4.31), as long as the clean coal has not yet been exploited, that is during an hypothetic initial phase of exclusive dirty coal consumption, we must have $\dot{\lambda}_S = \rho\lambda_S$, hence:

$$t \leq \underline{t}_c \;\Rightarrow\; \lambda_S(t) = \lambda_{S0}e^{\rho t}, \quad \text{where } \lambda_{S0} \equiv \lambda_S(0) \tag{4.33}$$

## 4.2 The case of a dominant scarcity effect

In the case of a dominant scarcity effect, the more the clean coal has been used in the past, the higher its present and future exploitation costs assuming that such exploitation is still possible, that is $S_c(t) < \bar{S}_c$. This suggests that $\lambda_S$ should be negative.

**Proposition 3** *Under a stock-dependent cost function CCS.2 with a dominant scarcity effect, assuming that the clean coal has to be exploited along the optimal path, the costate variable associated with the clean coal cumulated production is negative as long as its exploitation is not yet definitively closed:*

$$\forall t \geq 0: \quad \int_t^{\infty} x_c(\tau)d\tau > 0 \;\Rightarrow\; \lambda_S(t) < 0 \tag{4.34}$$

**Proof:** Solving the non-homogenous differential equation (4.31) results in:

$$\lambda_S(t) = \left\{\lambda_{S0} + \int_0^t [c_s'(S_c(\tau))x_c(\tau) + \nu_S(\tau)]e^{-\rho\tau}d\tau\right\}e^{\rho t} \tag{4.35}$$

where $\nu_S(t) \geq 0$. Next, using the transversality condition (4.32) and the condition $\lim_{t\uparrow\infty} S_c(t) \leq \bar{S}_c$ bounding $S_c(t)$ from above, we obtain the value of $\lambda_{S0}$:

$$\lambda_{S0} = -\int_0^{\infty} [c_s'(S_c(t))x_c(t) + \nu_S(t)]e^{-\rho t}dt$$

Substituting this value for $\lambda_{S0}$ in the above expression (4.35) of $\lambda_S(t)$, we finally get:

$$\lambda_S(t) = -\int_t^{\infty} [c_s'(S_c(\tau))x_c(\tau) + \nu_S(\tau)]e^{-\rho(\tau-t)}d\tau \tag{4.36}$$

which is negative under the qualifying assumption $\int_t^{\infty} x_c(\tau)d\tau > 0$ since $c_s'(S_c) > 0$ under CCS.2. ■

From (4.36), it should be clear that $\lambda_S(t)$ includes two components. Increasing at time $t$ the cumulated clean coal consumption by $x_c(t)$ units has two effects on the sum of the optimal future discounted[9] net surplus:

- first through the increase in the future sequestration costs by $c_s'(S_c(\tau))x_c(\tau)$, $\tau > t$;

- second through the tightening of the available capacity constraint restricting the size of the stock of carbon which could be stockpiled in the future, this second effect being captured by $\nu_S(\tau)$, $\tau > t$.

It remains to determine the behavior of $\lambda_S(t)$ once the qualifying condition (4.34) does not hold anymore, that is once the sequestration option is definitively closed, from time $t = \bar{t}_c$ onwards.

**Proposition 4** *Under a stock-dependent cleaning cost function with a dominant scarcity effect, once the sequestration is definitively closed:*

*- either the carbon reservoir capacity constraint is not binding at the closing time and then $\lambda_S(t) = 0$, more precisely:*

$$S_c(\bar{t}_c) < \bar{S}_c \;\Rightarrow\; \lambda_S(t) = 0, \quad t \geq \bar{t}_c \tag{4.37}$$

*- or the carbon stockpiling constraint is effective at the closing time and then:*

$$S_c(\bar{t}_c) = \bar{S}_c \;\Rightarrow\; \lambda_S(t) = -\int_t^{\infty} \nu_S(\tau)e^{-\rho(\tau-t)}d\tau, \quad t \geq \bar{t}_c \tag{4.38}$$

**Proof:** This result is an immediate implication of (4.36) which holds at any time. For all $t \geq \bar{t}_c$, $x_c = 0$. If first $S_c(\bar{t}_c) < \bar{S}_c$, then for all $t \geq \bar{t}_c$, $S_c(t) < \bar{S}_c$ hence $\nu_S(t) = 0$ and thus, from (4.36), $\lambda_S(t) = 0$. Second if $S_c(\bar{t}_c) = \bar{S}_c$ then $S_c(t) = \bar{S}_c$ for all $t \geq \bar{t}_c$ and, from (4.36) again, we get (4.38). ■

The important point is that even if sequestration is definitively closed , $\lambda_S(t)$ may be still strictly negative at least for some time. We shall come back soon on the meaning of

[9] Discounted in value at time $t$.

the analytical expression of $\lambda_S$ when the reservoir capacity constraint is tight at the closing date of the clean coal exploitation.

Since $\lambda_S(t) < 0$, at least as long as the sequestration is not definitively closed, then the full marginal cost of the clean coal amounts now to:

$$c_m^c(x_c(t)) = p^F(t) + c_s(S_c(t)) - \lambda_S(t) > p^F(t) + c_s(S_c(t)) \tag{4.39}$$

This suggests first that, along the optimal path, the clean coal exploitation cannot begin before having attained the pollution cap $\bar{Z}$ (Proposition 5) and, second, that if the clean coal has ever to be used, then its exploitation must be closed before the end of the period at the ceiling (Proposition 6).

**Proposition 5** *Under a stock-dependent CCS cost function with a dominant scarcity effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint is binding along the path, then its exploitation cannot begin before the ceiling constraint is binding, in brief:* $\underline{t}_c \geq \underline{t}_Z$.

**Proof:** Assume that the clean coal is exploited while the ceiling is not attained yet: $\underline{t}_c < \underline{t}_Z$. Then, either only the clean coal is used during the time interval $[\underline{t}_c, \underline{t}_Z]$, or there exists a subinterval $[t'_c, t'_Z]$, $\underline{t}_c \leq t'_c < t'_Z \leq \underline{t}_Z$, during which the both types of coal are exploited, or, last, there exists a subinterval $[t''_c, t''_Z]$, $\underline{t}_c \leq t''_c < t''_Z \leq \underline{t}_Z$, during which the clean coal and the solar energy are simultaneously exploited.

First, if only the clean coal is used during $[\underline{t}_c, \underline{t}_Z]$, then from $Z(\underline{t}_c) < \bar{Z}$ and $\dot{Z}(t) = -\alpha Z(t) < 0$ for $t \in [\underline{t}_c, \underline{t}_Z]$, we conclude that $Z(\underline{t}_Z) < \bar{Z}$, a contradiction.

Second, assume that the both types of coal are simultaneously exploited during $[t'_c, t'_Z]$. Then their full marginal costs must be equal. Since the ceiling is not attained yet, the dirty coal full marginal cost amounts to $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ while the clean coal full marginal cost amounts to $p^F(t) + c_s(S_c(t)) - \lambda_S(t)$, $\lambda_S(t) < 0$. Hence:

$$\lambda_S(t) = c_s(S(t)) - \zeta\lambda_{Z0}e^{(\rho+\alpha)t}, \quad t \in [t'_c, t'_Z] \tag{4.40}$$

Time differentiating the above equality leads to:

$$\dot{\lambda}_S(t) = c'_S(S(t))x_c(t) - \zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t}$$

Substituting the left-hand-side of (4.31) with $\nu_S = 0$ for $\dot{\lambda}_S(t)$, and simplifying, we obtain:

$$\rho\lambda_S(t) = -\zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t}$$

Last, substitute the right-hand-side of (4.40) for $\lambda_S(t)$ in the above equality and simplify to get:

$$0 < \rho c_s(S_c(t)) = -\alpha\zeta\lambda_{Z0}e^{(\rho+\alpha)t} < 0, \quad t \in [t'_c, t'_Z]$$

again a contradiction.

Last, we prove in Proposition 8 that clean coal and solar energy may never be simultaneously exploited during any time interval along the optimal path. ■

At this stage, we know that the clean coal exploitation cannot begin before the ceiling is reached. Proposition 6 below shows that it cannot either be introduced after the beginning of the ceiling period.

**Proposition 6** *Under a stock-dependent CCS cost function with a dominant scarcity effect, if clean coal has ever to be used along the optimal path, then its exploitation may not start after the beginning of the period at the ceiling:* $\underline{t}_c \leq \underline{t}_Z$.

**Proof:** Assume that $\underline{t}_Z \leq \underline{t}_c$, then during the time interval $[\underline{t}_Z, \underline{t}_c]$, either $y(t) = 0$ so that $x_d(t) = \bar{x}_d$, or $y(t) > 0$ and $y(t) + x_d(t) = y(t) + \bar{x}_d = \tilde{y}$, depending on wether $c_y \geq u'(\bar{x}_d)$ or $c_y < u'(\bar{x}_d)$, hence $p(t) = \min\{u'(\bar{x}_d), c_y\} \equiv \bar{p}$, $t \in [\underline{t}_Z, \underline{t}_c]$.

Since the clean coal is not competitive at $\underline{t}_Z$, its full marginal cost may not be lower than $\bar{p}$ at this time: $p^F(t)(\underline{t}_Z) + \underline{c}_s - \lambda_{S0}e^{\rho\underline{t}_Z} > \bar{p}$. Moreover, since $p^F(t)$ is increasing and $\lambda_{S0}$ is negative, we have: $p^F(t)(t) + \underline{c}_s - \lambda_{S0}e^{\rho t} > \bar{p}$, $\forall t \in [\underline{t}_Z, \underline{t}_c]$, so that the clean coal consumption cannot become competitive at $\underline{t}_c$, hence a contradiction. ■

Thus from Propositions 5 and 6 we conclude that the exploitation of the clean coal must begin when the ceiling is attained: $\underline{t}_c = \underline{t}_Z$. The following Proposition 7 shows that its exploitation must be closed before the end of the ceiling period.

**Proposition 7** *Under a stock-dependent CCS cost function with a dominant scarcity effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint be binding along the path, then its exploitation must be closed before the end of the period at the ceiling.*

**Proof:** Assume that at the end of the period at the ceiling, the both types of coal are simultaneously used, that is $x_c(\bar{t}_Z) > 0$ and $x_d(\bar{t}_Z) > 0$. At this date, we know from (3.22) that the shadow marginal cost of the pollution stock must be nil: $\lambda_Z(\bar{t}_Z) = 0$. Then the dirty coal full marginal cost amounts to $p^F(\bar{t}_Z)$ while the clean coal full marginal cost amounts to $p^F(\bar{t}_Z) + c_s(S(\bar{t}_Z)) - \lambda_S(\bar{t}_Z) > p^F(\bar{t}_Z)$. Since the marginal cost of the clean coal is larger than the cost of the dirty one, only the dirty one has to be used, hence a contradiction. ■

Last, Proposition 8 will permit, together with the above propositions, to fully characterize the optimal path provided that the ceiling constraint has to be effective. It shows that the clean coal and the solar energy may never be simultaneously exploited.

**Proposition 8** *Under a stock-dependent CCS.2 cost function with a dominant scarcity effect, the clean coal and the solar energy may never be exploited simultaneously along the optimal path.*

**Proof:** Let us assume that clean coal and solar energy are simultaneously used over some time interval. Their full marginal costs must be equal, that is: $c_y = c_x + \lambda_{X0}e^{\rho t} + c_s(S(t)) - \lambda_S(t)$. Time differentiating, substituting the RHS of (4.31) (with $\nu_S = 0$ since $S_c(t) < \bar{S}_c$) and simplifying, we get:

$$0 < \lambda_{X0}e^{\rho t} = \lambda_S(t) < 0$$

the RHS of this inequality directly coming from Proposition 3, hence a contradiction. ■

The Propositions 5, 6, 7 and 8 have different implications depending upon wether the cost of the solar energy is high or low.

### 4.2.1 The high solar cost case: $c_y > u'(\bar{x}_d)$

In this case, we may conclude from the above Propositions 5-8 that, if the ceiling constraint has to be effective and if the clean coal has to be exploited, then the period at the ceiling contains two phases, the first one being a phase during which the both types of coal are used and the second one a phase during which only the dirty coal is exploited. This is due to the fact that, at a price $c_y$ even if only the dirty coal were exploited then $x_d$ would be smaller than $\bar{x}_d$ hence the ceiling constraint could not be active.

A typical optimal path is a five-phases path as illustrated in Figure 6 for the energy price and in Figure 7 for the energy consumptions.[10]

[10] A full analytical characterization of the optimal path under CCS.2 is given in Appendix A.2 for the both cases of high and low solar costs.

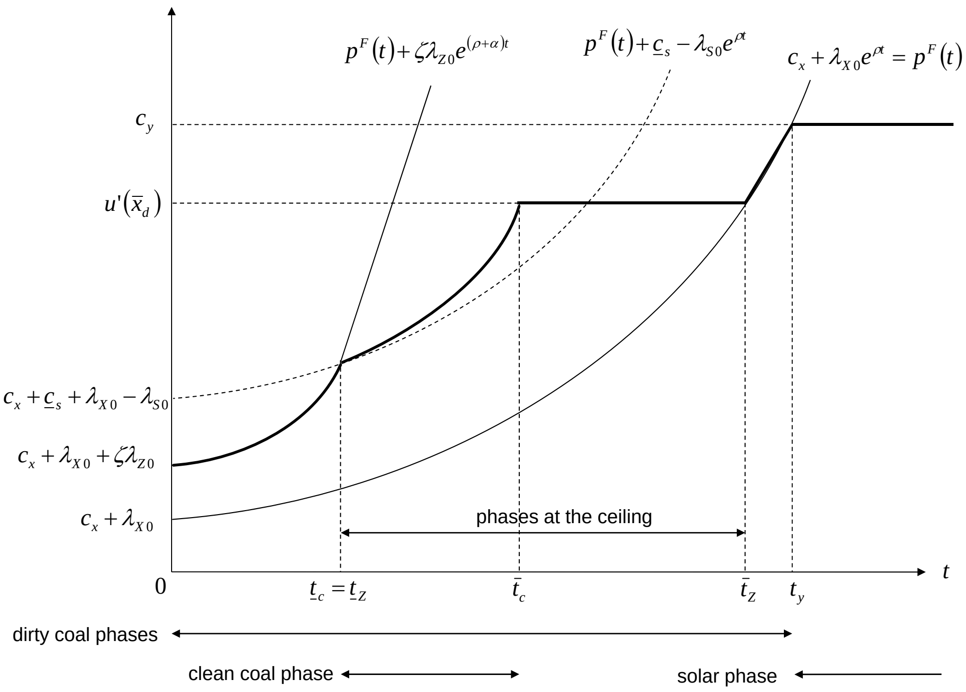


Figure 6: Optimal price path under stock-dependent CCS average costs, with a dominant scarcity effect. The high solar cost case: $c_y > u'(\bar{x}_d)$

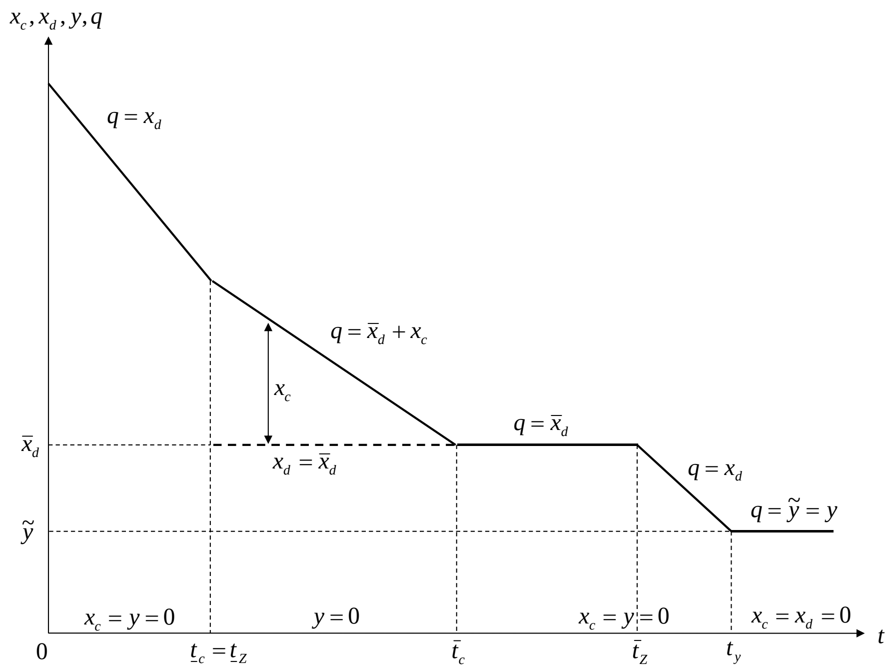


Figure 7: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant scarcity effect. The high solar cost case: $c_y > u'(\bar{x}_d)$

The first phase is a dirty coal phase during which the energy price is equal to $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. Since only the dirty coal is exploited, its full marginal cost must be lower than the full marginal cost of the clean one, that is:

$$p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} < p^F(t) + \underline{c}_s - \lambda_{S0}e^{\rho t}$$

Since $\lambda_Z(t)$ is growing at a higher proportional rate than $-\lambda_S(t)$, there exists some time $t = \underline{t}_c$ at which the both prices are equal. From Proposition 5, the ceiling constraint must begin to bind at this time, that is $\underline{t}_c = \underline{t}_Z$.

The second phase is a phase at the ceiling, the both types of coal being simultaneously used. During such a phase, the dirty coal production amounts to $x_d(t) = \bar{x}_d$. From the first-order-condition (4.30), the clean coal production must be such that $u'(x_c(t) + \bar{x}_d) = p^F(t) + c_s(S(t)) - \lambda_S(t)$. Time differentiating this expression and substituting the RHS of (4.31) for $\dot{\lambda}_S$ (with $\nu_S = 0$ since $S_c(t) < \bar{S}_c$), results in:

$$\dot{x}_c(t) = \frac{\rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)]}{u''(x_c(t) + \bar{x}_d)} < 0 \tag{4.41}$$

Clean coal consumption decreases during the phase. Since this consumption is nil during the preceding phase, such a result is possible if and only if the clean coal consumption jumps upwards at the beginning of the second phase, that is at time $t = \underline{t}_Z = \underline{t}_c$. Moreover, this upward jump must be balanced by a downward jump of the same magnitude in the dirty coal consumption trajectory to preserve the continuity of the price path, as illustrated in Figure 6. Such discontinuities can arise thanks to the assumptions of constant full marginal cost of both the clean and the dirty coals at any time, which is the main difference between the stock-dependent CCS cost structure of the present section, and the flow-dependent structure of the previous section.

Another important remark which must be pointed out is that, during this phase of simultaneous exploitation of the both types of coal, we have:

$$\dot{p}(t) = \frac{d}{dt}\left[p^F(t) + c_s(S(t)) - \lambda_S(t)\right] = \dot{p}^F(t) - \rho\lambda_S(t) > \dot{p}^F(t) \tag{4.42}$$

Moreover, since the energy price $p(t)$ equals $p^F(t) + \zeta\lambda_Z(t)$ from the first-order condition (3.14) relative to the dirty coal use, then $p^F(t) + \zeta\lambda_Z(t) = p^F(t) + c_s(S(t)) - \lambda_S(t)$, and from (4.42):

$$\dot{p}^F(t) - \rho\lambda_S(t) = \dot{p}^F(t) + \zeta\dot{\lambda}_Z(t) > \dot{p}^F(t) \Rightarrow \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) > 0 \tag{4.43}$$

However the instantaneous proportional growth rate of $\lambda_Z$ is now lower than $\rho+\alpha$ because the ceiling constraint is tight, hence $\nu_Z(t)>0$ (see (3.17)). Thus during this phase at the ceiling, the marginal social cost of the atmospheric carbon stock is growing as illustrated in Figure 6. However, the proportional growth rate of $\lambda_Z$ is lower at the beginning of this phase than at the end of the preceding one, so that $\lim_{t\uparrow \underline{t}_Z} \dot{p}(t) > \lim_{t\downarrow \underline{t}_Z} \dot{p}(t)$, as in the case of flow-dependent cost function when the ceiling is reached.

This second phase ends at time $t=\bar{t}_c$ when the energy price attains the level $u'(\bar{x}_d)$ and, simultaneously, the consumption of clean coal falls down to zero since $x_d(\bar{t}_c)=\bar{x}_d$.

The third phase is a phase at the ceiling during which only the dirty coal is used: $x_d(t)=\bar{x}_d$, $x_c(t)=0$. During this phase, $\lambda_Z(t)=u'(\bar{x}_d)-p^F(t)$ hence $\dot{\lambda}_Z(t)=-\rho\lambda_{X0}e^{\rho t}<0$. The marginal social cost of the pollution stock is now decreasing. The phase ends at the time $t=\bar{t}_Z$ when $\lambda_Z$ is nil.

From $\bar{t}_Z$ onwards, $\lambda_Z$ is always nil and the next phase is the standard Hotelling phase of exclusive exploitation of the dirty coal up to that time $t=t_y$ at which the increasing energy price attains the level $c_y$ allowing the solar energy to be a competitive substitute of the dirty coal and, simultaneously, the stock of coal is exhausted.

Note that, in this case, $\underline{t}_c=\underline{t}_Z$. Let us denote by $\underline{t}$ this common date: $\underline{t}\equiv\underline{t}_Z=\underline{t}_c$. Thus we have again seven endogenous variables to determine, as in the flow-dependent CCS cost case, but with one date missing and one more initial costate variable: $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$, $\bar{t}_c$, $\bar{t}_Z$ and $t_y$. The seven equations system they are solving is detailed in Appendix A.2.1.

*The value of $\lambda_S$ after the end of the sequestration phase:*

As pointed out in Proposition 4, when the stockpiling constraint is effective at the end of the sequestration phase, $\lambda_S(t)$ may then be still strictly negative for some time after the closing time of the clean coal exploitation. But how much time? It is clear that any additional stockpiling capacity which would be available only after $\bar{t}_Z$ would be worthless since the pollution ceiling constraint is not binding anymore from $\bar{t}_Z$ onwards. Let us show that the time period during which an additional stockpiling capacity would be exploited if it was available is shorter than $\bar{t}_Z-\bar{t}_c$.

Since we assume that the average CCS cost function is increasing in $S_c$, the reservoir capacity impacts the optimal scenarios by stopping the availability of stockpiling capacities at an average cost which is at least equal to $c_s(\bar{S}_c)$. The logic of the model would be to assume that any additional capacity $\Delta\bar{S}_c$ could be exploited at an average CCS cost $c_s(S_c)$ which is increasing over the interval $(\bar{S}_c, \bar{S}_c + \Delta\bar{S}_c)$. Over $[0, \bar{S}_c + \Delta\bar{S}_c]$, $c_s(S_c)$ should have the same general properties than over $[0, \bar{S}_c]$. However, in order to show that the time interval during which such an additional capacity has some value is shorter than $\bar{t}_Z - \bar{t}_c$, it is sufficient to show that this is the case even if the average CCS cost is the lowest one, that is equal to $c_s(\bar{S}_c)$.

From (3.14) and (4.30), the time $\tilde{t}$ at which the full marginal costs of the both types of coal would be equal while $\lambda_S(t) = 0$, is given as the solution of:

$$c_s(\bar{S}_c) = \zeta\lambda_Z(t)$$

From (3.14), since $u'(q(t)) = u'(\bar{x}_d)$ over the time interval $[\bar{t}_c, \bar{t}_Z]$, we have:

$$\zeta\lambda_Z(t) = u'(\bar{x}_d) - (c_x + \lambda_{X0}e^{\rho t}), \quad t \in [\bar{t}_c, \bar{t}_Z]$$

together with $\zeta\lambda_Z(\bar{t}_c) = c_s(\bar{S}_c) - \lambda_S(\bar{t}_c) > c_s(\bar{S}_c)$ and $\zeta\lambda_Z(\bar{t}_Z) = 0$. Thus there exists a unique time $\tilde{t}$: $\bar{t}_c < \tilde{t} < \bar{t}_Z$, at which $\zeta\lambda_Z(\tilde{t}) = c_s(\bar{S}_c)$ and from which any additional reservoir capacity is worthless.

### 4.2.2 The low solar cost case: $c_y < u'(\bar{x}_d)$

As in the case of flow-dependent costs, and for the same reasons, there may not exist a phase at the ceiling during which the dirty coal and only the dirty coal is exploited. Assuming that such a phase could exist, the energy price would have to be equal to $u'(\bar{x}_d)$, a price higher than the solar energy average cost $c_y$ meaning that this alternative energy primary source should have to be exploited, thus a contradiction.

We know from Proposition 5 that if clean coal has to be used, it may not be before the pollution cap $\bar{Z}$ is reached and, from Proposition 7, that clean coal and solar energy may never be exploited simultaneously. Furthermore from Proposition 6, the clean coal exploitation must be closed before the end of the period at the ceiling. Thus if clean coal has to be used and the ceiling constraint has to be active along the optimal path, then the only possible period at the ceiling is a two-phases period. During the first one, the both clean and dirty coals are simultaneously exploited and during the second period, both the dirty coal and the solar energy. Typical paths – four-phases paths in the current case – of energy price and the associated energy consumptions are illustrated in Figures 8 and 9 respectively.

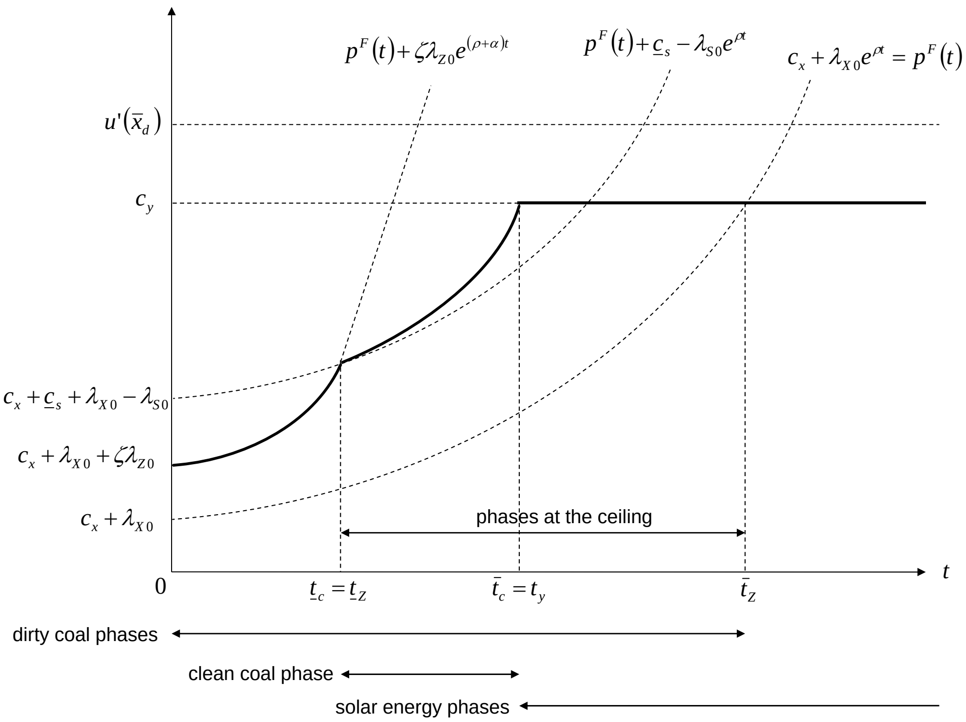


Figure 8: Optimal energy price path under stock-dependent CCS average costs, with a dominant scarcity effect. The low solar cost case: $c_y < u'(\bar{x}_d)$

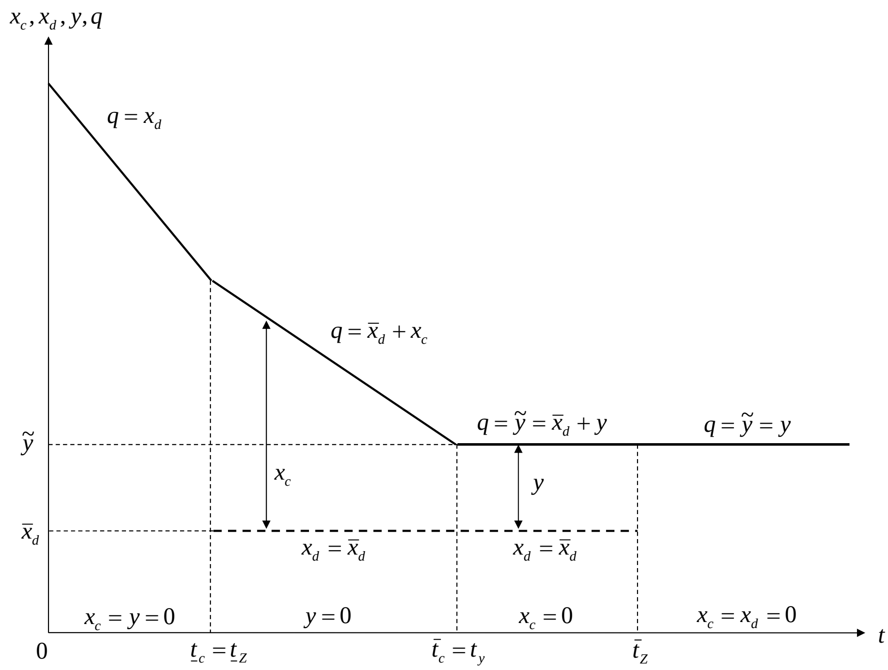


Figure 9: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant scarcity effect. The low solar cost case: $c_y < u'(\bar{x}_d)$

The two first phases are similar to the two first phases of the high solar cost case. The first phase is the usual phase of exclusive use of the dirty coal during which the atmospheric carbon stock grows up to the time $\underline{t}_Z$ at which the carbon cap is attained.

At time $\underline{t}_Z$, the clean coal becomes competitive, $\underline{t}_Z = \underline{t}_c$, and the resulting second phase is a phase of joint exploitation of the two types of coal while at the ceiling: $x_d(t) = \bar{x}_d$ and $x_c(t)$ is decreasing according to (4.41). Thus at time $t = \underline{t}_Z$, the dirty coal consumption is instantaneously reduced and this downward jump must be balanced by an upward jump of the same magnitude in the clean coal consumption. As in the high solar cost case during this phase:

$$\frac{d}{dt} = \left[p^F(t) + c_s(S(t)) - \lambda_S(t)\right] > \dot{p}^F(t) \quad \text{and} \quad \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) > 0$$

The argument is the same as the argument leading to expressions (4.42) and (4.43). The main difference with the high solar cost case is that now, the phase ends when the energy price is equal to $c_y$. At this point, the phases of competitiveness of the solar energy begin.

Just before this time $t_y$, since $c_y < u'(\bar{x}_d)$ and $x_d(t_y) = \bar{x}_d$, then $x_c(t) = \tilde{y} - \bar{x}_d > 0$. However, since the solar energy is competitive just after $t_y$ and, from Proposition 7, both clean coal and solar energy may not be simultaneously used, hence the exploitation of the clean coal must be closed so that $t_y = \bar{t}_c$. Thus the clean coal consumption falls from $\tilde{y} - \bar{x}_d$ down to 0 and the production of the solar energy jumps from 0 up to $\tilde{y} - \bar{x}_d$ to keep the continuity of the energy services consumption path. During this third phase, the production of dirty coal and solar energy are both constant, $x_d(t) = \bar{x}_d$ and $y(t) = \tilde{y} - \bar{x}_d$, while the pollution stock remains at the ceiling level $Z(t) = \bar{Z}$. The associated shadow cost declines: $\lambda_Z(t) = (c_y - c_x - \lambda_{X0}e^{\rho t})/\zeta$. The phase ends at time $t = \bar{t}_Z$ when $\lambda_Z$ has been reduced to 0, that is when $p^F(t) = c_y$. The exploitation of the dirty coal must be closed and simultaneously, the stock of coal must be exhausted.

The last phase from $\bar{t}_Z$ onwards is a phase of exclusive solar energy consumption, $q(t) = y(t) = \tilde{y}$. Then the pollution stock is gradually eliminated by natural absorption, $Z(t) = Z(\bar{t}_Z)e^{-\alpha(t-\bar{t}_Z)} = \bar{Z}e^{-\alpha(t-\bar{t}_Z)} < \bar{Z}$, $t \geq \bar{t}_Z$.

Note that in this low solar cost case, we have not only $\underline{t}_c = \underline{t}_Z (\equiv \underline{t})$, but also $\bar{t}_c = t_y$. Let us denote by $\hat{t}$ this other common date. Hence, only six variables have to be determined now: $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$, $\hat{t}$ and $\bar{t}_Z$. The system of six equations that they solve is exposed in Appendix A.2.2.

*The value of $\lambda_S$ after the end of the clean coal exploitation phase:*

Here again, $\lambda_S$ may be strictly negative over some time interval $(\bar{t}_c, \tilde{t})$, $\bar{t}_c = \underline{t}_Z < \tilde{t} < \bar{t}_Z$, occurring at the end of the clean coal exploitation phase when the carbon capture policy is restricted by the reservoir capacity. The argument runs along the same lines than the argument developed in the high solar cost case, but during the phase $[\bar{t}_c, \bar{t}_Z]$, the $\lambda_Z$-path is now established from $c_y$ instead of $u'(\bar{x}_d)$ since the energy price path is determined by $c_y$ during this time interval. More precisely, we have:

$$\zeta\lambda_Z(t) = c_y - (c_x + \lambda_{X0}e^{\rho t}), \quad t \in [\bar{t}_c, \bar{t}_Z]$$

together with $\zeta\lambda_Z(\bar{t}_c) = c_s(\bar{S}_c) - \lambda_S(\bar{t}_c) > c_s(\bar{S}_c)$ and $\zeta\lambda_Z(\bar{t}_Z) = 0$. Hence there exists a unique time $t = \tilde{t}$ solving $\zeta\lambda_Z(t) = c_s(\bar{S}_c)$ and defining the date from which $\lambda_S$ is nil forever.

## 4.3 The case of a dominant learning effect

Now, the more the clean coal has been used in the past, the lower its marginal additional cost as compared with the dirty coal. This suggests that $\lambda_S$ should be positive up to the time at which its exploitation is definitively closed.

**Proposition 9** *Under a stock-dependent CCS cost function with a dominant learning effect, assuming that the clean coal has to be exploited along the optimal path, the costate variable associated with the clean coal cumulated production is positive as long as its exploitation is not definitively closed:*

$$\forall t \geq 0 : \quad \int_t^\infty x_c(\tau)d\tau > 0 \Rightarrow \lambda_S(t) > 0 \tag{4.44}$$

**Proof:** This is a direct implication of (4.36) with $\nu_S = 0$ and $c_s' < 0$:

$$\lambda_S(t) = -\int_t^\infty c_s'(S_c(\tau))x_c(\tau)e^{-\rho(\tau-t)}d\tau > 0 \quad \blacksquare \tag{4.45}$$

Integrating by parts (4.45) we get the following alternative expression of $\lambda_S(t)$ which will turn out to be useful in the proof of Propositions 10, 11 and 12:

$$\lambda_S(t) = c_s(S_c(t)) - \rho\int_t^\infty c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau \tag{4.46}$$

Note that in the present case, once the exploitation of the clean coal is definitively closed, then $\lambda_S$ is nil:

$$\forall t \geq \bar{t}_c : \quad \lambda_S(t) = 0 \tag{4.47}$$

The following Propositions 10 and 11 show that, as in the case of a dominant scarcity effect, the exploitation of the clean coal cannot begin before the ceiling constraint is binding and must be closed before the end of the ceiling period in the case of a learning effect. However, as we shall see, it may happen that the optimal clean coal exploitation has to begin after the time at which the ceiling is attained. Under a dominant learning effect, the equivalent of Proposition 7 obtained under a dominant scarcity effect does not hold anymore.

**Proposition 10** *Under a stock-dependent CCS cost function with a dominant learning effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint be active along the path, then its exploitation may not begin before the ceiling constraint is binding:* $\underline{t}_c \geq \underline{t}_Z$.

**Proof:** The proof runs along the lines of the proof of Proposition 5, but some details of the arguments must be adapted. Assume that $\underline{t}_c < \underline{t}_Z$. First, if during the time interval $[\underline{t}_c, \underline{t}_Z]$ only the clean coal is used, then the argument is the same.

Second, assume that both the dirty and clean coals are exploited during some interval $[t'_c, t'_Z]$. Equating their respective full marginal costs results in:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)t} = c_s(S_c(t)) - \lambda_S(t), \quad t \in (t'_c, t'_Z)$$

Substituting the R.H.S. of (4.46) for $\lambda_S(t)$, we get:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)t} = \rho \int_t^{\infty} c_s(S_c(\tau)) e^{-\rho(\tau-t)} d\tau \tag{4.48}$$

Time differentiate to obtain:

$$\zeta(\rho+\alpha)\lambda_{Z0} e^{(\rho+\alpha)t} = -\rho c_s(S_c(t)) + \rho^2 \int_t^{\infty} c_s(S_c(\tau)) e^{-\rho(\tau-t)} d\tau$$

that is, taking (4.48) into account:

$$0 < \zeta \alpha \lambda_{Z0} e^{(\rho+\alpha)t} = -\rho c_s(S_c(t)) < 0, \quad t \in [t'_c, t'_Z]$$

hence a contradiction.

Last we show in Proposition 12 that clean coal and solar energy may never be exploited simultaneously. ■

**Proposition 11** *Under a stock-dependent CCS cost function with a dominant learning effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint be active along the path, then its exploitation must be closed before the end of the ceiling period.*

**Proof:** Assume that at $\bar{t}_Z$, the ending time of the ceiling period, the both types of coal are still used, that is $x_c(\bar{t}_Z) > 0$ and $x_d(\bar{t}_Z) = \bar{x}_d$. Equating their full marginal costs and taking into account that $\lambda_Z(\bar{t}_Z) = 0$ from (3.22), we get:

$$p^F(\bar{t}_Z) = p^F(\bar{t}_Z) + c_s(S_c(\bar{t}_Z)) - \lambda_S(\bar{t}_Z)$$

Substituting the R.H.S. of (4.46) for $\lambda_S(\bar{t}_Z)$ results in:

$$p^F(\bar{t}_Z) = p^F(\bar{t}_Z) + \rho \int_{\bar{t}_Z}^{\infty} c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau > p^F(\bar{t}_Z)$$

a contradiction. ■

The last common feature of the optimal paths for the both cases of scarcity and learning dominant effects stands in the impossibility of using simultaneously the clean coal and the solar energy. Here again, the proof has to be adapted from Proposition 8.

**Proposition 12** *Under a stock-dependent CCS cost function with a dominant learning effect, the clean coal and the solar energy may never be exploited simultaneously along the optimal path.*

**Proof:** Assume that the clean coal and the solar energy are simultaneously used during some interval $[t_1, t_2]$. Equating their full marginal costs results in:

$$c_y = c_x + \lambda_{X0}e^{\rho t} + c_s(S_c(t)) - \lambda_S(t), \quad t \in [t_1, t_2]$$

Substituting the R.H.S. of (4.46) for $\lambda_S(t)$, we get:

$$c_y - c_x = \lambda_{X0}e^{\rho t} + \rho \int_t^{\infty} c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau \tag{4.49}$$

Time differentiating, we obtain:

$$0 = \rho[\lambda_{X0}e^{\rho t} - c_s(S_c(t))] + \rho^2 \int_t^{\infty} c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau$$

and taking (4.49) into account:

$$0 = \rho[c_y - c_x] - \rho c_s(S_c(t))$$

Time differentiating again, we finally get:

$$0 = -\rho c_s'(S_c(t))x_c(t) > 0, \quad t \in [t_1, t_2]$$

a contradiction ■

Having reviewed the common features of the optimal paths in the cases of scarcity and learning dominant effects, let us turn now to their differences.

From Propositions 10, 11 and 12, the only kind of phases during which the clean coal is used is a phase of joint exploitation of the both types of coal while at the ceiling. Thus if the scarcity and learning dominant effects have different implications, and they should have at least in some cases, this may be because:

- either what happens during this kind of phase is different in the two cases,

- or the position of this phase within the optimal sequence of phases is different in the two cases,

- or the both.

Let us examine first the reasons for which what happens within this kind of phase could be different. During such a phase, $q(t) = x_c(t) + \bar{x}_d$, $t \in [\underline{t}_c, \bar{t}_c]$, and the time derivative of $x_c$ is given formally by (see (4.41)):

$$\dot{x}_c(t) = \frac{\rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)]}{u''(x_c(t) + \bar{x}_d)} \tag{4.50}$$

the difference with (4.41) being that we cannot conclude here about the sign of $\dot{x}_c(t)$ since $\lambda_S(t) > 0$. However, we can show that $x_c(t)$, hence $p(t)$, may follow two types of trajectories and only two during the phase.

First remark that, from (4.47), $\lambda_S(t)$ is tending to 0 at the end of the phase. Thus, since $\lambda_S(t)$ is necessarily continuous in such a model, there must exist some terminal interval $[\bar{t}_c - \Delta, \bar{t}_c]$, $0 < \Delta \leq \bar{t}_c - \underline{t}_c$, during which $\dot{x}_c(t)$ is negative and the energy price is increasing. We have now to determine what could happen at the beginning of the phase when this terminal interval is strictly shorter than the entire phase, that is when $\Delta < \bar{t}_c - \underline{t}_c$.

The following Proposition 13 states that the sign of $\dot{x}_c(t)$ may change at most only once within the phase.

**Proposition 13** *Under a stock-dependent CCS cost function with a dominant learning effect, assuming that there exists a phase during which the both types of coal are exploited while at the ceiling, then during such a phase:*

*- either the price of the energy services is monotonically increasing,*

*- or the price of the energy services is first decreasing and next increasing.*

**Proof:** Assume that $\lim_{t \downarrow \underline{t}_c} \dot{x}_c(t) > 0$. Define $t_0$ as the first date at which $\dot{x}_c(t)$ alternates in sign, since in this case the sign is changing at least once:

$$t_0 = \inf \{t : \dot{x}_c(t) \leq 0, t \in [\underline{t}_c, \bar{t}_c)\} \Rightarrow \dot{x}_c(t_0) = 0$$

From (4.30) and (4.31) respectively, we get:

$$\begin{aligned} u'(x_c(t) + \bar{x}_d) &= c_x + \lambda_{X0} e^{\rho t} + c_s(S_c(t)) - \lambda_S(t) \\ \dot{\lambda}_S(t) &= \rho \lambda_S(t) + c_s'(S_c(t)) x_c(t) = \rho \lambda_S(t) + \dot{c}_s(S_c(t)) \end{aligned}$$

with $\dot{c}_s(S_c(t)) < 0$. Time differentiating the first expression and using the second one, we get:

$$u''(x_c(t) + \bar{x}_d)\dot{x}_c(t) = \rho[\lambda_{X0} e^{\rho t} - \lambda_S(t)]$$

Define $\phi(t) = \lambda_{X0} e^{\rho t} - \lambda_S(t)$. Then $u'' < 0$ implies that:

$$\dot{x}_c(t) > / = / < 0 \Leftrightarrow \phi(t) < / = / > 0$$

Time differentiating $\phi(t)$ and using (4.31), we get:

$$\dot{\phi}(t) = \rho \lambda_{X0} e^{\rho t} - \rho \lambda_S(t) - \dot{c}_s(S_c(t)) = \rho \phi(t) - \dot{c}_s(S_c(t))$$

Integrating over $[t_0, t]$, $t_0 < t \leq \bar{t}_c$, and taking into account that $\phi(t_0) = 0$, we obtain:

$$\phi(t) = -e^{\rho t} \int_{t_0}^{t} \dot{c}_s(S_c(\tau)) e^{-\rho \tau} d\tau > 0, \quad t \in (t_0, \bar{t}_c]$$

We conclude that, if the sign of $\dot{\phi}(t)$, hence the sign of $\dot{x}_c(t)$ and $\dot{p}(t)$, is changing over $[\underline{t}_c, \bar{t}_c)$, it is only once. ■

The last common characteristics shared by all the paths is about their behavior during the pre-ceiling phase, hence also before the beginning of the clean coal exploitation according to Proposition 10, that is over the time interval $[0, \underline{t}_Z] \subseteq [0, \underline{t}_c]$. During this initial phase, from (4.35), the shadow full marginal cost of the clean coal amounts to:

$$c_m^c = c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0}) e^{\rho t}$$

which may be either increasing or decreasing depending on whether the shadow marginal cost of coal $\lambda_{X0}$ is larger or smaller than the shadow marginal value of the cumulated experience in cleaning some part of its available stock, $\lambda_{S0}$. Such a formulation could prove to be paradoxical since no experience has been yet accumulated. But this is the marginal value of a zero-experience and this marginal value may be very high.

The sign of $\lambda_{X0} - \lambda_{S0}$, which is endogenous, determines the position of the phase of simultaneous exploitation of the both types of coal in the optimal sequence of phases. However, as in the case of a dominant scarcity effect, the types of optimal sequences are depending upon whether the solar energy cost is high or low.

### 4.3.1 The high solar cost case: $c_y > u'(\bar{x}_d)$

We examine the different possible types of paths according to the sign of $\lambda_{X0} - \lambda_{S0}$.

**- Case where $\lambda_{X0} > \lambda_{S0}$**

In this case, the shadow marginal value of the experience is relatively low as compared with the coal scarcity rent and the structure of the optimal path is strongly determined by the dominance of this scarcity effect.

Since $\lambda_{X0} > \lambda_{S0}$ and provided that there exists a phase of joint use of the both types of coal while at the ceiling, the clean coal exploitation must precisely begin at the time at which the pollution cap $\bar{Z}$ is reached. The argument is given by Figure 10. At the crossing point of the trajectories $p^F(t) + \bar{c}_s - \lambda_{S0}e^{\rho t}$ and $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ (remind that $p^F(t) = c_x + \lambda_{X0}e^{\rho t}$), either the common full marginal cost is lower than $u'(\bar{x}_d)$ as illustrated in Figure 10, or it is higher (not depicted) so that the clean coal is never competitive. Thus the unique possible optimal sequence of phases is: i) only dirty coal up to the time at which the ceiling is attained and, simultaneously, the clean coal becomes competitive, ii) both the dirty and clean coals while at the ceiling, iii) only dirty coal while at the ceiling, iv) again dirty coal only during a post-ceiling phase, and v) the infinite phase of solar energy use.

The other implication of $\lambda_{X0} > \lambda_{S0}$ is that at time $\underline{t}_c^+$, at the beginning of the phase of joint exploitation of the both types of coal, due to the continuity of $\lambda_S(t)$ in the present case, then:

$$\lambda_{X0}e^{\rho \underline{t}_c^+} - \lambda_S(\underline{t}_c^+) \simeq (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c^+} > 0 \tag{4.51}$$

From (4.50) we conclude that $\dot{x}_c(\underline{t}_c^+) < 0$, hence from Proposition 13, that $\dot{x}_c(t) < 0$ for all $t$ during the phase and the energy price is increasing.

Although the optimal price path depicted by Figure 10 could look quite similar to the optimal price path determined in the case of a dominant scarcity effect with high solar cost as illustrated in Figure 6, these two cases notably differ during the phase of a joint

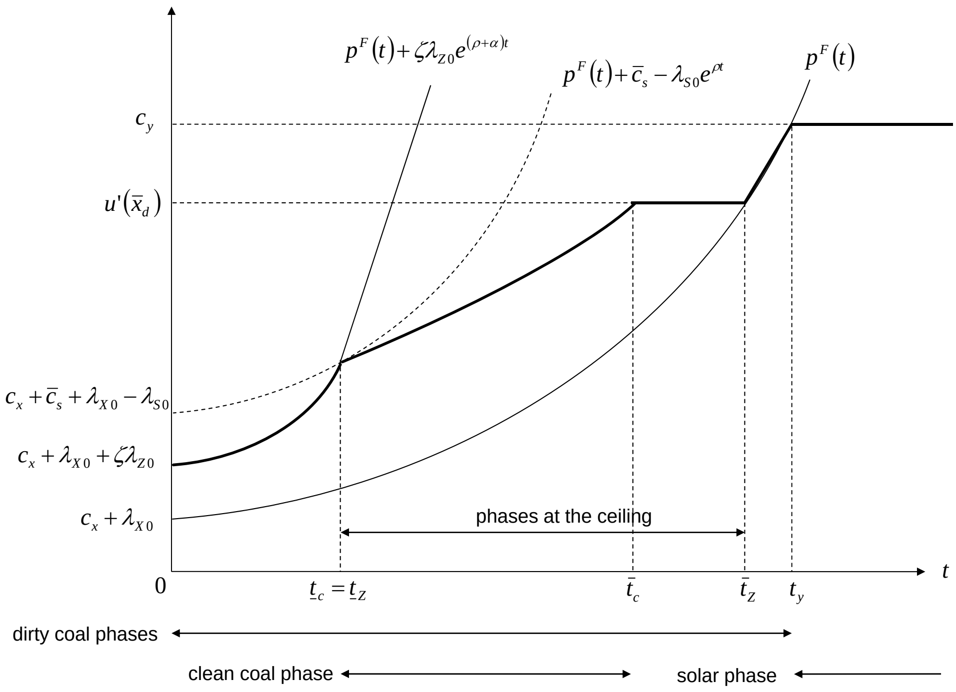


Figure 10: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} > \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$

exploitation of the two types of coal while at the ceiling. In the both cases, we have $\dot{x}_c(t) < 0$ hence $\dot{p}(t) > 0$, but contrary to the case of a dominant scarcity effect, here the shadow marginal cost of the pollution stock $\lambda_Z(t)$ decreases during this phase. From (4.42) and (4.43), we obtain now:

$$\dot{p}(t) = \frac{d}{dt}\left[p^F(t) + c_s(S(t)) - \lambda_S(t))\right] = \dot{p}^F(t) - \rho\lambda_S(t) < \dot{p}^F(t) \qquad (4.52)$$

and:

$$\dot{p}^F(t) - \rho\lambda_S(t) = \dot{p}^F(t) + \zeta\lambda_Z(t) < \dot{p}^F(t) \;\Rightarrow\; \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) < 0 \qquad (4.53)$$

However, the qualitative properties of the energy consumption paths (not illustrated) are almost the same as the ones depicted in Figure 7.

**- Case where $\lambda_{X0} < \lambda_{S0}$**

In this case, the shadow marginal value of the CCS experience is higher than the scarcity rent of coal. This gives rise to some new types of optimal paths, not only because what is happening during the phase of joint exploitation of the both types of coal is different, but also because the position of this phase within the optimal sequence of phases may be different.

Figures 11 and 12 illustrate why the time profile of the energy price and the energy consumption paths are different within this phase of joint exploitation although the optimal sequence of phases is the same as the sequence of the previous subcase $(\lambda_{X0} - \lambda_{S0}) > 0$.

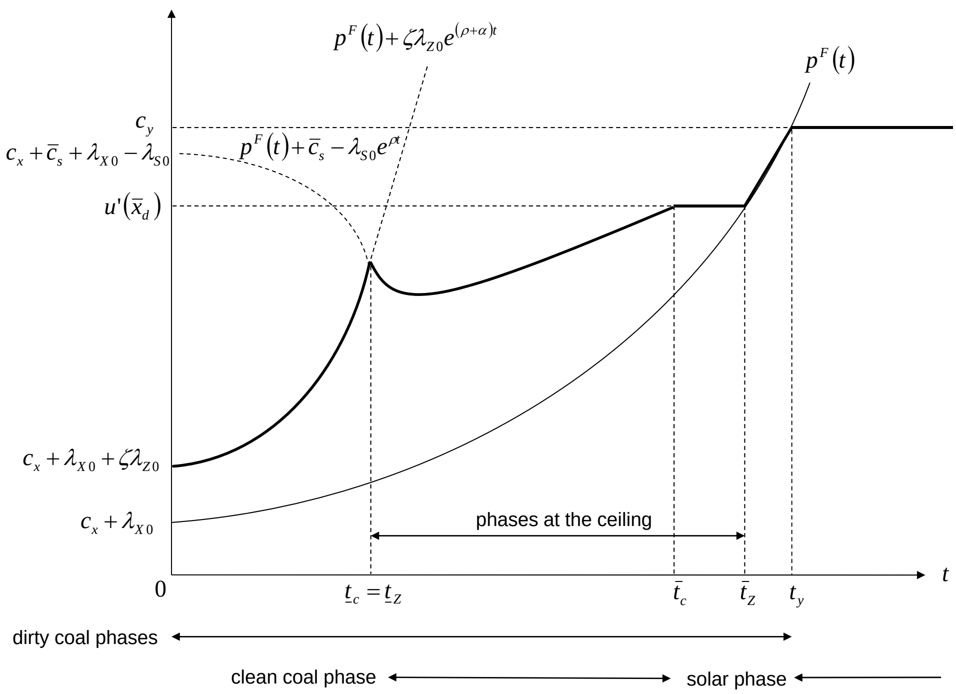


Figure 11: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

Since $(\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} < 0$, then at the beginning of the joint exploitation phase we may have $\lambda_{X0}e^{\rho \underline{t}_c^+} - \lambda_S(\underline{t}_c^+) < 0$ so that $\dot{x}(\underline{t}_c^+) > 0$. From Proposition 13 we know that, in this case, the energy price must be first decreasing and next increasing as illustrated in Figure 11, implying an unusual increase in the total coal consumption once the pollution cap is attained to capitalize on the learning effects. In fact, at the time $\underline{t}_Z = \underline{t}_c$ at which the ceiling is reached, the clean coal becomes also competitive thus triggering a shock – an instantaneous upward jump – in the allocation of its cumulated consumption, contrary to the dominant scarcity effect case.

The other main characteristics of this phase of joint exploitation of the two kinds of coal while at the ceiling is the pattern of the shadow marginal cost of the pollution stock. Clearly, since the price of the energy services is decreasing at the beginning of the phase,

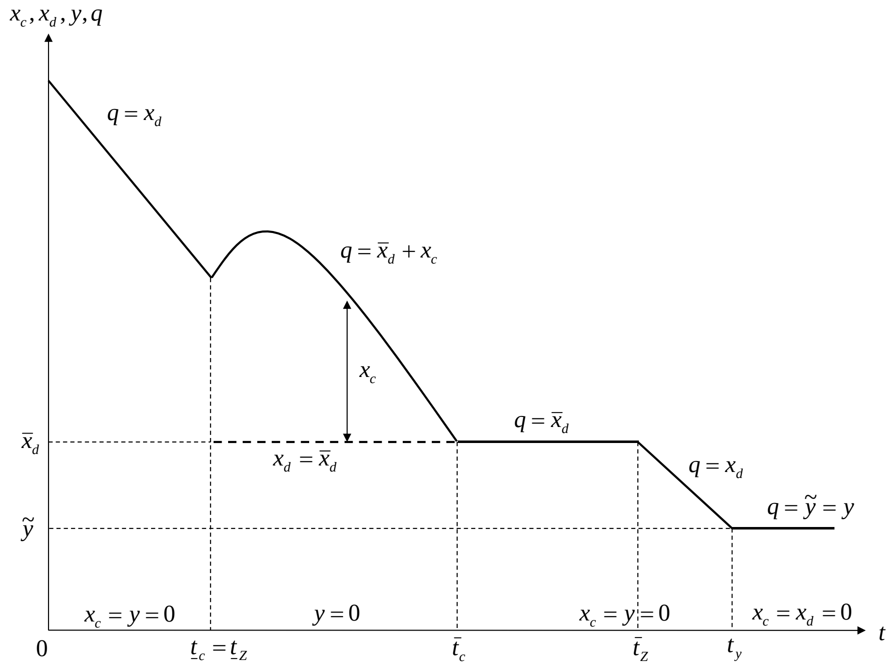


Figure 12: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

then $\lambda_Z(t)$ must be initially decreasing. But an important point is that $\lambda_Z(t)$ also decreases during the second part of the phase when the energy price increases again. The formal argument is the argument developed to obtain the above relationships (4.52) and (4.53), argument which holds whatever is the sign of $\lambda_{X0} - \lambda_{S0}$.

Finally, a last case has to be considered. In Figures 13 and 14, the optimal sequence of phases is modified in the following terms. The clean coal begins to be competitive after the beginning of the period at the ceiling so that $\underline{t}_c$ does not coincide anymore with $\underline{t}_Z$. Consequently, the phase of joint exploitation of the both types of coal takes place within the period at the ceiling and it is flanked by two phases of exclusive dirty coal use: $\underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$.

Contrary to the above cases of stock-dependent average cost functions, the exploitation of the clean coal begins here smoothly: $\lim_{t \downarrow \underline{t}_c} x_c(t) = 0$. Hence, there is not an abrupt change anymore in the total coal consumption use at time $\underline{t}_c$, contrary to the case where $\underline{t}_c = \underline{t}_Z$.

The system of equations from which the endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}_Z$, $\underline{t}_c$,

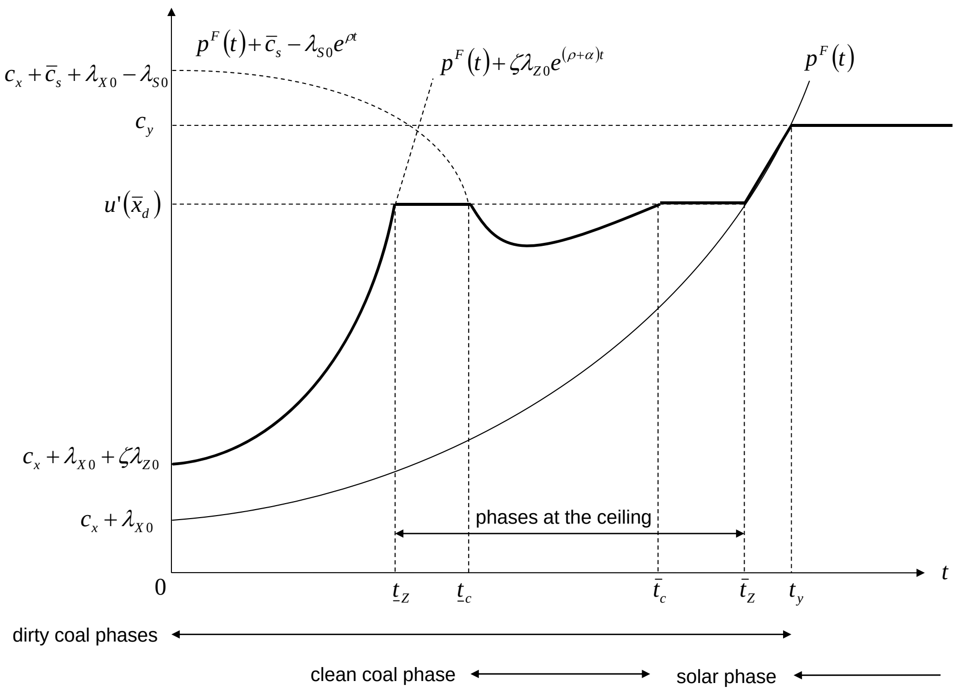


Figure 13: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

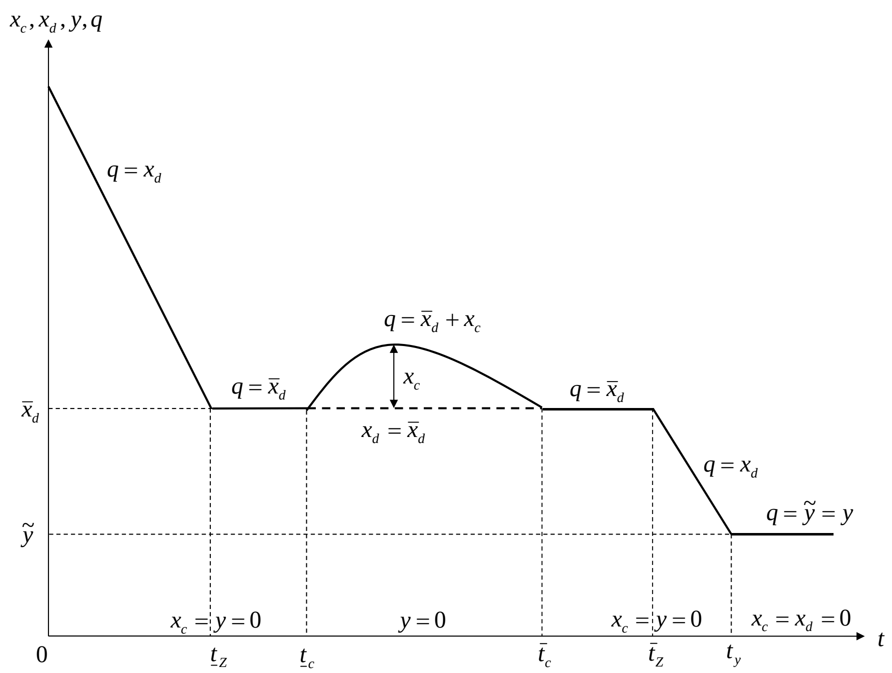


Figure 14: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

$\bar{t}_c$, $\bar{t}_Z$ and $t_y$ can be extracted in the high solar cost case is detailed in Appendix A.3.1 for the both subcases $\lambda_{X0} > \lambda_{S0}$ and $\lambda_{X0} < \lambda_{S0}$. This system contains seven equations when $\underline{t}_Z = \underline{t}_c \equiv \underline{t}$, and eight equation when $\underline{t}_Z < \underline{t}_c$.

### 4.3.2 The low solar cost case: $c_y < u'(\bar{x}_d)$

As in the high solar cost case, various types of optimal paths can appear according to whether $(\lambda_{X0} - \lambda_{S0})$ is positive or negative.

**- Case where $\lambda_{X0} > \lambda_{S0}$**

Qualitatively, this case is similar to the case in which the scarcity effect dominates and the solar cost is low. According to the arguments developed in the previous paragraph, the phase of joint exploitation of the two types of coal must begin when the ceiling is attained and the energy price must be increasing during this phase although the shadow marginal cost of the pollution stock is decreasing, up to the time at which this price equals $c_y$ instead of $u'(\bar{x}_d) < c_y$, time at which the solar energy becomes competitive. Then, from Proposition 12, the exploitation of the clean coal must cease at this time. The production of solar energy thus substitutes for the production of clean coal while staying at the ceiling up to the time at which $p^F(t) = c_y$. Last the dirty coal exploitation is closed, the coal reserves must be exhausted and the solar energy supplies to totality of the energy needs. Consequently, the price and consumption paths are qualitatively similar to the paths illustrated in Figures 8 and 9 respectively.

**- Case where $\lambda_{X0} < \lambda_{S0}$**

First, the period of joint exploitation of the two types of coal may precede the period of competitiveness of the solar energy. The associated price and consumption paths are illustrated in Figures 15 and 16 respectively.

However, as illustrated in Figure 17, the phase of competitiveness of the clean coal may also take place once the solar energy is competitive, that is at a date at which the solar energy is already exploited from some time: $t_y = \underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$. In this case, the exploitation of the solar energy must be interrupted since the energy price falls down the trigger price $c_y$ during the time interval $[\underline{t}_c, \bar{t}_c]$ of joint exploitation of the both kinds of coal. At time $t = \bar{t}_c$, the solar energy becomes competitive again and its production replaces the production of the clean coal. Then, the dirty coal and the solar energy are

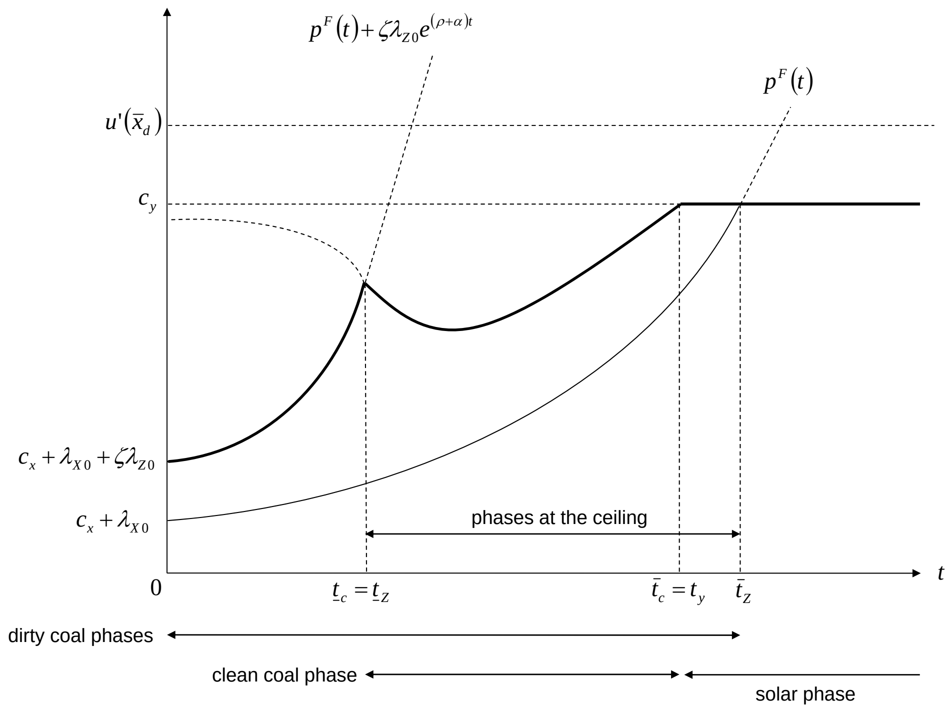


Figure 15: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

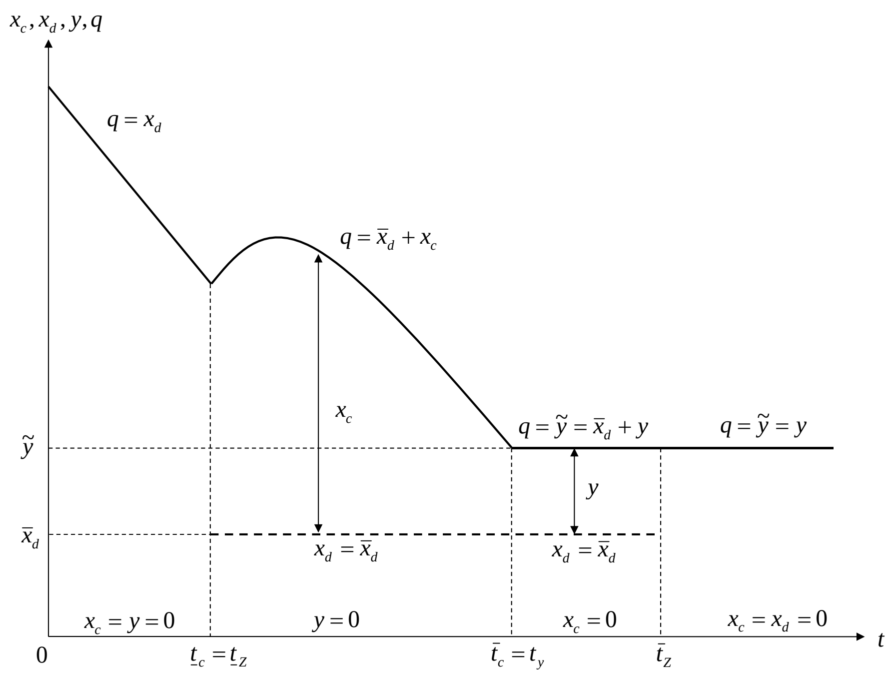


Figure 16: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

simultaneously exploited, as in the first phase at the ceiling, up to the time $t = \bar{t}_Z$ at which $p^F(t) = c_y$ and at which the stock of coal is exhausted. The associated energy consumption paths are illustrated in Figure 18.

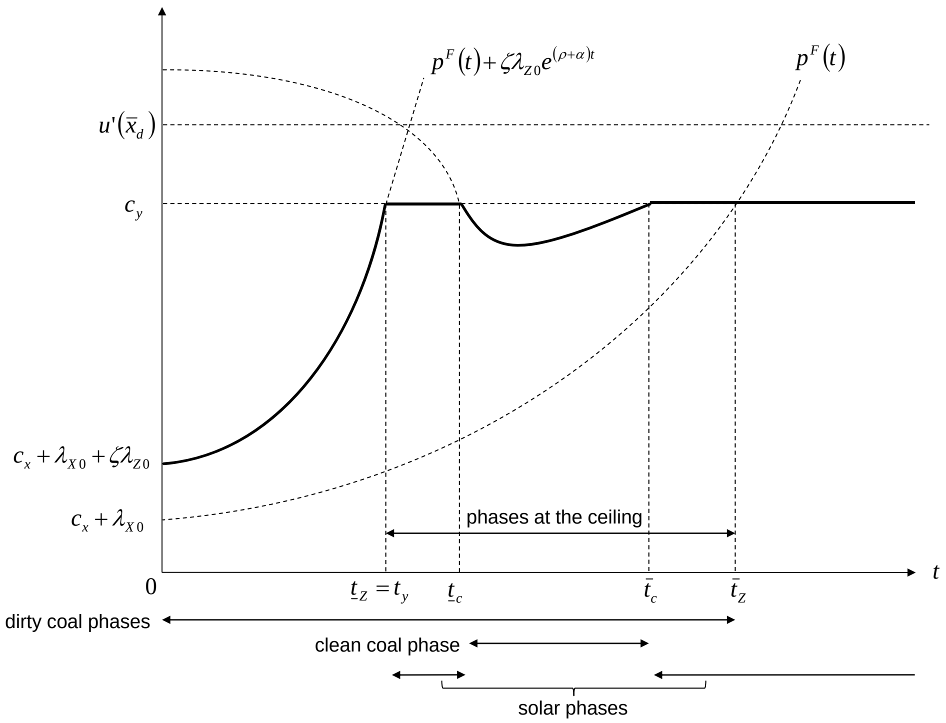


Figure 17: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

Last, the full characterization of the optimal path under a CCS.3 cost function in the low solar cost case, that is the determination of the endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}_Z$, $\underline{t}_c$, $\bar{t}_c$, $\bar{t}_Z$ and $t_y$, is developed in Appendix A.3.2 for the both subcases $\lambda_{X0} > \lambda_{S0}$ and $\lambda_{X0} < \lambda_{S0}$.

# 5 Optimal time profile of the carbon tax

The main tax of this model is the carbon tax, the duty having to be charged per unit of carbon emission released into the atmosphere when some part of the energy services are produced from dirty coal.

Whatever the assumptions about the CCS cost functions and about the level of the solar energy cost, the time profile of this tax is, qualitatively, roughly the same: first increasing from some positive level and next declining down to zero at time $\bar{t}_Z$, the end of

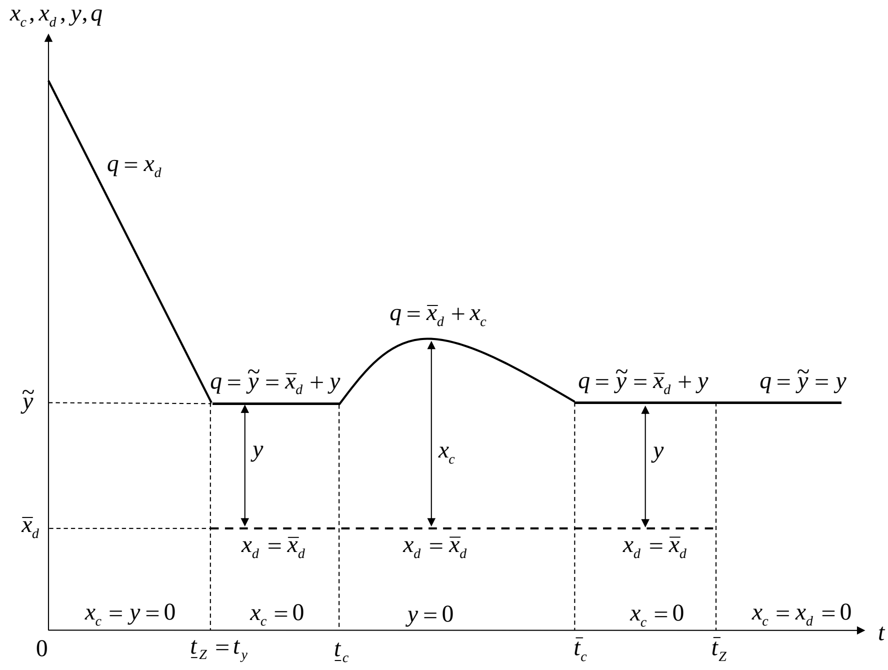


Figure 18: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

the period during which the ceiling constraint is binding (see (3.22)). However, the date at which the maximum is attained is not necessary the same under all the assumptions. The various possibilities are illustrated in Figure 19 where case a. depicts the flow-dependent CCS cost case, case b. the stock-dependent cost case with a dominant scarcity effect, case c. the stock-dependent cost case with a dominant learning effect when $\underline{t}_Z = \underline{t}_c$ whatever is the sign of $\lambda_{X0} - \lambda_{S0}$ and, last, case d. the stock-dependent cost case with a dominant learning effect when $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_c > \underline{t}_Z$.

Concerning this date at which the carbon tax reaches its peak, the case of a stock-dependent CCS cost function with a dominant scarcity effect must be contrasted from the other cases. In all the cases, the carbon tax is increasing at the instantaneous proportional rate $(\rho + \alpha)$ up to time $\underline{t}_Z$ at which the ceiling constraint begins to be tight (see (3.21). But in the case of a stock-dependent CCS cost function with a dominant scarcity effect, the tax is still increasing even after $\underline{t}_Z$, that is during some part of the period at the ceiling although at a lower instantaneous proportional rate (see Figure 19, case b.), contrary to the other cases in which the tax rate begins to decrease once the ceiling is attained (cases a., c. and d.).

The other differences bear on the behavior of the carbon tax rate during the clean coal exploitation period. In the case of a flow-dependent CCS cost function, the tax rate reaches its maximum during this period of clean coal use (case a. in Figure 19), in the case of stock-dependent CCS cost function with a dominant scarcity effect the tax rate is increasing during the phase of clean coal exploitation (case b.) while the rate is declining under stock-dependent cost functions with a dominant learning effect (cases c. and d.).

The last characteristics having to be pointed out is that, as far as the main qualitative properties of the carbon tax trajectory are at stake, the cost of the solar energy, either high or low, does not play an essential role. We conclude that what is really determining this time profile is the nature of the CCS cost function.

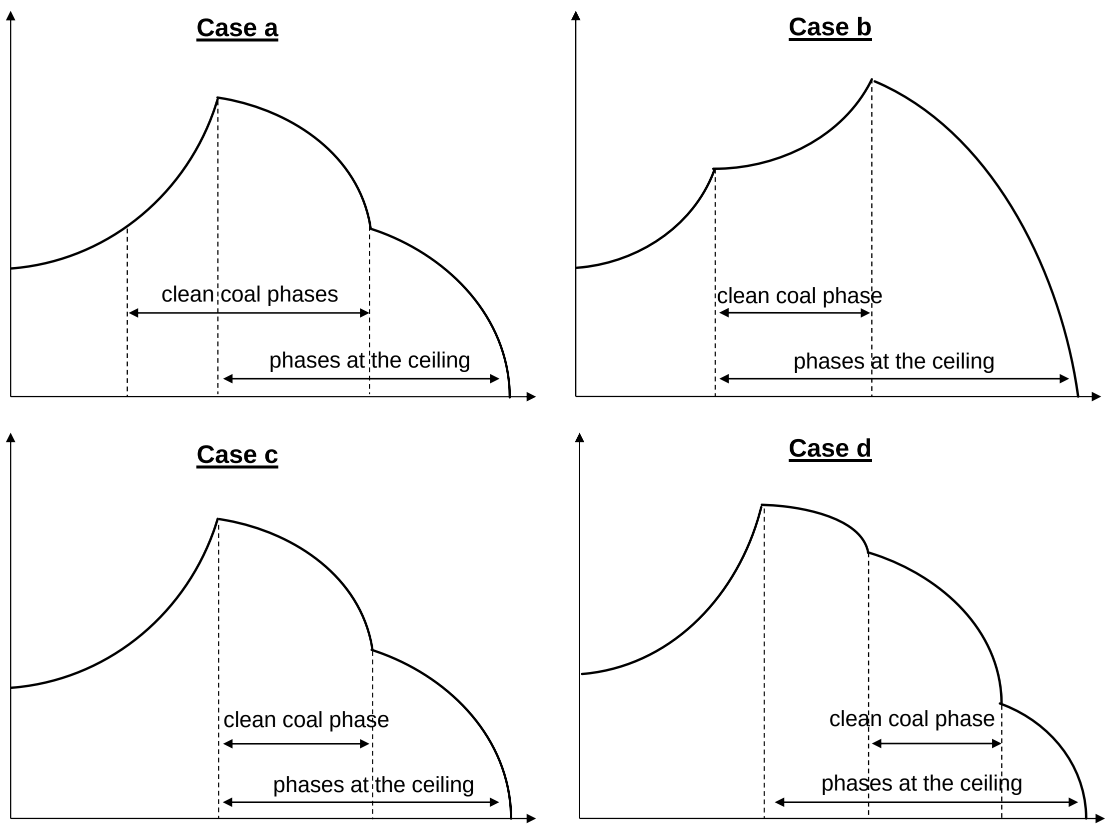


Figure 19: The various optimal time profiles of thee carbon tax.

## 6 Conclusion

In a Hotelling model, we have characterized the optimal geological carbon sequestration policies for alternative sequestration cost function and thus generalized the study by Lafforgue et al. (2008). The key features of the model were the following. i) The energy needs can be supplied by three types of energy resources that are perfectly substitutable: dirty coal (depletable and carbon-emitting), clean coal (also depletable but carbon-free thanks to a CCS device) and solar energy (renewable and carbon-free). ii) The atmospheric carbon stock cannot exceed some given institutional threshold as in Chakravorty et al. (2006). iii) The CCS cost function depends either on the flow of clean coal consumption or on its cumulated stock. In the later case, the marginal cost function can be either increasing in the stock (dominant scarcity effect) or decreasing (dominant learning effect).

Within this framework, we have shown that, under a stock-dependent CCS cost function, the clean coal exploitation must begin at the earliest when the carbon cap is reached while it must begin before under a flow-dependent cost function. Under stock-dependent cost function with a dominant learning effect, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation. Last under stock-dependent cost function, even if the qualitative properties of the price path can be roughly similar in some cases whatever be the dominant effect – scarcity or learning – they can imply some contrasting repercussions on the social marginal cost of the pollution stock. In particular, the scarcity effect can lead to a carbon tax trajectory which is still increasing even after the ceiling has been reached while, in this kind of ceiling models, the tax generally begins to decrease precisely at this date.

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# Appendix

## A.1 Full characterization of the optimal price path under CCS.1

### A.1.1 The high solar cost case: $u'(\bar{x}_d) < c_y$

Let us denote by $x_c^1(t, \lambda_{Z0})$ and $x_c^2(t, \lambda_{X0})$ the clean coal consumption during the phases $[\underline{t}_c, \underline{t}_Z)$ and $[\underline{t}_Z, \bar{t}_c)$, respectively. During the phase $[\underline{t}_c, \underline{t}_Z)$, $x_c^1(t, \lambda_{Z0})$ reads as the solution of:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)t} = c_s(x_c) + c_s'(x_c) x_c$$

and during the phase $[\underline{t}_Z, \bar{t}_c)$, $x_c^2(t, \lambda_{X0})$ solves:

$$u'(x_c + \bar{x}_d) = c_x + \lambda_{X0} e^{\rho t} + c_s(x_c) + c_s'(x_c) x_c$$

When the atmospheric carbon cap $\bar{Z}$ is sufficiently high and the initial pollution stock $Z^0$ is sufficiently low so that there exists an initial phase of dirty coal consumption without CCS, then the optimal path is the six-phase path as illustrated in Figure 1. To fully characterize this optimal path, the seven variables $\lambda_{X0}$, $\lambda_{Z0}$, $\underline{t}_c$, $\underline{t}_Z$, $\bar{t}_c$, $\bar{t}_Z$ and $t_y$ have to be determined. They solve the following system of seven equations:

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned} \int_0^{\underline{t}_Z} q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) dt + \int_{\underline{t}_Z}^{\bar{t}_c} x_c^2(t, \lambda_{X0}) dt & \\ + \bar{x}_d [\bar{t}_Z - \underline{t}_Z] + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0} e^{\rho t}) dt &= X^0 \end{aligned} \tag{6.54}$$

- The atmospheric carbon stock continuity equation at $\underline{t}_Z$:

$$\begin{aligned} Z^0 + \zeta \int_0^{\underline{t}_c} q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) e^{\alpha t} dt & \\ + \zeta \int_{\underline{t}_c}^{\underline{t}_Z} \left[ q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) - x_c^1(t, \lambda_{Z0}) \right] e^{\alpha t} dt &= \bar{Z} e^{\alpha \underline{t}_Z} \end{aligned} \tag{6.55}$$

- The full marginal costs equality equation at the beginning time $\underline{t}_c$ of clean coal exploitation:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_c} = c_s(0) \tag{6.56}$$

- The continuity equation of the energy price path at the date $\underline{t}_Z$ at which the ceiling constraint is binding:

$$c_x + \lambda_{X0} e^{\rho \underline{t}_Z} + \zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_Z} = u'\left(x_c^2(\underline{t}_Z, \lambda_{X0}), \bar{x}_d\right) \Leftrightarrow x_c^1(\underline{t}_Z, \lambda_{Z0}) = x_c^2(\underline{t}_Z, \lambda_{X0}) \tag{6.57}$$

- The continuity equation of the energy price path at the closing time $\bar{t}_c$ of clean coal exploitation:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_c} + c_s(0) = u'(\bar{x}_d) \;\Leftrightarrow\; x_c^2(\bar{t}_c, \lambda_{X0}) = 0 \tag{6.58}$$

- The continuity equation of the energy price path at the date $\bar{t}_Z$ at which the ceiling constraint ends to be active:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_Z} = u'(\bar{x}_d) \tag{6.59}$$

- The continuity equation of the energy price path at the time $t_y$ at which solar energy becomes competitive:

$$c_x + \lambda_{X0} e^{\rho t_y} = c_y \tag{6.60}$$

For any set $\{\lambda_{X0}, \lambda_{Z0}, \underline{t}_c, \underline{t}_Z, \bar{t}_c, \bar{t}_Z, t_y\}$ satisfying the above system of seven equations and such that $\zeta \lambda_{Z0} < c_s(0)$, then the necessary conditions (3.13)-(3.17) are satisfied. Since the problem is strictly convex, these conditions are also sufficient.

When the initial pollution stock $Z^0$ is sufficiently close to $\bar{Z}$ so that the clean coal exploitation must be started immediately, i.e. $\underline{t}_c = 0$, only six variables have to be determined. The equation (6.55) must be modified as follows:

$$Z^0 + \zeta \int_0^{\underline{t}_Z} \left[ q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) - x_c^1(t, \lambda_{Z0}) \right] e^{\alpha t} dt = \bar{Z} e^{\alpha \underline{t}_Z} \tag{6.61}$$

and the equation (6.56) must be suppressed.

### A.1.2 The low solar cost case $u'(\bar{x}_d) > c_y$

Now $x_c^2(t, \lambda_{X0})$ as defined in the previous paragraph is the clean coal consumption during the phase $[\underline{t}_Z, t_y)$, and we define $x_c^3(t, \lambda_{X0})$, the clean coal consumption during the phase $[t_y, \bar{t}_c)$, as the solution of the following equation:

$$c_y = c_x + \lambda_{X0} e^{\rho t} + c_s(x_c) + c_s'(x_c) x_c$$

First, when $Z^0$ is large enough and/or $c_y$ is large enough so that the optimal price path is the six-phase path illustrated in Figure 3, the same seven variables $\lambda_{X0}$, $\lambda_{Z0}$, $\underline{t}_c$, $\underline{t}_Z$, $t_y$, $\bar{t}_c$ and $\bar{t}_Z$ have to be determined. The system of seven equations they solve now becomes:

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned} \int_0^{\underline{t}_Z} q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) dt + \int_{\underline{t}_Z}^{t_y} x_c^2(t, \lambda_{X0}) dt & \\ + \int_{t_y}^{\bar{t}_c} x_c^3(t, \lambda_{X0}) dt + \bar{x}_d[\bar{t}_Z - \underline{t}_Z] &= X^0 \end{aligned} \tag{6.62}$$

- The equation (6.55) for the continuity of the atmospheric pollution stock at $\underline{t}_Z$.

- The equations (6.56) and (6.57) for the price path continuity at $\underline{t}_c$ and $\underline{t}_Z$, respectively.

- The continuity equation of the energy price path at $t_y$:

$$u'(x_c^2(t_y, \lambda_{X0}), \bar{x}_d) = c_y \;\Leftrightarrow\; x_c^2(t_y, \lambda_{X0}) = x_c^3(t_y, \lambda_{X0}) \tag{6.63}$$

- The continuity equation of the energy price path at $\bar{t}_c$:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_c} + c_s(0) = c_y \;\Leftrightarrow\; x_c^3(\bar{t}_c, \lambda_{X0}) = 0 \tag{6.64}$$

- The continuity equation of the energy price path at $\bar{t}_Z$:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_Z} = c_y \tag{6.65}$$

Again, when $Z^0$ is sufficiently close to $c_y$, it is necessary to immediately begin the CCS activity at $t = 0$, in which case equation (6.62) has to be substituted for (6.55) and equation (6.56) has to be deleted.

## A.2 Full characterization of the optimal price path under CCS.2

When the scarcity effect is purely dominant, and whatever the level of the average solar cost $c_y$ as compared with $u'(\bar{x}_d)$, two cases have to be considered depending on whether the reservoir capacity constraint is binding or not at the closing time of the clean coal exploitation (see Proposition 4). This implies that four cases have to be investigated.

### A.2.1 The high solar cost case $u'(\bar{x}_d) < c_y$

*a. Case where $S_c(\bar{t}_c) < \bar{S}_c$*

In this case, the capacity constraint on the cumulated clean coal exploitation is never binding, thus implying that $\nu_S(t) = 0$ for any $t \geq 0$ and that $\lambda_S(t) = 0$ for $t \geq \bar{t}_c$. The expression (4.36) of the costate variable of the cumulated clean coal production can be simplified into:

$$\lambda_S(t) = -e^{\rho t} \int_t^{\bar{t}_c} c_s'(S_c(\tau)) x_c(\tau) e^{-\rho \tau} d\tau$$

Integrating by parts the above expression results in:

$$\lambda_S(t) = c_s(S_c(t)) - e^{\rho t} \left[ c_s(S_c(\bar{t}_c)) e^{-\rho \bar{t}_c} + \rho \int_t^{\bar{t}_c} c_s(S_c(\tau)) e^{-\rho \tau} d\tau \right] \tag{6.66}$$

The seven endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\bar{t}_c$, $\bar{t}_Z$ and $t_y$ solve the following system of seven equations:

- The initial condition on the costate variable $\lambda_S(t)$ which, from (6.66), results in:

$$\lambda_{S0} = \lambda_S(0) = \underline{c}_s e^{-\rho \underline{t}} - c_s(S_c(\bar{t}_c))e^{-\rho \bar{t}_c} - \rho \int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t} dt \tag{6.67}$$

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned} \int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}}^{\bar{t}_c} q(c_m^c(x_c(t)))dt & \\ + \bar{x}_d[\bar{t}_Z - \bar{t}_c] + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0}e^{\rho t})dt &= X^0 \end{aligned} \tag{6.68}$$

where, from (6.66), the full marginal cost $c_m^c(x_c(t))$ of the clean coal amounts to:

$$c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + e^{\rho t}\left[c_s(S_c(\bar{t}_c))e^{-\rho \bar{t}_c} + \rho \int_t^{\bar{t}_c} c_s(S_c(\tau))e^{-\rho \tau} d\tau\right], \quad t \in [\underline{t}, \bar{t}_c)$$

- The atmospheric carbon stock continuity equation at time $\underline{t}$:

$$Z^0 + \zeta \int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})e^{\alpha t} dt = \bar{Z}e^{\alpha \underline{t}} \tag{6.69}$$

- The continuity equation of the energy price path at the date $\underline{t}$ at which the ceiling constraint is binding and, simultaneously, the clean coal exploitation begins:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}} = \underline{c}_s - \lambda_{S0}e^{\rho \underline{t}} \tag{6.70}$$

- The continuity equation of the energy price path at the closing time $\bar{t}_c$ of the clean coal exploitation:

$$c_x + \lambda_{X0}e^{\rho \bar{t}_c} + c_s(S_c(\bar{t}_c)) = u'(\bar{x}_d) \tag{6.71}$$

- The equations (6.59) and (6.60) for the continuity of the energy price path at times $\bar{t}_Z$ and $t_y$, respectively.

*b. Case where $S_c(\bar{t}_c) = \bar{S}_c$*

In this case, the reservoir is fulfilled at time $\bar{t}_c$ implying $\lambda_S(\bar{t}_c) < 0$. Here we cannot deduce $\lambda_{S0}$ from the general expression of $\lambda_S(t)$ as in the previous case. This missing information must be replaced by an additional terminal condition on the cumulated clean coal production: $S_c(\bar{t}_c) = \bar{S}_c$.

Integrating by parts (4.36), we have now:

$$\lambda_S(t) = c_s(S_c(t)) - e^{\rho t}\left[c_s(\bar{S}_c)e^{-\rho\bar{t}_c} + \rho\int_t^{\bar{t}_c} c_s(S_c(\tau))e^{-\rho\tau}d\tau + \int_t^{\infty}\nu_S(\tau)e^{-\rho\tau}d\tau\right] \quad (6.72)$$

thus implying:

$$\lambda_{S0} = \underline{c}_s e^{-\rho\underline{t}} - c_s(\bar{S}_c)e^{-\rho\bar{t}_c} - \rho\int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt - \int_{\bar{t}_c}^{\infty}\nu_S(t)e^{-\rho t}dt \quad (6.73)$$

Replacing into (6.72) the term $\int_t^{\infty}\nu_S(t)e^{-\rho t}dt$ by its expression coming from (6.73), with $\nu_S(t) = 0$ for $t \in [0, \bar{t}_c)$, we obtain after simplifications:

$$\forall t \in [\underline{t}, \bar{t}_c): \quad \lambda_S(t) = c_s(S_c(t)) - e^{\rho t}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{t} c_s(S_c(\tau))e^{-\rho\tau}d\tau - \lambda_{S0}\right] \quad (6.74)$$

$$\text{at time } \bar{t}_c: \quad \lambda_S(\bar{t}_c) = c_s(\bar{S}_c) - e^{\rho\bar{t}_c}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt - \lambda_{S0}\right] \quad (6.75)$$

The seven endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\bar{t}_c$, $\bar{t}_Z$ and $t_y$ are determined as the solution of the following seven-equations system:

- The continuity equation of the cumulated clean coal production at $\bar{t}_c$:

$$\int_{\underline{t}}^{\bar{t}_c} x_c(t)dt = \int_{\underline{t}}^{\bar{t}_c} q(c_m^c(x_c(t)))dt - \bar{x}_d[\bar{t}_c - \underline{t}] = \bar{S}_c \quad (6.76)$$

where, from (6.74), the full marginal cost $c_m^c(x_c(t))$ of the clean coal is now equal to:

$$c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + e^{\rho t}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{t} c_s(S_c(\tau))e^{-\rho\tau}d\tau - \lambda_{S0}\right], \quad t \in [\underline{t}, \bar{t}_c)$$

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \bar{x}_d[\bar{t}_Z - \underline{t}] + \bar{S}_c + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0}e^{\rho t})dt = X^0 \quad (6.77)$$

- The equation (6.69) for the continuity of the atmospheric carbon stock at $\underline{t}$.

- The continuity equation of the energy price path at $\underline{t}$:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}} = \underline{c}_s - \lambda_{S0}e^{\rho\underline{t}} \quad (6.78)$$

- The continuity equation of the energy price path at $\bar{t}_c$ which, using (6.75), implies:

$$\begin{aligned} c_x + \lambda_{X0}e^{\rho\bar{t}_c} + c_s(\bar{S}_c) - \lambda_S(\bar{t}_c) &= u'(\bar{x}_d) \\ \Rightarrow c_x + \lambda_{X0}e^{\rho\bar{t}_c} + e^{\rho\bar{t}_c}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt - \lambda_{S0}\right] &= u'(\bar{x}_d) \quad (6.79) \end{aligned}$$

- The equations (6.59) and (6.60) for the continuity of the energy price path at times $\bar{t}_Z$ and $t_y$, respectively.

### A.2.2 The low solar cost case $u'(\bar{x}_d) > c_y$

*a. Case where* $S_c(\bar{t}_c) < \bar{S}_c$

As explained in Section 4.2.2, only the six endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\hat{t}$ (with $\hat{t} = \bar{t}_c = t_y$) and $\bar{t}_Z$ have now to be determined. They solve the following system of six equations:

- The equation (6.67) for the initial condition on $\lambda_S(t)$, with $\bar{t}_c = \hat{t}$.

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}}^{\hat{t}} q(c_m^c(x_c(t)))dt + \bar{x}_d[\bar{t}_Z - \hat{t}] = X^0 \qquad (6.80)$$

where, the full marginal cost $c_m^c(x_c(t))$ has the same expression as in the corresponding high solar cost case for $t \in [\underline{t}, \hat{t})$.

- The equation (6.69) for the continuity of the atmospheric carbon stock at $\underline{t}$.

- The equation (6.70) for the continuity of the energy price path at time $\underline{t}$.

- The continuity equation of the energy price path at time $\hat{t}$:

$$c_x + \lambda_{X0}e^{\rho\hat{t}} + c_s(S_c(\hat{t})) = c_y \qquad (6.81)$$

- The equation (6.65) for the continuity of the energy price path at time $\bar{t}_Z$.

*b. Case where* $S_c(\bar{t}_c) = \bar{S}_c$

The six endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$, $\hat{t}$ and $\bar{t}_Z$ are determined as the solution of the following six-equations system:

- The equation (6.76) for the continuity of the cumulated clean coal production at $\hat{t}$, with $\hat{t} = \bar{t}_c$.

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \bar{x}_d[\bar{t}_Z - \underline{t}] + \bar{S}_c = X^0 \qquad (6.82)$$

- The equation (6.69) for the continuity of the atmospheric carbon stock at $\underline{t}$.

- The equation (6.78) for the continuity of the energy price path at $\underline{t}$.

- The equation (6.79) for the continuity of the energy price path at $\hat{t}$, with $\hat{t} = \bar{t}_c$.

- The equation (6.65) for the continuity of the energy price path at time $\bar{t}_Z$.

## A.3 Full characterization of the optimal price path under CCS.3

Under a stock-dependent CCS cost function with a dominant learning effect, the expression of the costate variable of the cumulated clean coal production is given by (4.46). Expanding the integral term and simplifying, it comes:

$$\lambda_S(t) = c_s(S_c(t)) - e^{\rho t}\left[c_s(S_c(\bar{t}_c))e^{-\rho\bar{t}_c} + \rho\int_t^{\bar{t}_c} c_s(S_c(\tau))e^{-\rho\tau}d\tau\right] \tag{6.83}$$

which the same expression as (6.66) obtained in the dominant scarcity effect case. However, the initial value of $\lambda_S$ slightly differs since the CCS cost function is now decreasing in $S$:

$$\lambda_{S0} = \bar{c}_s e^{-\rho\underline{t}_c} - c_s(S_c(\bar{t}_c))e^{-\rho\bar{t}_c} - \rho\int_{\underline{t}_c}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt \tag{6.84}$$

Finally, since in this case the reservoir that hosts the sequestered carbon emissions is not constrained by any limit in capacity, the associated costate variable must be nil at the closing time of the clean coal exploitation, as specified by (4.47): $\lambda_S(t) = 0\ \forall t \geq \bar{t}_c$.

### A.3.1 The high solar cost case $u'(\bar{x}_d) < c_y$

*a. Case where* $\lambda_{X0} > \lambda_{S0}$

As mentioned in Section 4.3.1, the energy price and consumption paths are qualitatively very similar to the ones obtained in the dominant scarcity effect case with high solar cost when the capacity constraint on the cumulated clean coal production is never binding. Hence, the seven endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\bar{t}_c$, $\bar{t}_Z$ and $t_y$ solve almost the same seven-equations system as in Appendix A.2.1.a:

- The equation (6.84) for the initial condition on $\lambda_S(t)$.

- The equation (6.68) for the cumulated coal consumption/coal endowment balance.

- The equation (6.69) for the continuity of the atmospheric carbon stock at time $\underline{t}$.

- The continuity equation of the energy price path at time $\underline{t}$:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}} = \bar{c}_s - \lambda_{S0}e^{\rho\underline{t}} \tag{6.85}$$

- The equation (6.71) for the continuity of the energy price path at time $\bar{t}_c$.

- The equations (6.59) and (6.60) for the continuity of the energy price path at times $\bar{t}_Z$ and $t_y$, respectively.

*b. Case where* $\lambda_{X0} < \lambda_{S0}$

As seen in Section 4.3, when $\lambda_{X0} - \lambda_{S0} < 0$ two subcases have to be considered according to whether the dates at which the carbon cap is reached and at which the clean coal exploitation begins coincide are not.

First, if $\underline{t}_Z = \underline{t}_c \equiv \underline{t}$, then the seven variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$, $\bar{t}_c$, $\bar{t}_Z$ and $t_y$ exactly solve the same system of equations than the previous one (see Appendix A.3.1 case a.).

Second, if $\underline{t}_Z < \underline{t}_c \equiv \underline{t}$, then we have now to determine eight endogenous variables: $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}_Z$, $\underline{t}_c$, $\bar{t}_c$, $\bar{t}_Z$ and $t_y$. They solve the following system of seven equations:

- The equation (6.84) for the initial condition on $\lambda_S(t)$.

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned}\int_0^{\underline{t}_Z} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}_c}^{\bar{t}_c} q(c_m^c(x_c(t)))dt \\ +\bar{x}_d[(\bar{t}_Z - \underline{t}_Z) - (\bar{t}_c - \underline{t}_c)] + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0}e^{\rho t})dt \quad = \quad X^0 \qquad (6.86)\end{aligned}$$

where, $c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + c_s(S_c(t)) - \lambda_S(t)$, with $\lambda_S(t)$ given by (6.83).

- The atmospheric carbon stock continuity equation at time $\underline{t}_Z$:

$$Z^0 + \zeta \int_0^{\underline{t}_Z} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})e^{\alpha t}dt = \bar{Z}e^{\alpha \underline{t}_Z} \qquad (6.87)$$

- The continuity equation of the energy price path at time $\underline{t}_Z$:

$$c_x + \lambda_{X0}e^{\rho \underline{t}_Z} + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} = u'(\bar{x}_d) \qquad (6.88)$$

- The continuity equation of the energy price path at time $\underline{t}_c$:

$$c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} = u'(\bar{x}_d) \qquad (6.89)$$

- The equation (6.71) for the continuity of the energy price path at time $\bar{t}_c$.

- The equations (6.59) and (6.60) for the continuity of the energy price path at times $\bar{t}_Z$ and $t_y$, respectively.

### A.3.2 The low solar cost case $u'(\bar{x}_d) > c_y$

*a. Cases where $\lambda_{X0} > \lambda_{S0}$ or where $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_Z = \underline{t}_c$*

The six endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}$, $\hat{t}$ and $\bar{t}_Z$ are determined as the solution of the following six-equations system:

- The equation (6.84) for the initial condition on $\lambda_S(t)$.

- The equation (6.80) for the cumulated coal consumption/coal endowment balance.

- The equation (6.69) for the continuity of the atmospheric carbon stock at time $\underline{t} = \underline{t}_Z = \underline{t}_c$.

- The equation (6.85) for the continuity of the energy price path at time $\underline{t}$.

- The equation (6.81) for the continuity of the price path at time $\hat{t} = \bar{t}_c = t_y$.

- The equation (6.65) for the continuity of the price path at time $\bar{t}_Z$.

*b. Case where $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_Z < \underline{t}_c$*

In this last case, the seven endogenous variables $\lambda_{X0}$, $\lambda_{Z0}$, $\lambda_{S0}$, $\underline{t}_Z = t_y$, $\underline{t}_c$, $\bar{t}_c$ and $\bar{t}_Z$ solve the following system:

- The equation (6.84) for the initial condition on $\lambda_S(t)$.

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}_Z} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}_c}^{\bar{t}_c} q(c_m^c(x_c(t)))dt + \bar{x}_d[(\bar{t}_Z - \underline{t}_Z) - (\bar{t}_c - \underline{t}_c)] = X^0 \quad (6.90)$$

where, $c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + c_s(S_c(t)) - \lambda_S(t)$, with $\lambda_S(t)$ given by (6.83).

- The equation (6.69) for the continuity of the atmospheric carbon stock at time $\underline{t}_Z$.

- The continuity equation of the energy price path at time $\underline{t}_Z = t_y$:

$$c_x + \lambda_{X0}e^{\rho \underline{t}_Z} + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} = c_y \quad (6.91)$$

- The continuity equation of the energy price path at time $\underline{t}_c$:

$$c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} = c_y \tag{6.92}$$

- The continuity equation of the energy price path at time $\bar{t}_c$:

$$c_x + \lambda_{X0}e^{\rho \bar{t}_c} + c_s(S_c(\bar{t}_c)) = c_y \tag{6.93}$$

- The equation (6.65) for the continuity of the price path at time $\bar{t}_Z$.

# Triggering the Technological Revolution in Carbon Capture and Sequestration Costs

## I/ The Polluting Resource is Abundant *

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# Triggering the Technological Revolution in Carbon Capture and Sequestration Costs

## I: The Polluting Resource is Abundant

**Abstract**

The nature of optimal environmental policies able to induce sufficient technical progress in pollution abatement technologies has raised a vivid debate between economics over the last decade. Some emphasize the importance of learning-by-doing on these technologies, an argument in favor of early action. Other insisted upon the time needed for R&D to identify the best abatement options, an incentive to delay action in the future. Either triggering technical progress from learning effects of research, all analysis conclude to ambiguous effects of environmental policies on the speed of technical change. One strong limitation of previous approaches is that they do not endogenize the best ways to improve the efficiency of abatement technologies, either through learning on existing techniques or through research to discover new ones. We consider an economy that can trigger some cost breakdown in CCS costs thanks to both learning and R&D. We first reconsider the results of the literature about the extreme cases of a pure learning induced technical revolution and a pure R&D induced cost breakdown in the context of an atmospheric carbon ceiling framework. We show how this setting helps to clarify the existing results from the literature and remove some of their ambiguities. In particular we perform a sensitivity analysis of the optimal policies with respect to relevant parameters, providing strong intuitions about the various effects affecting their dynamics. We next examine the case of a combined learning and R&D policy. We show that the economy may initially perform only research efforts or rely only upon learning to trigger the cost breakdown. A combined policy may only follow pure R&D or learning policies. Combining learning and R&D requires to increase both research efforts and the use of the abatement technology, but the growth rate of pollution abatement must be higher than the growth rate of the research efforts. Contrarily to what is commonly observed in models with constant average and marginal costs of abatement, the use of cleaning technologies may begin before the atmospheric constraint begins to bind. In such situation, the time constraints upon technological development outweighs the environmental constraints and result in early introduction of abatement technologies. But the contrary may also be optimal and we provide a complete discussion of the relevance of these various scenarios.

**Keywords:** Carbon capture and storage; Energy substitution; Learning-by-doing; Research and development; Carbon stabilization cap.

**JEL classifications:** Q32, Q42, Q54, Q55, Q58.

# Contents

# 1 Introduction

Technology plays a prominent role in all reflections about finding ways out of the global warming problem. This issue has been forcefully raised by Scott Barret in several occasions (Barret, 2006). For him, instead of seeking for an almost impossible international agreement on carbon emissions mitigation, governments should better cooperate over a common target of triggering a technological revolution in clean energy generation. The help of technical progress is particularly expected in three domains: the enhancement of the productive efficiency of fossil fuels, the development of non carbon based energy production techniques and the improvement of the efficiency of carbon pollution abatement technologies, the future of carbon capture and storage (CCS) technologies appearing as an important issue in this respect. If the economic literature fully agrees to this general statement, it largely diverges in assessing both the policy implications of technical progress opportunities in carbon emissions mitigation and the effects of environmental policies upon technical change in the energy production and consumption sectors. These issues have provoked a vivid debate among economist during the last decade.

Two main reasons may explain this difficulty to reach an agreement about the nature of the relationships between environmental policy and technological development. First, the topic of technical change, or more precisely of endogenous technical change, has emerged only recently in the economic literature. A lot remains to be done on this issue, especially to build a consistent view of the various advances coming both from the microeconomic approaches developed in industrial organization economics and the macroeconomic approaches of the endogenous growth literature. Second, global warming and technical progress are two dynamical processes with their own drivers and constraints, and reaching a reasonable understanding of the time links between these two processes is a modeling challenge, both on the theoretical and on the empirical side.

A first motive of dissent relies in the desirable speed of action to introduce pollution abatement technologies. One main set of arguments in favor of delaying abatement roots in discounting arguments, the abatement options being today typically costly and thus be favorably delayed in the future (Wigley *et al.*, 1996). A second set of arguments advances that in their present sate, existing abatement technologies are too costly, and time should be given to research to develop new and more affordable technical options.

This line of thought has been heavily criticized by Van der Zwaan *et al.*, (2002), and Kverndokk and Rosendhal (2007), among others, because it does not take into account the potential of experience and learning-by-doing in pollution abatement technologies. Taking benefit of such learning opportunities rather requires early action than delay. The argument is reinforced by a capital accumulation motive, the replacement of old and costly vintages by new and cheaper one is a costly process requiring a significant time. This time to build issue appears to be particularly relevant for CCS technologies, their development being submitted to costly capacity expansion constraints. The early action these is also endorsed by industrial organization views. By announcing sufficiently early a credible path of action, in terms of an announced increasing time schedule of a carbon tax for example, the industry will react to this incentive scheme by investing today in abatement technologies, the uncertainty about what the regulator plans to do in the future being removed.

However, learning-by-doing is not the only way to induce technical advances. Another main option is R&D. R&D has two main advantages with respect to learning. First, it does not require to actually use the technology. With sufficient time and effort, it is possible to achieve in the lab potential cost cuts without bearing the high initial cost of using non mature technologies. Second, R&D can span much more potential technical options than actual use, which requires specific technical choices before beginning the exploitation of a given technique, the risk being to be trapped into inferior options or inappropriate initial choices.

It appears immediately that in an R&D induced technical change world, early development of infant abatement technologies may be counterproductive. In policy terms, this means that subsidizing non mature abatement technologies in the hope that learning can reduce their costs in the future may be suboptimal. It would be better to give more time to research to assert the economic potential of different technological options. This issue has been examined carefully by Goulder and Matthai (2000). Comparing a learning induced technical change model with a R&D induced technical change model, they conclude that in a R&D world, delaying actual abatement is optimal while the interest to advance or delay a policy action promoting pollution abatement is usually ambiguous in a learning world.

Induced technical change in carbon emission mitigation technologies is only one aspect of a more general problem involving also alternative clean energy production, like solar energy. These alternative energy sources may also benefit from both learning and R&D cost cutting progress. Such technical advances possibilities should modify the timing of optimal transitions between energy sources, as shown by Chakravorty *et al.* in a recent paper (Chakravorty, 2012) in the case of learning-by-doing. The same type of conclusions emerges from the study of Henriet (2012) for a R&D induced technical breakthrough in a clean energy source production cost (Henriet, 2012).

As remarked by Gerlagh *et al.*(2009), the Goulder-Matthai analysis does not exhaust the concern expressed by Jaffe, Newell and Stavins (2001) concerning the need of a better understanding of the impact of environmental policies upon the nature of induced technological change and the feedback effect of technical change upon the environment itself and thus upon environmental policies.

Contributing to this understanding is the main objective of the present work. Many confusions arise in the previous literature because of the usual incremental way of modeling technical progress. This is especially true when comparing learning-by-doing and research induced technical change. In an incremental model, technical change is a sequence of small improvements progressively reducing the cost of the abatement technology. But incremental actual cost cuts achieved through learning and potential cost cuts achieved by research activity are not really comparable. This is one of the main reason for the ambiguous effects of an environmental policy in a learning-by-doing model shown by Goulder and Matthai.

To escape this difficulty, we adopt a drastic view of technical change, more in line of the Barret initial proposal. Thanks to a combination of R&D activity and learning-by-doing, it is possible to increase over time some know-how index. Once the index has reached a given target, it induces an abrupt revolution in abatement technologies, taking the form of a cost drop from a high level to a low level. To simplify, we assume only one revolution of this kind, meaning that future learning or research activity will become worthless after the revolution.

We make a parallel simplification concerning the dynamics of the environment. Most papers model the environmental dynamics as a progressive accumulation of carbon into the atmosphere, the size of the carbon stock generating welfare damages at each point of time. These damages are increasing with the size of the pollution stock. We depart from this approach by using an alternative route pioneered by Chakravorty *et al.* (2006, 2008) . We assume that the atmospheric carbon stock does not harm directly welfare, but be crossed over some critical threshold level in carbon concentration, earth climate conditions would become catastrophic. This echoes the current policy proposals of targeting a temperature rise of no more than $2^0$ C, that is actually trying to stabilize the carbon concentration to a constant level by the end of the century. Hence the environmental policy takes the form of a given mandate over the maximum level of the atmospheric carbon stock.

To simplify farther, we assume in the present paper that fossil fuels are not exhaustible. Introducing depletion constraints over fossil fuels will result in complex Hotelling effects affecting both the timing of carbon accumulation and the timing of technological development. We deserve the study of these issues to a companion paper (Amigues *et al.*, 2012).

One drawback of the Goulder-Matthai analysis is that they focus on the polar cases of learning and R&D induced technical change, but these polar cases are extreme situations where the economy would be constrained to use only one device to trigger technological advances. We encompass this limitation by examining a model where both activities contribute to technical progress. This will allow a much better understanding of the delay problem raised in the earlier literature. In particular we show how can be endogenously determined time periods during which the economy should perform only R&D to enhance the technical efficiency of pollution abatement and time periods during which a combination of learning processes and research activity is optimal.

The model is laid down in section 2. In order to drive interesting comparisons with previous results of the literature, we study in section 3 the case of a pure learning-by-doing induced technological revolution and in section 4 the case of a pure R&D induced technological break. We improve on earlier studies by performing rather systematically a sensitivity analysis of the main variables. Usually, one finds such sensitivity analysis in simulation models, but their results are typically hard, if not impossible, to interpret. Our simple setting allows us to derive sensitivity results in the analytical domain, providing strong intuitions on our findings. In particular, we shall exhibit the similarities and the differences between the cases of a learning induced or a R&D induced technical change. In section 5, we examine the general case of a combined learning and R&D process. We derive the implications of such a process for an optimal environmental policy. We also describe the optimal technological development policy which may be of a combined type, a pure learning or a pure R&D type depending upon the model fundamentals. The last section 6 concludes.

## 2 The model

The economy has access to two primary energy sources. The first one is a polluting resource (let say coal). We assume an infinite supply of this resource, meaning that it will never be exhausted, that is we treat coal as a kind of a renewable polluting resource, or equivalently assume that coal is abundant. Let $x(t)$ be the rate of coal extraction. The second energy source is a clean renewable resource (let say solar) and we denote by $y(t)$ the used flow of solar energy.

Assuming for the sake of simplicity a one to one transformation process of primary energy sources units into energy services units, the production of solar energy services bears a cost $c_y y(t)$. The processing of coal into the generation of energy services may take two forms. Coal may be processed without consideration for the environmental consequences of burning this fossil fuel to produce energy. We call dirty coal processing this energy generation process and $x_d(t)$ denotes the corresponding coal energy services generation rate. The cost of dirty coal processing is $c_x x_d(t)$. It results into a pollution flow $\zeta x_d(t)$ assumed proportional to the dirty coal energy generation. Under our one to one transformation process assumption, $x_d$ is also that fraction of coal extraction involved into dirty processing and $\zeta$ is the polluting content of coal.

The pollution flow accumulates into the environment and $Z(t)$ is the pollution stock size at time $t$. There exists a self-cleaning capacity of the environment, assumed to simplify proportional to the pollution stock size, so that the motion of $Z(t)$ over time is given by:

$$\dot{Z}(t) \quad = \quad \zeta x_d(t) - \alpha Z(t) \quad .$$

The initial pollution stock is $Z(0) = Z^0$.

Coal may be also processed through a clean energy generation process, thanks for example to CCS effort, resulting in no carbon emissions into the atmosphere. $x_c(t)$ is the rate of clean coal services generation and $(c_x + c_s(t))x_c$ is the cost of clean coal energy services. Under our transformation assumption, $x_c(t)$ is also that fraction of coal extraction involved in clean coal energy production, so that: $x(t) = x_d(t) + x_c(t)$.

Energy services differ by their primary sources (coal or solar) and by their type of coal processing (either dirty or clean coal energy generation) but they are perfect substitutes for the final users. Let $q(t) \equiv x_d(t) + x_c(t) + y(t)$ be the instantaneous consumption rate of energy services. This consumption generates a gross surplus, $u(q)$, assumed increasing and concave and satisfying the first Inada condition: $\lim_{q \downarrow 0} u'(q) = +\infty$.

As in Chakravorty *et al.* (2006), we assume that pollution does not harm directly welfare but be crossed over some critical threshold $\bar{Z}$, earth climate conditions would become catastrophic. Thus the society decides to maintain the carbon concentration below this critical level. To give content to the problem we have to assume that $Z^0 \leq \bar{Z}$.

Operating clean coal energy production equipments benefits both from learning-by-doing and dedicated research efforts. The cost reduction that may be achieved through these two processes may be defined in different ways. Here we adopt a drastic view of technical progress. The combination of the accumulation of experience with R&D efforts results into a technological revolution in the clean coal energy generation process. To describe this combined process, we adopt the simplest formulation able to retain the main aspects of the problem. Both learning-by-doing and R&D contribute to the accumulation over time of some stock of know-how. Let $A(t)$ be the level of this stock at time $t$. $A(t)$ grows over time at a rate depending upon the production scale of clean coal energy, $x_c(t)$, and upon the R&D effort rate, $r(t)$, through the following relation:

$$\dot{A}(t) = a(x_c(t), r(t)) \ .$$

$a(x_c, r)$ is twice continuously differentiable and both $a_c \equiv \partial a/\partial x_c > 0$, $a_r \equiv \partial a/\partial r > 0$. Know-how may be increased through only learning or R&D, that is: $a(0, r) > 0$ if $r > 0$ together with $a(x_c, 0) > 0$ if $x_c > 0$, while $a(0, 0) = 0$. Assume that $A(0) = 0$, that is normalize to zero the initial know-how index. Once some sufficient level of know-how, $\bar{A}$, has been attained, the technological revolution occurs, resulting into a sudden drop down of the cost of clean coal energy generation, from a high level $\bar{c}_s$, to a low level $\underline{c}_s$. Thus the additional clean coal energy cost is a function of $A(t)$, $c_s(A(t))$, such that:

$$c_s(A(t)) = \begin{cases} \bar{c}_s & \text{if} \quad A(t) < \bar{A} \\ \\ \underline{c}_s & \text{if} \quad A(t) \geq \bar{A} \end{cases} .$$

We assume to simplify only one technological revolution of this kind, meaning that future learning will be worthless after the revolution and that further R&D efforts will not allow for future cost breaks.

R&D activity has a cost $C_r(r)$, a cost function we assume twice continuously differentiable over $r \in (0, \infty)$, increasing and convex in $r$ while $C_r(0) = 0$. So the marginal cost function $c_r(r) \equiv dC_r(r)/dr$ defined over $(0, \infty)$ verifies: $c_r(r) \geq 0$, $c_r'(r) > 0$ and in addition $\lim_{r \downarrow 0} c_r(r) = c_r^0 \geq 0$, the right end limit of the marginal R&D cost at zero is not necessarily zero.

The society has to determine a primary resources policy use, a split between dirty and clean coal energy generation, together with a R&D policy maximizing a discounted sum of instantaneous net surpluses, $\rho > 0$ being the constant level of the social discount rate, while taking into account the atmospheric carbon concentration constraint, $Z \leq \bar{Z}$.

This problem may be given different formulations depending upon the model fundamentals. If the cost of the clean solar energy is lower than the cost of dirty coal generation, then coal is never used and the pollution problem disappears. So we assume that $c_x < c_y$. It may be the case that clean coal energy generation is so costly even after the revolution that the society will prefer to produce only dirty coal energy services. In such a case, there will be no learning about the clean coal technology and R&D efforts will be worthless and thus no cost breakthrough can occur. This scenario where $x(t) = x_d(t)$ has been already studied by Chakravorty *et al.*(2006). It may also be the case that the pollution ceiling is never attained, a scenario where the more costly clean coal option would never be engaged.

In order to drive an interesting discussion, we assume first that the ceiling constraint binds eventually along the optimal path and, second, that the clean cost option is not too costly to be used at least over some time interval, maybe only after the technological revolution. We shall be more precise about the relevant assumptions for that to be the case in the sequel. If clean coal energy generation is profitable it will be used permanently after its introduction inside the energy mix. Either as a pure consequence of learning-by-doing, in case of no R&D efforts, or as a result of the combination of learning and R&D, the level of know-how will permanently rise, triggering the revolution at some time, $\bar{t}_A$. Then the optimal program may be designed as a sequential optimal control problem composed of two phases: a first phase $[0, \bar{t}_A)$ before the break and a second phase $[\bar{t}_A, \infty)$ after the break.

An optimal policy is hence a solution of the following program $OP$:

$$\begin{aligned}
\max_{x_c, x_d, y, r, \bar{t}_A} \quad & \int_0^{\bar{t}_A} \left[ u(q(t)) - c_x x(t) - \bar{c}_s x_c(t) - c_y y(t) - C_r(r(t)) \right] e^{-\rho t} dt + e^{-\rho \bar{t}_A} \bar{V} \\
s.t. \quad & \dot{Z}(t) = \zeta x_d(t) - \alpha Z(t) \quad Z(0) = Z^0 \text{ given} \\
& \dot{A}(t) = a(x_c(t), r(t)) \quad A(0) = 0 \\
& x_c(t) \geq 0 \ , \ x_d(t) \geq 0 \ , \ y(t) \geq 0 \ , \ r(t) \geq 0 \\
& x_c(t) + x_d(t) \leq x(t) \\
& Z(t) \leq \bar{Z} \\
& A(\bar{t}_A) \geq \bar{A} \ .
\end{aligned}$$

$\bar{V}$ is the continuation value obtained by solving the following continuation problem after the technological revolution:

$$\begin{aligned}
\max_{x_c, x_d, y} \quad & \int_{\bar{t}_A}^{\infty} \left[ u(q(t)) - c_x x(t) - \underline{c}_s x_c(t) - c_y y(t) \right] e^{-\rho (t - \bar{t}_A)} dt \\
s.t. \quad & \dot{Z}(t) = \zeta x_d(t) - \alpha Z(t) \quad Z(\bar{t}_A) = Z^A \text{ given} \\
& x_c(t) \geq 0 \ , \ x_d(t) \geq 0 \ , \ y(t) \geq 0 \\
& x_c(t) + x_d(t) \leq x(t) \\
& Z(t) \leq \bar{Z} \ .
\end{aligned}$$

Before examining the policies solving the program $OP$, it is useful to consider as benchmarks two polar cases, the case of a pure learning-by doing know-how generation and the case of a pure R&D generation of know-how. We devote the next two sections to these polar cases before turning to the general case.

# 3 Technological revolution induced by learning

Assume no R&D opportunities. The economy has to rely only upon learning-by-doing, that is on experience accumulation, to trigger the technological revolution. The simplest way to define experience, and thus here the know-how index, is to identify it with the cumulated number of clean coal energy services units generated since the beginning of clean coal energy production. Let $\underline{t}_c$ be the beginning time of clean coal energy production, then:

$$A(t) \equiv \int_{\underline{t}_c}^{t} x_c(\tau)d\tau \ .$$

Denote by $\lambda_A(t)$ and $\lambda_Z(t)$ respectively the costate variables associated to the state variables $A(t)$ and $Z(t)$. Denote also by $\nu_{xc}$, $\nu_{xd}$, $\nu_y$, the Lagrange multipliers associated to the positivity constraints over $x_c$, $x_d$, $y$, respectively, and by $\nu_Z$, the Lagrange multiplier associated to the constraint $Z(t) \leq \bar{Z}$; The optimality conditions over the time interval $[0, \bar{t}_A)$ are:

$$u'(q) = c_x + \bar{c}_s - \lambda_A - \nu_{xc} \tag{3.1}$$

$$u'(q) = c_x + \zeta\lambda_Z - \nu_{xd} \tag{3.2}$$

$$u'(q) = c_y - \nu_y \tag{3.3}$$

$$\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \tag{3.4}$$

$$\dot{\lambda}_A = \rho\lambda_A \ . \tag{3.5}$$

Let us first sketch as a benchmark the optimal policy absent any learning abilities, the extra cost of producing clean coal energy with respect to dirty coal energy being $c_s$ . A relevant scenario involves hitting the ceiling at some finite time $\underline{t}_Z$. We have to consider two possibilities. Either $c_y > c_x + c_s$, the high solar cost case, either $c_y < c_x + c_s$, the low solar cost case.

In the high solar cost case, solar energy is never introduced inside the energy mix. When at the ceiling, dirty coal energy generation is constrained by the natural regeneration capacity, so that the production of dirty coal energy is given by $\bar{x}_d \equiv \alpha\bar{Z}/\zeta$. Let $\bar{p} \equiv u'(\bar{x}_d)$. If $\bar{p} < c_x + c_s < c_y$, then the economy prefers to rely only upon dirty coal energy generation and never uses either clean coal energy generation or solar energy generation. The optimal path is a two phases path. During the first phase $[0, \underline{t}_Z)$, the economy produces only dirty coal energy, pollution accumulates and $x_d(t)$ is the solution of $u'(x) = c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$, where $\lambda_{Z0} = \lambda_Z(0)$. This phase ends when the pollution stock reaches the ceiling $\bar{Z}$. Then begins a phase of infinite duration, $[\underline{t}_Z, \infty)$ of dirty coal generation at the level $\bar{x}_d$.

If $c_x + c_s < \bar{p}$, clean coal energy is never introduced before the beginning of the ceiling phase. Identifying (3.2) and (3.1), we get during any time interval below the ceiling where dirty and clean coal generation would be simultaneously operated: $\zeta\lambda_Z(t) = c_s$, which is incompatible with $\lambda_Z(t)$ growing at the rate $\rho + \alpha$. But since solar energy is even more costly than clean coal energy, solar energy is also never used before the ceiling. Thus, the economy produces only dirty energy until the ceiling constraint begins to be binding. During this first phase $[0, \underline{t}_Z)$, the implicit energy price $p(t) \equiv u'(q(t)$ is defined through (3.2) and (3.4) by: $p(t) = c_x + \lambda_{Z0}e^{(\rho+\alpha)t}$. Thus, the implicit energy price increases over time while dirty coal energy generation is progressively reduced. When at the ceiling, the economy starts to produce clean coal energy. The energy implicit price $p(t)$ is now constant and equal to the marginal cost of clean coal energy production $c_x + c_s$. Since solar energy is more expensive than clean coal energy, it is also never used during this time phase. The optimal policy when at the ceiling combines the production of dirty coal energy at the constant rate $\bar{x}_d$ with the production of clean coal energy at the constant rate $\bar{x}_c$, solution of $u'(\bar{x}_d + x_c) = c_x + c_s$. Note that the energy price continuity at $\underline{t}_Z$ requires that the dirty coal energy production path jumps down at $\underline{t}_Z$ from the level $\bar{x}_d + \bar{x}_c$ to the level $\bar{x}_d$ while the clean coal energy production rate jumps up from 0 to $\bar{x}_c$.

Last, in the low solar cost case, we have to distinguish the possibilities $c_y < \bar{p}$ and $\bar{p} < c_y$. If $c_y < \bar{p}$, solar energy is introduced when the ceiling constraint becomes to be binding and clean coal energy generation is never put in operation. Thus after $\underline{t}_Z$, energy production combines dirty coal energy generation at the rate $\bar{x}_d$ and solar energy generation at the rate $\bar{y}$, solution of $u'(\bar{x}_d + y) = c_y$. If $\bar{p} < c_y$, the economy prefers to rely only upon dirty coal energy generation and never uses clean energy in any form: clean coal energy or solar energy.

We turn now to a sensitivity analysis of the optimal policy with respect to some relevant parameters. Consider the optimal scenario in the high solar cost case with $c_x + c_s < \bar{p}$. To completely characterize the optimal policy, one has to identify two variables, $\lambda_{Z0}$, the initial level of the pollution opportunity cost, and $\underline{t}_Z$, the time at which the ceiling is attained. Let $x_d(t, \lambda_{Z0})$ be implicitly defined as the solution of $u'(x) = c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. $(\lambda_{Z0}, \underline{t}_Z)$ are solutions of the following system of conditions:

$$\begin{aligned} \zeta\lambda_{Z0}e^{(\rho+\alpha)t} &= c_s \\ \bar{Z}e^{\alpha\underline{t}_Z} &= Z^0 + \zeta\int_0^{\underline{t}_Z} x_d(t, \lambda_{Z0})e^{\alpha t}dt \ . \end{aligned}$$

We concentrate upon the parameters $c_s$, $Z^0$ and $\bar{Z}$.

Let $I_Z^Z \equiv -\int_0^{\underline{t}_Z}(1/u''(q(t))e^{(\rho+2\alpha)t}dt$ and $\Delta_0 \equiv \zeta\left[\zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z + x_c(\underline{t}_Z)e^{\alpha\underline{t}_Z}\right]$. Then it is easily verified that:

$$\begin{aligned}
\frac{d\lambda_{Z0}}{dc_s} &= \frac{x_c(\underline{t}_Z)e^{-\rho\underline{t}_Z}}{\Delta_0} > 0 \quad ; \quad \frac{d\underline{t}_Z}{dc_s} = \frac{\zeta I_Z^Z e^{-(\rho+\alpha)\underline{t}_Z}}{\Delta_0} > 0 \\
\frac{d\lambda_{Z0}}{dZ^0} &= \frac{(\rho+\alpha)\lambda_{Z0}}{\Delta_0} > 0 \quad ; \quad \frac{d\underline{t}_Z}{dZ^0} = -\frac{1}{\Delta_0} < 0 \\
\frac{d\lambda_{Z0}}{d\bar{Z}} &= -\frac{(\rho+\alpha)\lambda_{Z0}e^{\alpha\underline{t}_Z}}{\Delta_0} < 0 \quad ; \quad \frac{d\underline{t}_Z}{d\bar{Z}} = \frac{e^{\alpha\underline{t}_Z}}{\Delta_0} > 0\ .
\end{aligned}$$

As expected, a higher clean coal energy cost translates into a larger opportunity cost of pollution. This is an immediate consequence of the fact that a higher clean coal cost means a lower clean coal energy production and thus a lower energy consumption rate when at the ceiling. Since the energy price level at the ceiling (equal to the clean coal marginal cost) is increased while the rise of the opportunity cost of pollution makes increase the energy price also before the ceiling, the overall effect over the time length before the ceiling could be ambiguous. However, the analysis shows that it must increase, the direct effect over the energy price at the ceiling being larger than the indirect effect over the energy price before the ceiling. The effects of either a larger initial pollution stock or a stricter ceiling constraint are straightforward. Both result in an increased opportunity cost of pollution and a faster attainment of the ceiling.

Next, we examine the changes introduced by learning abilities to this benchmark scenario. Learning abilities do not modify our original result that clean coal energy is never introduced before the ceiling phase. They do not change either our conclusion that solar energy is never introduced inside the energy mix if $c_x + \bar{c}_s < c_y$ and eliminates the clean coal energy option in the reverse case. Let us thus assume that solar energy is more costly than clean coal energy and that $c_x + \bar{c}_s < \bar{p}$.

The optimal path is composed of three phases. During a first phase $[0, \underline{t}_Z)$, the economy produces only dirty coal energy at a declining rate, the energy price growing at the rate $(\rho+\alpha)$. The pollution threshold $\bar{Z}$ is attained at $\underline{t}_Z$, the end of this phase. Then begins a second phase $[\underline{t}_Z, \bar{t}_A)$ during which the environmental constraint binds, the economy combines dirty coal energy generation constrained by the constant rate $\bar{x}_d = \alpha\bar{Z}/\zeta$ and clean coal energy generation. (3.5) defines $\lambda_A(t) = \lambda_{A0}e^{\rho t}$, $\lambda_{A0} = \lambda_A(0)$, $t \le \bar{t}_A$. Then, $u'(\bar{x}_d + x_c(t)) = c_x + \bar{c}_s - \lambda_{A0}e^{\rho t}$ defines implicitly $x_c(t, \lambda_{A0})$ during the time interval $[\underline{t}_Z, \bar{t}_A)$

and $\dot{x}_c(t) = -\rho\lambda_{A0}e^{\rho t}/u''(\bar{x}_d + x_c) > 0$. Clean coal energy generation increases before the cost break while the implicit energy price decreases. The use of the pollution abatement technology results in experience accumulation up to the level $\bar{A}$, attained at time $\bar{t}_A$, at which the technological revolution occurs and the pollution abatement cost falls from the level $\bar{c}_s$ down to $\underline{c}_s$. Last, the economy enters an infinite duration phase $[\bar{t}_A, \infty)$ combining dirty and clean coal energy generation, the energy price being constant and equal to the post-revolution clean coal energy marginal cost $c_x + \underline{c}_s$.

After the cost breakdown, the economy produces clean coal energy at the constant rate $\underline{x}_c$ solution of: $u'(\bar{x}_d + x_c) = \underline{c}_s$. Thus, $\bar{V}$ the continuation value after the cost break in current terms at $\bar{t}_A$ is given by:

$$\bar{V} \quad = \quad \frac{1}{\rho}\left[u(\bar{x}_d + \underline{x}_c) - c_x(\bar{x}_d + \underline{x}_c) - \underline{c}_s\underline{x}_c\right] \ .$$

$\bar{t}_A$ must verify the following transversality condition:

$$\mathcal{H}(\bar{t}_A) \quad = \quad -\frac{\partial}{\partial \bar{t}_A}\bar{V}e^{-\rho\bar{t}_A} \ .$$

Since the economy is blockaded at the ceiling during the first phase of clean coal energy generation $[\underline{t}_Z, \bar{t}_A)$, $\dot{Z}(\bar{t}_A) = 0$. Denote by $\lim_{t\uparrow\bar{t}_A} x_c(t) = x_c^-$, the above condition is equivalent to:

$$\begin{aligned} u(\bar{x}_d + x_c^-) - c_x(\bar{x}_d + x_c^-) - \bar{c}_s x_c^- + \lambda_A(\bar{t}_A)x_c^- \quad &= \quad u(\bar{x}_d + \underline{x}_c) - c_x(\bar{x}_d + \underline{x}_c) \\ &\quad\quad -\underline{c}_s\underline{x}_c \ . \end{aligned}$$

Simplifying the $c_x\bar{x}_d$ term on both sides and taking into account (3.1): $u'(\bar{x}_d + x_c^-) = c_x + \bar{c}_s - \lambda_A(\bar{t}_A)$ while $u'(\bar{x}_d + \underline{x}_c) = c_x + \underline{c}_s$, we get:

$$u(\bar{x}_d + x_c^-) - u'(\bar{x}_d + x_c^-)x_c^- \quad = \quad u(\bar{x}_d + \underline{x}_c) - u'(\bar{x}_d + \underline{x}_c)\underline{x}_c \ .$$

Let $\Gamma(x) \equiv u(\bar{x}_d + x) - u'(\bar{x}_d + x)x$. Then $d\Gamma(x)/dx = -u''(\bar{x}_d + x)x > 0$ shows that $\Gamma(x)$ is a monotonously increasing function of $x$, hence is bijective, showing that $x_c^- = \underline{x}_c$. The clean coal energy generation rate is a continuous time function at $\bar{t}_A$. The Figure 1 illustrates the dynamics of the corresponding energy price path.

During the first phase $[0, \underline{t}_Z)$ before the ceiling constraint begins to be binding, only dirty coal energy is produced and the energy price rises at the rate $\rho + \alpha$, just as in the

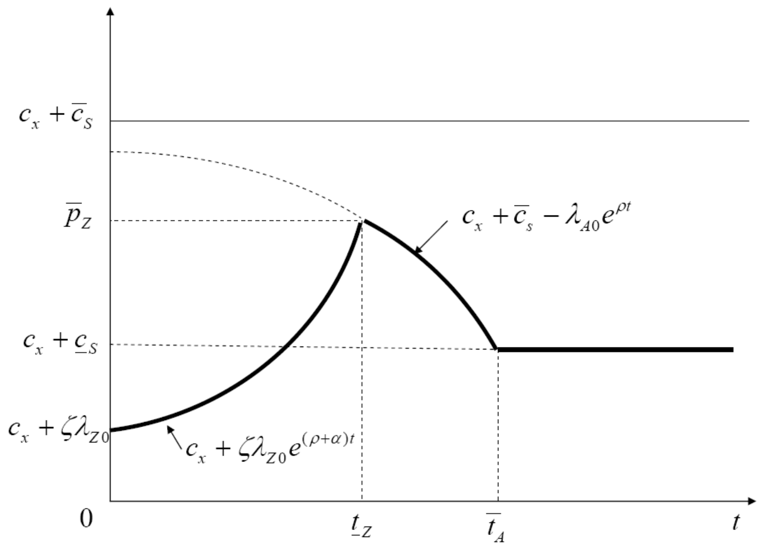


Figure 1: **Price Dynamics in the Pure Learning Case**

no learning abilities benchmark. But because of the learning process, the energy price at the beginning of the ceiling phase, $p(\underline{t}_Z) \equiv \bar{p}_Z$ is now lower than $c_x + \bar{c}_s$. The energy price next decreases before the cost breakthrough until the break occurs and the price stabilizes forever at the level $c_x + \underline{c}_s$. The energy production rates experience the same kind of jumps described in the benchmark scenario without learning abilities. The use of clean coal energy jumps from 0 up to $x_c(\underline{t}_Z) \equiv x_c^Z$, solution of $u'(\bar{x}_d + x_c) = \bar{p}_Z$. The production of dirty coal energy makes a parallel jump down, from the level $\bar{x}_d + x_c^Z$ to the level $\bar{x}_d$.

In terms of policy tools implementation, an optimal account of learning abilities requires two instruments. The first one is a carbon tax (or a carbon price in a cap and trade system) upon dirty coal energy generation. The tax must be rising at the rate $\rho + \alpha$ before the ceiling begins to be binding and clean energy generation is introduced inside the energy mix. Then the carbon tax should decline over time before stabilizing at the level $\underline{c}_s$ after the cost breakdown occurs. The use of clean energy must be subsidized at the consumption stage during the first phase of clean coal energy generation $[\underline{t}_Z, \bar{t}_A)$. The clean energy production sector supplies clean energy at its marginal cost $c_x + \bar{c}_s$ during this phase, resulting into a clean energy production price equal to this marginal cost. The subsidy is given by $\lambda_{A0}e^{\rho t}$ and allows for an energy price reduction at the consumption stage. Since the carbon tax is equal to $(\bar{c}_s - \lambda_{A0}e^{\rho t})/\zeta$, the dirty coal consumption is maintained to its mandated level

$\bar{x}_d$. The subsidy increases over time, allowing for a permanent increase of clean coal energy consumption until the cost breakdown occurs. After $\bar{t}_A$, the subsidy is removed and the production and consumption prices of clean energy are identical and equal to $c_x + \underline{c}_s$. The maximum level of the subsidy, attained at time $\bar{t}_A$ is the cost gap $\bar{c}_s - \underline{c}_s$. The current value level of the subsidy is thus $(\bar{c}_s - \underline{c}_s)e^{-\rho(\bar{t}_A - t)}$ at time $t$, $\underline{t}_Z \leq t \leq \bar{t}_A$.

We have shown previously that without learning abilities, the economy prefers to rely only upon dirty coal generation if $\bar{p} < c_x + c_s$. The same happens with learning abilities if $\bar{p} < c_x + \underline{c}_s$. In this situation, the technological breakthrough is unable to induce a sufficiently low level of the pollution abatement marginal cost to justify beginning clean coal energy generation. In the intermediate case: $c_x + \underline{c}_s < \bar{p} < c_x + \bar{c}_s$, a new possible scenario emerges. In this scenario, clean coal energy use is delayed after the attainment of the ceiling until some time $\underline{t}_c$. Then clean coal energy generation expands until the cost breakdown occurs. Remark that there should be no quantity discontinuity in this scenario. The use of dirty coal energy is maintained to the level $\bar{x}_d$ while the use of clean coal energy is initially nill at $\underline{t}_c$. However, Appendix A.1 shows that the economy cannot improve over a policy based upon the sole use of dirty coal energy by adopting such a combined policy.

Let us retain the case $c_x + \bar{c}_s < \bar{p}$. To characterize the optimal policy with learning abilities, we have to identify four variables, the initial values of $\lambda_Z$ and $\lambda_A$ together with the optimal time to attain the ceiling $\underline{t}_Z$ and the optimal time to trigger the technological revolution, $\bar{t}_A$. Let $x_d(t, \lambda_{Z0})$ be implicitly defined by $u'(x) = c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ over the time interval $[0, \underline{t}_Z)$ and $x_c(t, \lambda_{A0})$ be implicitly defined by $u'(\bar{x}_d + x_c) = c_x + \bar{c}_s - \lambda_{A0}e^{\rho t}$ over the time interval $[\underline{t}_Z, \bar{t}_A)$. $(\lambda_{Z0}, \lambda_{A0}, \underline{t}_Z, \bar{t}_A)$ are solutions of the following system of four conditions:

- The ceiling attainment condition, $Z(\underline{t}_Z) = \bar{Z}$:

$$\bar{Z}e^{\alpha \underline{t}_Z} \quad = \quad Z^0 + \zeta \int_0^{\underline{t}_Z} x_d(t, \lambda_{Z0})e^{\alpha t}dt$$

- The critical experience level attainment condition at the revolution time, $A(\bar{t}_A) = \bar{A}$:

$$\bar{A} \quad = \quad \int_{\underline{t}_Z}^{\bar{t}_A} x_c(t, \lambda_{A0})dt$$

- The price continuity requirement at $\underline{t}_Z$:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} \quad = \quad \bar{c}_s - \lambda_{A0}e^{\rho \underline{t}_Z}$$

- The price continuity requirement at the cost break time:

$$\lambda_{A0}e^{\rho\bar{t}_A} = \bar{c}_s - \underline{c}_s$$

Denote by:

$$\begin{aligned}
& I_Z^Z \equiv -\int_0^{\underline{t}_Z} \frac{e^{(\rho+2\alpha)t}}{u''(q(t))}dt \ > \ 0 \ ; \ I_A \equiv -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{e^{\rho t}}{u''(q(t))}dt \ > \ 0 \\
& J_A^c \equiv -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{dt}{u''(q(t))} \ > \ 0 \ ; \ x_c^Z \equiv x_c(\underline{t}_Z) \ ; \ x_c^A \equiv x_c(\bar{t}_A) \\
& T_A \equiv \bar{t}_A - \underline{t}_Z \ ; \ \pi_Z \equiv \zeta(\rho+\alpha)\lambda_{Z0}e^{\alpha\underline{t}_Z} + \rho\lambda_{A0} \\
& \Delta_0 \equiv \zeta\left[\zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z + x_c^Z e^{\alpha\underline{t}_Z}\right] \ .
\end{aligned}$$

We refer to Appendix A.2 for calculation details. We show the following effects of a higher initial pollution stock over the optimal path features:

$$\frac{d\lambda_{Z0}}{dZ^0} = \frac{(\rho+\alpha)\lambda_{Z0}}{\Delta_0} > 0 \quad ; \quad \frac{d\underline{t}_Z}{dZ^0} = \frac{d\bar{t}_A}{dZ^0} = -\frac{1}{\Delta_0} < 0$$
$$\frac{d\lambda_{A0}}{dZ^0} = \frac{\rho\lambda_{A0}}{\Delta_0} > 0$$

The impact of a higher $Z^0$ is qualitatively the same as in the case without learning abilities: the pollution opportunity cost is higher and the attainment of the ceiling is accelerated. We remark that even if the levels of the variables are of course different, the qualitative expressions of the partial derivatives are the same with and without learning abilities. This is a consequence of the fact that a higher initial pollution stock mainly affects the optimal path before the ceiling phase. Computing the effect of a higher $Z^0$ over the price level at the beginning of the ceiling phase, $\bar{p}_Z$, confirms this result:

$$\begin{aligned}
\frac{d\bar{p}_Z}{dZ^0} &= \left[\frac{d\lambda_{Z0}}{dZ^0} + (\rho+\alpha)\lambda_{Z0}\frac{d\underline{t}_Z}{dZ^0}\right]e^{(\rho+\alpha)\underline{t}_Z} \\
&= \left[(\rho+\alpha)\lambda_{Z0} - (\rho+\alpha)\lambda_{Z0}\right]\frac{e^{(\rho+\alpha)\underline{t}_Z}}{\Delta_0} = 0
\end{aligned}$$

A higher initial pollution stock has no effect over the energy price at the beginning of the ceiling phase, and thus no effect over the production rate of clean coal energy at $\underline{t}_Z$. Furthermore, $d\underline{t}_Z/dZ^0 = d\bar{t}_A/dZ^0 < 0$ shows that the first phase at the ceiling keeps the same length, thus the production plan of clean coal energy is just translated sooner in time by a higher initial pollution level. This property induces an upper shift of the initial level of the learning rent $\lambda_A$ completely neutral in terms of current value subsidy levels throughout the phase $[\underline{t}_Z, \bar{t}_A)$.

One could be tempted to think that a stricter ceiling constraint would have the same qualitative effects than a higher initial pollution stock. But the induced change over dirty coal energy production modifies the comparative advantage of dirty coal versus clean coal energy generation and thus the value of experience acquisition. More precisely:

$$\begin{aligned}
\frac{d\lambda_{Z0}}{d\bar{Z}} &= -\frac{(\rho+\alpha)\lambda_{Z0}e^{\alpha\underline{t}_Z}}{\Delta_0} + \frac{\alpha\rho\lambda_{A0}T_A}{\zeta\Delta_0} \ ? \\
\frac{d\underline{t}_Z}{d\bar{Z}} &= \frac{e^{\alpha\underline{t}_Z}}{\Delta_0} + \frac{\alpha\rho\lambda_{A0}T_A I_Z^Z}{x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} > 0 \\
\frac{d\lambda_{A0}}{d\bar{Z}} &= -\frac{\rho\lambda_{A0}e^{\alpha\underline{t}_Z}}{\Delta_0} - \frac{\alpha\rho\lambda_{A0}T_A(\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z})}{x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} < 0 \\
\frac{d\bar{t}_A}{d\bar{Z}} &= \frac{e^{\alpha\underline{t}_Z}}{\Delta_0} + \frac{\alpha T_A(\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z})}{x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} > 0
\end{aligned}$$

A stricter ceiling constraint ($d\bar{Z} < 0$) has an ambiguous effect over the initial pollution opportunity cost. This translates into an ambiguous consequence over the energy implicit price path before the beginning of the ceiling phase. However the analysis shows a faster attainment of the ceiling, a higher learning rent together with a sooner technological revolution time. The effect of a stricter environmental standard combines two components shown in the above expressions. The first component is the effect of a change of $\bar{Z}$ for a given clean energy production path. As expected, this component works in the same direction as the effect of higher initial $Z^0$. It increases the pollution opportunity cost, fastens the ceiling attainment, increases the value of learning and fastens the revolution. The second component expresses the induced effect of a stricter ceiling upon $x_c(t)$, the clean coal energy production rate, during the pre-revolution phase at the ceiling, $[\underline{t}_Z, \bar{t}_A)$. This effect is depending upon $T_A$, the length of this time phase. This effect has a negative impact over the pollution opportunity cost, it reduces $\underline{t}_Z$, makes increase the value of learning and fastens the revolution. Thus these two effects work in the same direction for $\underline{t}_Z, \lambda_{A0}$ and $\bar{t}_A$ but in an opposite direction for $\lambda_{Z0}$.

This does not mean that the effect of a ceiling modification over the energy price at the beginning of the ceiling phase, $\bar{p}_Z$, and upon $T_A$, the time length of the first phase of clean coal production, is indeterminate. More precisely:

$$\begin{aligned}\frac{d\bar{p}_Z}{d\bar{Z}} &= \frac{\alpha\rho\lambda_{A0}T_A}{\zeta x_c^Z}e^{\rho\underline{t}_Z} > 0 \\ \frac{dT_A}{d\bar{Z}} &= \frac{\alpha T_A}{\zeta x_c^Z} > 0\ .\end{aligned}$$

Thus a stricter ceiling constraint results into a lower energy price at the beginning of the ceiling phase together with a shorter time period before the revolution once clean coal production begins. This implies that the production of clean coal energy is increased by a stricter ceiling constraint and hence that the energy implicit price is lower during the time phase $[\underline{t}_Z, \bar{t}_A)$. To this lower price level correspond both a higher subsidy level to clean coal energy and a lower carbon price. Note that a lower $\bar{p}_Z$ and a lower $\underline{t}_Z$ are compatible with either a higher or a lower level of $\zeta\lambda_{Z0}$, the initial level of the carbon price. Note also that learning abilities reverses the usual result that a stricter environmental constraint should translate into a higher optimal carbon tax. The analysis shows to the contrary that, in between the beginning of the ceiling period and the technological revolution, the carbon tax level is lowered by a stricter ceiling constraint.

We have shown also that a stricter ceiling means an increased use of clean coal energy and thus a faster technological revolution. This is reminiscent of the Porter, Van der Linde hypothesis (1995). Following Michael Porter, imposing 'tight' environmental regulation (that is 'more' than Pigouvian) should spur more R&D efforts from the energy industry. In the present context, improving the efficiency of pollution abatement requires an increased use of the clean coal energy generation technology. A stricter environmental standard, though perfectly 'Pigouvian', achieves this outcome quite naturally, by reducing the comparative advantage of dirty energy with respect to clean energy. However, this does not mean that before the beginning of clean coal energy use, a stricter environmental standard should translate into a higher carbon tax.

Turn to the consequences of a higher experience threshold triggering the technological revolution in clean coal energy generation. After computations, we get:

$$\begin{aligned}&\frac{d\lambda_{Z0}}{d\bar{A}} = \frac{\rho\lambda_{A0}}{\Delta_0} > 0 \quad ; \quad \frac{d\lambda_{A0}}{d\bar{A}} = -\frac{\rho\lambda_{A0}}{x_c^Z}\left[1 + \frac{\zeta\rho\lambda_{A0}I_Z^Z}{x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}}\right] < 0 \\ &\frac{d\underline{t}_Z}{d\bar{A}} = \frac{\zeta\rho\lambda_{A0}I_Z^Z}{x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} > 0 \quad ; \quad \frac{d\bar{t}_A}{d\bar{A}} = \frac{1}{x_c^Z}\left[1 + \frac{\zeta\rho\lambda_{A0}I_Z^Z}{\Delta_0 e^{\alpha\underline{t}_Z}}\right] > 0 \\ &\frac{d\bar{p}_Z}{d\bar{A}} = \frac{\rho\lambda_{A0}e^{\rho\underline{t}_Z}}{x_c^Z} > 0 \quad ; \quad \frac{dT_A}{d\bar{A}} = \frac{1}{x_c^Z} > 0\ .\end{aligned}$$

As expected, a higher experience requirement to trigger the cost break results into a lower level of the learning rent and a longer time before the cost break. The opportunity cost of pollution is increased together with the energy price at the beginning of the ceiling phase but because the learning rent is decreased in a higher proportion than $\lambda_Z$ is increased, the attainment of the ceiling is delayed.

The sensitivity analysis of a small increase of the clean energy cost before the break is more intricate since it affects simultaneously the price convergence condition towards the ceiling, the relative profitability of clean coal energy before the break and the value of learning in getting an increased cost cut. Denote by:

$$I_c \equiv \int_{\underline{t}_Z}^{\bar{t}_A} x_c(t) e^{-\rho t} dt > 0 \ .$$

Then we get first:

$$\begin{aligned}
\frac{d\lambda_{Z0}}{d\bar{c}_s} &= \frac{\rho I_c}{\Delta_0} > 0 \ ; \\
\frac{d\underline{t}_Z}{d\bar{c}_s} &= \frac{\zeta \rho I_Z^Z I_c}{x_c^Z \Delta_0 e^{\alpha \underline{t}_Z}} > 0 \ ; \\
\frac{d\bar{p}_Z}{d\bar{c}_s} &= \zeta e^{(\rho+\alpha)\underline{t}_Z} \left[ \frac{d\lambda_{Z0}}{d\bar{c}_s} + (\rho+\alpha)\lambda_{Z0} \frac{d\underline{t}_Z}{d\bar{c}_s} \right] > 0 \ .
\end{aligned}$$

Hence a higher clean energy cost induces a higher pollution opportunity cost together with a delayed attainment of the ceiling. The energy price at the beginning of the ceiling is also increased. These results are expected, note that the direct effect of the cost increase dominates the indirect effect over the energy price before the ceiling phase, resulting into a slower move towards the ceiling.

The effect of a higher initial clean coal energy cost over the learning rent is indeterminate but it is possible to show that:

$$\begin{aligned}
\frac{d\bar{t}_A}{d\bar{c}_s} &= \frac{\zeta}{x_c^Z \Delta_0 e^{\alpha \underline{t}_Z}} \left[ (\pi_Z I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z})(J_A^c - I_A e^{-\rho \bar{t}_A}) + x_c^Z I_Z^Z (e^{-\rho \underline{t}_Z} - e^{-\rho \bar{t}_A}) \right] > 0 \\
\frac{dT_A}{d\bar{c}_s} &= \frac{J_A^c - I_A e^{-\rho \bar{t}_A}}{x_c^Z} > 0
\end{aligned}$$

Hence a higher initial clean coal energy cost means a longer period before the cost breakdown. Since the energy price is shifted upward at the beginning of the ceiling phase, the exploitation of clean coal energy is reduced, implying a slower learning process and thus a delayed, although more significant, cost cut.

Last, considering the consequences of a lower clean energy cost after the break, we find that:

$$\frac{d\lambda_{Z0}}{d\underline{c}_s} > 0 \quad ; \quad \frac{d\lambda_{A0}}{d\underline{c}_s} < 0$$
$$\frac{d\underline{t}_Z}{d\underline{c}_s} > 0 \quad ; \quad \frac{d\bar{t}_A}{d\underline{c}_s} > 0$$
$$\frac{d\bar{p}_Z}{d\underline{c}_s} > 0 \quad ; \quad \frac{dT_A}{d\underline{c}_s} > 0$$

These effects fit the intuition. A higher clean energy cost level after the break, that is a lower cost cut thanks to learning, results into a higher pollution opportunity cost together with a delayed attainment of the ceiling. On the other hand, the learning rent is reduced and the time length before the cost break to occur is enlarged.

The following propositions summarize our findings:

**Proposition 1** *(i) With only learning abilities, clean coal energy generation is never introduced before the atmospheric ceiling constraint begins to be binding. If solar energy production is cheaper than clean coal energy, it eliminates this option and is itself eliminated in the reverse case. The optimal energy policy is a three phases path composed of a first phase of only dirty energy generation at a declining rate until the carbon ceiling level is attained. Then clean coal energy generation begins at an increasing rate while the production of dirty energy is constrained by the ceiling. This second phase ends at the technological revolution time. After the revolution, the economy stays permanently at the ceiling and produces a constant rate of clean coal energy at the post-revolution low marginal cost.*

*(ii) The possibility of a learning induced technological revolution in CCS lowers the carbon opportunity cost before the attainment of the ceiling, this cost being rising exponentially at the rate $(\rho + \alpha)$. During the pre-revolution phase at the ceiling, the energy implicit price decreases over time. There is no price cut at the technological breakthrough. After the break, the energy price remains constant and equal to the clean coal energy marginal production cost.*

*(iii) The implementation of the optimal policy requires to combine a carbon tax (or a carbon price in a cap and trade system) and a subsidy at the consumption stage to*

*clean energy use. The carbon tax increases before the atmospheric ceiling constraint begins to be binding , decreases during the second phase and stays constant after the cost revolution. The subsidy is introduced whence the ceiling is attained, it increases exponentially at the rate $\rho$ before the revolution and is suppressed after the cost break-through. Its current value level at time $t$ is given by $(\bar{c}_s - \underline{c}_s)e^{-\rho(\bar{t}_A - t)}$, the cost gap in present value from $t$.*

Concerning the sensitivity analysis with respect to some relevant parameters, we show that:

**Proposition 2** *(i) The pollution opportunity cost, or the optimal carbon tax, is increased by a higher initial pollution stock, a higher know-how requirement to trigger the technological revolution or a higher CCS cost before the revolution. A stricter environment standard, that is a lower $\bar{Z}$, has an ambiguous effect over the carbon price before the ceiling constraint begins to be binding and lowers this price whence the ceiling is binding.*

*(ii) The learning rent, or equivalently the subsidy needed to induce the optimal level of clean coal energy generation, is reduced by a higher initial pollution stock or a higher required know-how level to trigger the technological revolution. It is increased by a stricter environmental standard while a higher pre-revolution CCS cost has ambiguous effects over the learning rent.*

*(iii) The ceiling constraint binds earlier if the initial pollution stock is higher, the ceiling constraint more stringent, the know-how requirement to trigger the cost cut in CCS operation less stringent or the abatement cost before the revolution less expensive.*

*(iv) The length of the learning phase between the beginning of the ceiling phase and the revolution time is independent from the initial pollution stock. It decreases with a stricter ceiling constraint, a less stringent know-how target and a lower pre-revolution CCS abatement cost.*

# 4 The R&D induced technical revolution in abatement

Consider the reverse case of no learning abilities. The technical revolution only results from sufficient efforts in R&D. Take the simplest form for the consequences of such efforts over the accumulation of know-how, that is assume that $\dot{A} = r$. Then the optimality conditions before the revolution become:

$$u'(q) = c_x + \bar{c}_s - \nu_{xc} \quad (4.1)$$

$$u'(q) = c_x + \zeta\lambda_Z - \nu_{xd} \quad (4.2)$$

$$u'(q) = c_y - \nu_y \quad (4.3)$$

$$\lambda_A = c_r(r) - \nu_r \quad (4.4)$$

$$\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \quad (4.5)$$

$$\dot{\lambda}_A = \rho\lambda_A \; . \quad (4.6)$$

Assume that $c_y > \bar{p} > c_x + \bar{c}_s$, hence solar energy never enters the energy mix. As in the preceding case, clean coal energy generation should not be introduced before the beginning of the ceiling phase. Since R&D only affects the time of the revolution, whatever be the cost level, the pre-revolution high cost level or the post-revolution low cost level, the constancy of unit costs is incompatible with the rise of the pollution opportunity cost before the ceiling.

This in turn implies that the revolution should not occur strictly before the beginning of the ceiling phase. Assume to the contrary that $\bar{t}_A < \underline{t}_Z$. Then clean coal energy production begins at $\underline{t}_Z$ with the best technology of cost $\underline{c}_s$. Since there are no learning abilities, we are in the benchmark case exposed at the beginning of section 3. After reaching the ceiling, the energy price is constant and given by $c_x + \underline{c}_s$. Before $\underline{t}_Z$, the energy price is given by: $p(t) = c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ until $\bar{p}_Z = c_x + \underline{c}_s$ is attained at time $\underline{t}_Z$. Consider a small decrease of the research effort $dr < 0$ at each time during the time interval $[0, \bar{t}_A)$. The revolution time $\bar{t}_A$ would be slightly delayed but for a sufficiently low $dr$, the revolution occurs before $\underline{t}_Z$. Thus nothing would be changed to the resource use policy before and after $\underline{t}_Z$, the only consequence being a reduction in the R&D cost. Thus such a reduction would be beneficial, resulting in an optimal time of the revolution happening at least whence the ceiling has been attained, that is $\underline{t}_Z \le \bar{t}_A$ in any optimal scenario.

Next let us assume that the know-how requirement is sufficiently stringent to have $\underline{t}_Z < \bar{t}_A$. We shall be more precise later over the conditions for the revolution to occur only strictly after the beginning of the ceiling phase. After the revolution, the economy remains blockaded at the ceiling, the energy price is given by $c_x + \underline{c}_s$, the production of dirty coal energy by $\bar{x}_d$ and the production of clean coal energy by $\underline{x}_c$, thus, $\bar{V}$, the continuation value after the revolution is the same as before and given by:

$$\bar{V} \quad = \quad \frac{1}{\rho}\left[u(\bar{x}_d + \underline{x}_c) - c_x(\bar{x}_d + \underline{x}_c) - \underline{c}_s\underline{x}_c\right] \ .$$

Denote $h^- \equiv \lim_{t\uparrow\bar{t}_A} h(t)$ for any time function $h(t)$. The transversality condition at $\bar{t}_A$ is now expressed as:

$$u(\bar{x}_d + x_c^-) - c_x(\bar{x}_d + x_c^-) - \bar{c}_s x_c^- - C_r(r^-) + \lambda_A^- r^- \quad = \quad u(\bar{x}_d + \underline{x}_c) - c_x(\bar{x}_d + \underline{x}_c) - \underline{c}_s\underline{x}_c \ .$$

Taking (4.1) and (4.4) into account, this is equivalent to:

$$u(q^-) - u'(q^-)x_c^- - C_r(r^-) + c_r(r^-)r^- \quad = \quad u(\bar{x}_d + \underline{x}_c) - u'(\bar{x}_d + \underline{x}_c)\underline{x}_c$$

Denote by: $\Gamma_r(r) \equiv c_r(r)r - C_r(r)$. Since $C_r(0) = 0$, $\Gamma_r(0) = 0$ and $\Gamma_r' = c_r'(r)r > 0$ under our cost convexity assumption. Thus $\Gamma_r(r) > 0$ if $r > 0$. Denote by $\Gamma(x_c) \equiv u(\bar{x}_d + x_c) - u'(\bar{x}_d + x_c)x_c$, an increasing function of $x_c$, as shown before. Then the transversality condition is equivalent to:

$$\Gamma(\underline{x}_c) - \Gamma(x_c^-) \quad = \quad \Gamma_r(r^-) > 0$$

At the revolution time, $\bar{t}_A$, the energy price jumps down from the level $c_x + \bar{c}_s$ to the level $c_x + \underline{c}_s$. This corresponds to an upward jump of clean coal energy generation from the level $x_c^-$, solution of $u'(\bar{x}_d + x_c) = c_x + \bar{c}_s$, up to the level $\underline{x}_c$, itself solution of: $u'(\bar{x}_d + x_c) = c_x + \underline{c}_s$. The transversality condition shows that to this quantity jump corresponds a unique level of $r^-$, the research effort just before the revolution. Since $\lambda_A^-(\bar{t}_A) = c_r(r^-)$, the terminal level of the R&D knowledge rent is thus also determined. Let $\bar{\lambda}_A$ be this level, then: $\lambda_{A0}e^{\rho\bar{t}_A} = \bar{\lambda}_A$, taking (4.6) into account.

This shows a first difference between the R&D strategy to trigger the technological revolution and the learning strategy. Inducing the right level of experience acquisition, and thus the right level of clean coal energy production, required a specific subsidy in the preceding case. Such a subsidy device is no more needed to induce the optimal level of R&D investments, the carbon tax being a sufficient tool to implement the optimal scenario. The optimal time profile of the subsidy also resulted into a continuous energy price trajectory despite the cost revolution. This is no more the case under a R&D induced technological revolution and one obtains the usual conclusion that energy services are permanently priced at their marginal cost, the cost break resulting into a price breakdown at the revolution time.

A second difference appears in the computation of the R&D rent. While the learning rent simply identified with the cost gap in the preceding section, the R&D rent at the revolution time is defined through the transversality condition by a complex relation depending upon not only the shape of the R&D cost function but also upon the energy gross surplus function.

Third, the relative independency between the dynamics of energy use and the dynamics of know-how induced by the R&D policy widens the space of possible energy scenarios. Assume that the solar energy cost, $c_y$, is such that: $\underline{c}_s < c_y < \bar{c}_s$. In the learning induced revolution framework, solar energy would eliminate the use of clean coal energy and thus the possibility of a revolution. In the R&D induced revolution framework, the corresponding optimal scenario is the following. During a first phase $[0, \underline{t}_Z)$, the economy only relies upon the use of dirty coal energy until the ceiling is attained. Then solar energy is introduced in combination with dirty coal energy generation up to a level $\bar{y}$ solution of: $u'(\bar{x}_d + y) = c_y$. Thus, before the technological revolution, that is during the time interval $[\underline{t}_Z, \bar{t}_A)$, the economy is constrained by the ceiling but never produces clean coal energy, this one being more costly than solar energy. After the revolution, solar energy is eliminated from the energy mix and the economy combines the production of dirty energy at the rate $\bar{x}_d$ and of clean coal energy at the rate $\underline{x}_c$. Thus the energy transition scenario is composed of a first phase using only coal, a second phase using both coal and solar energy and a third phase using only coal but with a positive amount of clean coal energy.

Next, let us turn to the description of the optimal R&D policy. If $c_r(0) = c_r^0 > 0$, R&D investment may be delayed. Taking (4.4) into account, $\lambda_A(t) \geq c_r^0$ appears as a necessary condition for strictly positive R&D efforts, $r(t) > 0$. Since $\lambda_{A0} = \lambda_A(0)$ is determined by the whole model structure, this may or not be the case at time $t = 0$. If $\lambda_{A0} > c_r^0$, R&D effort is set immediately at a positive level. Since $c_r'(r) > 0$, the optimal R&D effort is implicitly defined by $\lambda_A(t) = c_r(r(t))$ as an increasing function of $\lambda_A$ and thus an increasing function of time since $\lambda_A(t)$ is growing exponentially. Let $r(t) = r_A(\lambda_A(t))$ be that function. Then $r(t)$ grows permanently over time. If $\lambda_{A0} < c_r^0$, R&D investments are delayed until $\underline{t}_A$ solution of $\lambda_A(t) = c_r^0$. At $\underline{t}_A$, R&D activity shows a smooth start from a zero level and then increases permanently over time as in the preceding case. One may observe that this feature goes in the opposite direction of many endogenous growth models where R&D efforts should be set initially at a high level and then be decreased. This is because these models usually assume the existence of increasing returns to scale in the knowledge generation process, returns to scale resulting from an inheritance effect of previously accumulated knowledge. Such effects are absent in the present model and we obtain the usual conclusion that because of discounting, R&D costs should be delayed in time. The result is an increasing R&D effort path, maybe from a zero initial level after some time period without research activity.

Let us first consider the optimal scenario in a situation where $\lambda_{A0} > c_r^0$. If $c_r^0 = 0$, this is the only optimal solution. It is identified by computing the vector of variables $(\lambda_{Z0}, \lambda_{A0}, \underline{t}_Z, \bar{t}_A)$, a vector solution of the following set of conditions:

- The ceiling attainment condition, $Z(\underline{t}_Z) = \bar{Z}$:

$$\bar{Z}e^{\alpha \underline{t}_Z} = Z^0 + \zeta \int_0^{\underline{t}_Z} x_d(t)e^{\alpha t}dt \ .$$

- The know-how requirement condition, $A(\bar{t}_A) = \bar{A}$:

$$\bar{A} = \int_0^{\bar{t}_A} r_A(\lambda_{A0}e^{\rho t})dt \ .$$

- The price continuity requirement at $\underline{t}_Z$:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_Z} = \bar{c}_s$$

- The R&D rent condition at $\bar{t}_A$:

$$\Gamma(\underline{x}_c) - \Gamma(x_c^-) = \Gamma_r(r_A(\lambda_{A0}e^{\rho \bar{t}_A})) \ .$$

It is worth contrasting the pure learning and pure R&D induced technological break-through models through a parallel comparative dynamics exercise to the one performed in

Section 3. The computations details are presented in Appendix A.3. The main conclusions are the following.

Denote as before $\Delta_0 = \zeta\left[\zeta(\rho+\alpha)\lambda_{Z0} + x_c^Z e^{\alpha \underline{t}_Z}\right]$ and by: $\bar{r} = \lim_{t\uparrow\bar{t}_A} r(t)$ and $r_0 \equiv r(0)$.

The effects of a larger initial pollution stock $Z^0$ or a stricter ceiling constraint are the following:

$$\begin{aligned}
&\frac{d\lambda_{Z0}}{dZ^0} = -\frac{d\lambda_{Z0}}{d\bar{Z}}e^{-\alpha\underline{t}_Z} = \frac{(\rho+\alpha)\lambda_{Z0}}{\Delta_0} > 0\ ;\ \frac{d\underline{t}_Z}{dZ^0} = -\frac{d\underline{t}_Z}{d\bar{Z}}e^{-\alpha\underline{t}_Z} = -\frac{1}{\Delta_0}\\
&\frac{d\lambda_{A0}}{dZ^0} = \frac{d\bar{t}_A}{dZ^0} = 0\\
&\frac{d\lambda_{A0}}{d\bar{Z}} = -\frac{\bar{r}\rho\lambda_{A0}}{\bar{r}-r_0}\frac{d\bar{t}_A}{d\bar{Z}} = -\frac{\alpha(\bar{c}_s-\underline{c}_s)e^{-\rho\bar{t}_A}}{\zeta r_0} < 0
\end{aligned}$$

The decoupling of the know-how dynamics from the economic arbitrages driving the energy policy removes the indeterminacy problem identified in the learning model. It appears clearly that a larger initial pollution stock or a stricter ceiling constraint have the same qualitative effects over the energy implicit price trajectory. Both make rise the pollution opportunity cost, and thus the energy price before the ceiling, and fasten the attainment of the ceiling.

The differences between a stricter ceiling an a higher initial pollution stock appear when considering the R&D optimal policy. There is no effect of the initial pollution stock over the R&D policy. Since the economy is permanently constrained by the ceiling after $\underline{t}_Z$, a stricter ceiling results into a higher R&D effort and thus in earlier technological revolution. We observed a similar accelerating effect in the learning model. Imposing a stricter environmental standard makes rise the R&D effort to trigger the cost revolution.

The consequences of a higher know-how requirement, $\bar{A}$, to trigger the technical revolution offer another illustration of the relative independency between the R&D policy and the energy policy. After computations, we get:

$$\begin{aligned}
&\frac{d\lambda_{Z0}}{d\bar{A}} = \frac{d\underline{t}_Z}{d\bar{A}} = 0\\
&\frac{d\lambda_{A0}}{d\bar{A}} = -\rho\lambda_{A0}\frac{d\bar{t}_A}{d\bar{A}} = -\frac{\rho\lambda_{A0}}{r_0} < 0
\end{aligned}$$

A higher knowledge target induces a slow down of the research efforts and thus a delayed technical revolution. A larger know-how requirement has no effect over the energy consumption policy, the ceiling being attained at the same time and the pollution opportunity cost being unaffected by a higher $\bar{A}$.

The independency feature disappears when considering the additional cost of clean coal energy production before the revolution, since this cost both affects the convergence condition of the energy price towards its ceiling level and the size of the cost breakdown. The calculus shows that:

$$\frac{d\lambda_{Z0}}{d\bar{c}_s} = \frac{x_c^z}{\Delta_0} e^{-\rho \underline{t}_Z} > 0 \quad ; \quad \frac{d\underline{t}_Z}{d\bar{c}_s} = \frac{\zeta I_Z^Z}{\Delta_0} e^{-(\rho+\alpha)\underline{t}_Z} > 0$$
$$\frac{d\lambda_{A0}}{d\bar{c}_s} = \frac{x_c^-}{r_0} e^{-\rho \bar{t}_A} > 0 \quad ; \quad \frac{d\bar{t}_A}{d\bar{c}_s} = -\frac{x_c^-(\bar{r}-r_0)}{r_0 \bar{r} \rho \lambda_{A0}} e^{-\rho \bar{t}_A} < 0$$

As in the pure learning model, a higher initial clean coal energy cost means a higher pollution opportunity cost together with a delayed arrival at the ceiling. As before, the direct effect over the energy price after $\underline{t}_Z$ resulting from a higher $\bar{c}_s$ dominates the indirect effect over the energy price of a higher pollution opportunity cost resulting in a longer time before the beginning of the ceiling phase. Contrarily to the learning model where the time length between the beginning of the learning process and the revolution time was enlarged by a higher $\bar{c}_s$, R&D is accelerated by the perspective of a larger cost breakthrough and the revolution comes earlier.

Since the clean coal cost level affects only the post-revolution phase, one should expect that a higher $\underline{c}_s$ has no effect upon the energy use before the revolution. Furthermore, the perspective of a smaller cost breakthrough should discourage research and delay the revolution time. The calculus confirms these straightforward intuitions.

Up to now we considered only scenarios where $\lambda_{A0} > c_r^0$, but we need to make precise the domain of validity of such policies. Let us consider an R&D policy starting at time 0 from $r(0) = 0$, that is $\lambda_{A0} = c_r^0$. Then the cost breakthrough occurs at a time $\bar{T}_A$ solution of:

$$\bar{A} \quad = \quad \int_0^T r_A(c_r^0 e^{\rho t}) dt$$

$\bar{T}_A$ is the maximum time delay to get the breakthrough since the economy starts from the lowest possible level of research efforts. Let $(\lambda_{Z0}^0, \underline{t}_Z^0)$ be defined as the solutions of:

$$\begin{aligned} \bar{Z} e^{\alpha \underline{t}_Z} &= Z^0 + \zeta \int_0^{\underline{t}_Z} x_d(t) e^{\alpha t} dt \\ \zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_Z} &= \underline{c}_s \ . \end{aligned}$$

$(\lambda_{Z0}^0, \underline{t}_Z^0)$ are the optimal initial levels of the pollution opportunity cost and time delay before the ceiling in a situation where the cost breakthrough would occur just when the ceiling constraint begins to be binding. If $\underline{t}_Z^0 > \bar{T}_A$, the active R&D phase has to be delayed until $\underline{t}_A = \underline{t}_Z^0 - \bar{T}_A$ as noticed before. In this scenario, clean energy production is introduced at $\underline{t}_Z^0$ with the best technology, the cost breakthrough occurring at $\underline{t}_Z^0$.

In the contrary case, triggering the technological revolution when attaining the ceiling requires to set $\lambda_{A0}$ above $c_r^0$ and thus $r(0) > 0$. To $\underline{t}_Z^0$ corresponds a unique value of $\lambda_{A0}$, we denote by $\lambda_{A0}^0$ solution of:

$$\bar{A} \quad = \quad \int_0^{\underline{t}_Z^0} r_A(\lambda_{A0} e^{\rho t}) dt \ .$$

Let $\bar{\lambda}_A^0 \equiv \lambda_{A0}^0 e^{\rho \underline{t}_Z^0}$.

We have to consider the transversality condition while taking explicitly into account the constraint: $\underline{t}_Z \leq \bar{t}_A$, which requires to modify this condition as such. Denote by $\mu_Z$, the Lagrange multiplier associated to the constraint $\underline{t}_Z \leq \bar{t}_A$. Then optimality requires that:

$$\mathcal{H}(\bar{t}_A) + \mu_Z \quad = \quad -\frac{\partial}{\partial \bar{t}_A} \bar{V} e^{-\rho \bar{t}_A}$$

with $\mu_Z \geq 0$ and $\mu_Z(\bar{t}_A - \underline{t}_Z) = 0$. This is equivalent to:

$$\bar{\Gamma} \ \equiv \ \Gamma(\underline{x}_c) - \Gamma(x_c^-) \ = \ \Gamma_r(r^-) + \mu_Z \ \geq \ \Gamma_r(r^-)$$

Note that $\bar{\Gamma}$ is given by the cost parameters and the energy demand shape, and is independent from either the R&D policy or the ceiling attainment condition. Let $\bar{r}$ be the solution of $\bar{\Gamma} = \Gamma_r(r)$ and $\bar{\lambda}_A = c_r(\bar{r}) \equiv \bar{\lambda}_A(\bar{\Gamma})$. Then, $\Gamma_r(r)$ being an increasing function of $r$, $\bar{r}$ is an increasing function of $\bar{\Gamma}$ and hence $\bar{\lambda}_A(\bar{\Gamma})$ is an increasing function of $\bar{\Gamma}$. This implies that if $\bar{\lambda}_A(\bar{\Gamma}) < \bar{\lambda}_A^0$, the constraint $\underline{t}_Z \leq \bar{t}_A$ does not bind, while it is binding in the reverse case.

Consider the case of a binding constraint, that is $\bar{\lambda}_A(\bar{\Gamma}) \geq \bar{\lambda}_A^0$. Let $r_0(t) \equiv r_A(\lambda_{A0}^0 e^{\rho t})$, the optimal R&D policy may be of two types. If $\underline{t}_Z^0 < \bar{T}_a$, it is defined by $r_0(t)$ over the time interval $[0, \underline{t}_Z^0)$. The research effort is initially strictly positive ($r_0(0) > 0$) and the cost break occurs at $\underline{t}_Z^0$. If $\underline{t}_Z^0 > \bar{T}_a$, then the active research phase is delayed until some time $\underline{t}_A = \underline{t}_Z^0 - \bar{T}_A$.

Turn now to the case of a non binding constraint: $\bar{\lambda}_A(\bar{\Gamma}) < \bar{\lambda}_A^0$. Then $\bar{\lambda}_A(\bar{\Gamma}) < \bar{\lambda}_A^0$ is equivalent to $r(\bar{t}_A) < r(\underline{t}_Z^0)$, implying that $\bar{t}_A > \underline{t}_Z^0$ to satisfy the knowledge accumulation constraint. Hence, $\lambda_{A0} < \lambda_{A0}^0$. The economy follows a less active R&D policy. To lower levels of $\bar{\Gamma}$ correspond lower levels of $\bar{\lambda}_A(\bar{\Gamma})$ and thus lower paths of R&D efforts. If $\bar{\Gamma}$ is such that $\bar{\lambda}_A(\bar{\Gamma}) < c_r^0$, R&D efforts become unprofitable and there is no cost breakthrough. We conclude that the optimal policy is one of the four possible types described in the following Proposition:

**Proposition 3** 1. *If $\bar{\lambda}_A(\bar{\Gamma}) < c_r^0$, there is no R&D activity and trivially the cost break never occurs, the society prefers to use clean coal energy when at the ceiling at the high cost level.*

2. *If $c_r^0 < \bar{\lambda}_A(\bar{\Gamma}) < \bar{\lambda}_A^0$, the active R&D policy starts immediately at time 0. R&D efforts increase over time and the cost breakthrough occurs strictly after the beginning of the ceiling phase, resulting in a time phase $[\underline{t}_Z, \bar{t}_A)$ where the economy uses the clean coal energy technology at its highest cost $\bar{c}_s$.*

3. *If $\bar{\lambda}_A^0 < \bar{\lambda}_A$ and $\underline{t}_Z^0 < \bar{T}_A$, then the economy starts to perform R&D efforts right from $t = 0$, the optimal R&D effort is given by $r_0(t)$ resulting in a cost breakthrough occurring just at the time $\underline{t}_Z^0$ when the ceiling constraint begins to bind.*

4. *If $\bar{\lambda}_A^0 < \bar{\lambda}_A$ and $\underline{t}_Z^0 > \bar{T}_A$, then the active R&D phase is delayed until some time $\underline{t}_A$ such that $\underline{t}_A = \underline{t}_Z^0 - \bar{T}_A$, also triggering the technological revolution just at the arrival at the ceiling.*

Concerning the sensitivity of the optimal path to some relevant parameters in the case $c_r^0 < \bar{\lambda}_A(\bar{\Gamma}) < \bar{\lambda}_A^0$, we have shown that:

**Proposition 4** *(i) The pollution opportunity cost before the ceiling constraint begins to bind, or equivalently the optimal carbon tax, is increased by a larger initial pollution stock, a stricter ceiling constraint or a higher CCS cost before the technological revolution. It is unaffected by the know-how target to trigger the revolution.*

*(ii) The ceiling constraint binds earlier with a higher initial pollution stock, a stricter ceiling constraint, a higher CCS cost before the revolution and is unaffected by the revolution know-how target.*

*(iii) The R&D rent, or equivalently the intensity of R&D efforts, is unaffected by the initial pollution stock. It is increased by a stricter ceiling constraint, a less stringent know-how requirement or a higher CCS cost before the revolution.*

*(iv) The technological breakthrough is delayed by a less stringent ceiling constraint, a more stringent know-how requirement to trigger the cost break or a lower initial CCS cost. The revolution time is independent from the initial pollution stock.*

It is interesting to contrast the sensitivity analysis of the learning induced and the R&D induced technological revolution. The following Table 1 summarizes our main findings, the pure R&D case qualitative effects appearing between parenthesis in the table.

| | $d\lambda_{Z0}$ | $d\underline{t}_Z$ | $d\lambda_{A0}$ | $d\bar{t}_A$ |
|---|---|---|---|---|
| $dZ^0$ | $+$ $(+)$ | $-$ $(-)$ | $+$ $(0)$ | $-$ $(0)$ |
| $d\bar{Z}$ | ? $(-)$ | $+$ $(+)$ | $-$ $(-)$ | $+$ $(+)$ |
| $d\bar{A}$ | $+$ $(0)$ | $+$ $(0)$ | $-$ $(-)$ | $+$ $(+)$ |
| $d\bar{c}_s$ | $+$ $(+)$ | $+$ $(+)$ | ? $(-)$ | $+$ $(-)$ |

Table 1: Comparing the learning and R&D sensitivity analysis

The table shows that the two technological breakthrough triggering devices, learning-by-doing or R&D, behave more or less the same in qualitative terms. A part from the independency property of the R&D way to trigger the revolution with respect to the energy policy we already noticed, we remark that one important difference lies in the effect of the initial CCS cost. A higher initial cost delays the revolution in a learning model while it accelerates it in a R&D model. This is a fairly straightforward consequence of the fact that a learning-by-doing process is dependent upon the profitability conditions over the use of clean coal energy, a higher CCS cost reducing the use of clean energy and thus delaying the revolution, while a higher initial CCS cost widens the cost gap that may be achieved thanks to R&D, thus creating an incentive to trigger the revolution sooner in time.

## 5 Combining learning and R&D to trigger the technological revolution

To characterize the optimal policy, we put more structure upon the know-how accumulation process. Assume that:

**Assumption 1** *1. $a(x_c, r)$ is a concave function of $(x_c, r)$, that is:*

$$a_{cc} \equiv \partial^2 a/\partial x_c^2 < 0 \qquad ; \qquad a_{rr} \equiv \partial^2 a/\partial r^2 < 0$$
$$a_{cc}a_{rr} - (a_{cr})^2 > 0 \quad \textit{where} \quad a_{cr} \equiv \partial^2 a/\partial x_c \partial r$$

*2. $x_c$ and $r$ are weak complements: $a_{cr} \geq 0$.*

*3. $a(x_c, r)$ exhibits non increasing returns to scale, that is:*

$$a(c_x, r) \;\; \geq \;\; a_c(x_c, r)x_c + a_r(x_c, r)r$$

The current value Lagrangian of the first phase problem $OP$ defined in section 2 (dropping the time index for the ease of reading) is:

$$\begin{aligned}\mathcal{L} \;\; = \;\; & u(x_c + x_d + y) - c_x(x_c + x_d) - \bar{c}_s x_c - c_y y - C_r(r) - \lambda_Z(\zeta x_d - \alpha Z) \\ & + \lambda_A a(x_c, r) + \nu_{xc}x_c + \nu_{xd}x_d + \nu_y y + \nu_r r + \nu_Z(\bar{Z} - Z) \;.\end{aligned}$$

The optimal policy must be a solution of the following set of conditions:

$$u'(q) \;\; = \;\; c_x + \bar{c}_s - \lambda_A a_c(x_c, r) - \nu_{xc} \tag{5.1}$$
$$u'(q) \;\; = \;\; c_x + \zeta\lambda_Z - \nu_{xd} \tag{5.2}$$
$$u'(q) \;\; = \;\; c_y - \nu_y \tag{5.3}$$
$$\lambda_A a_r(x_c, r) \;\; = \;\; c_r(r) - \nu_r \tag{5.4}$$
$$\dot{\lambda}_Z \;\; = \;\; (\rho + \alpha)\lambda_Z - \nu_Z \tag{5.5}$$
$$\dot{\lambda}_A \;\; = \;\; \rho\lambda_A \;. \tag{5.6}$$

To these conditions must be added the usual complementary slackness conditions and a transversality condition at $\bar{t}_A$ that we discuss later.

The main difference with the preceding sections is that it is now possible to begin the production of clean coal energy before the ceiling constraint begins to bind. This is a consequence of the non linear link between knowledge accumulation and the intensity of learning or R&D efforts triggering the revolution together with the complementarity effects between learning and R&D. Let us concentrate upon the high solar cost case: $c_y > c_x + \bar{c}_s$, so that coal is the only exploited primary energy source. First, we prove the following important result.

**Proposition 5** *Along an optimal energy and know-how accumulation policy, the cost breakthrough happens either strictly after the ceiling has been attained or either at the time when the ceiling begins to bind, that is $\underline{t}_Z \leq \bar{t}_A$ in all optimal scenarios.*

**Proof:** Assume to the contrary that $\bar{t}_A < \underline{t}_Z$. Over a time interval $[t_0, \bar{t}_A)$ we may be in three possible situations: either the cost break is triggered only by research, either it is triggered only through learning, or either it is triggered by a combination of research and learning. In the first case, slowing down slightly the research effort is beneficial, as noticed in section 3. In the second and third cases, $\lambda_A$ will be zero after the cost break and since the production of clean coal energy is positive in these two cases, $\zeta\lambda_Z = \underline{c}_s$ is incompatible with $\lambda_Z$ growing exponentially during the time interval $[\bar{t}_A, \underline{t}_Z)$. Hence clean coal exploitation should be interrupted after the cost break, meaning that delaying the revolution by reducing the use of clean coal energy before the revolution is beneficial.

### 5.1 Know-how accumulation scenarios

An optimal policy of knowledge accumulation is a sequence of time phases composed of the three following types of transitory tails:

(i) *A Combined tail* during which both research and learning are used to accumulate know-how. Let $\mathcal{T}^C$ be such a time phase, then $x_c(t) > 0$ and $r(t) > 0$, $t \in \mathcal{T}^C$.

(ii) *A Pure R&D tail* during which there is no exploitation of clean coal energy and the economy performs only research activity. Let $\mathcal{T}^R$ be such a time phase, then $x_c(t) = 0$ and $r(t) > 0$, $t \in \mathcal{T}^R$.

(iii) *A Pure learning tail* during which there is no research activity and know-how accumulates only because of learning. Let $\mathcal{T}^L$ be such a time phase, $x_c(t) > 0$ and $r(t) = 0$, $t \in \mathcal{T}^R$.

Such transitory phases can happen indifferently before or during the ceiling phase. The main complexity now is that there is no more a simple link between the energy production path, and thus the timing of the pre-ceiling and ceiling phases, and the structure of the combined learning and R&D know-how accumulation path.

We thus have to describe the main features of the possible transitory tails in the two cases of a pre-ceiling phase and a ceiling phase.

*Combined tails*

Assume first that $\mathcal{T}^C \subset [0, \underline{t}_Z)$. During this time phase, $(x_c(t), x_d(t), r(t))$ are defined as functions of $(\lambda_Z(t), \lambda_A(t))$ by (5.1), (5.2) and (5.4) with $\nu_{xd} = \nu_{xc} = \nu_r = 0$. Let $x_c(\lambda_Z, \lambda_A)$, $x_d(\lambda_Z, \lambda_A)$, $r(\lambda_Z, \lambda_A)$ be the corresponding implicit functions. Let $\delta \equiv a_{cc}a_{rr} - (a_{cr})^2$. $\delta > 0$ through the concavity assumption over $a(x_c, r)$ and let $\Delta_1 \equiv -u''(q(t))\lambda_A(t)\left[\lambda_A(t)\delta - a_{cc}c_r'\right] > 0$. Differentiating the relevant optimality conditions and dropping the arguments of the functions for the ease of reading gets:

$$\begin{aligned}
\frac{\partial x_d}{\partial \lambda_Z} &= -\frac{\zeta}{\Delta_1}\left[u''(\lambda_A a_{rr} - c_r') + \lambda_A^2\delta - \lambda_A a_{cc}c_r'\right] < 0\ ; &(5.7)\\
\frac{\partial x_d}{\partial \lambda_A} &= -\frac{u''}{\Delta_1}\left[a_c(\lambda_A a_{rr} - c_r') - \lambda_A a_r a_{cr}\right] < 0\ ; &(5.8)\\
\frac{\partial x_c}{\partial \lambda_Z} &= \frac{\zeta u''}{\Delta_1}\left(\lambda_A a_{rr} - c_r'\right) > 0\ ; &(5.9)\\
\frac{\partial x_c}{\partial \lambda_A} &= -\frac{\partial x_d}{\partial \lambda_A} > 0\ ; &(5.10)\\
\frac{\partial r}{\partial \lambda_Z} &= -\frac{\zeta u''}{\Delta_1}\lambda_A a_{cr} > 0\ ; &(5.11)\\
\frac{\partial r}{\partial \lambda_A} &= -\frac{u''\lambda_A}{\Delta_1}\left[a_c a_{cr} - a_r a_{cc}\right] > 0\ . &(5.12)
\end{aligned}$$

Since $\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}$ and $\lambda_A(t) = \lambda_{A0}e^{\rho t}$ during the time phase $[0, \underline{t}_Z)$, we conclude that both $\lambda_Z(t)$ and $\lambda_A(t)$ are time increasing and thus:

$$\frac{dx_d}{dt} < 0\ ;\ \frac{dx_c}{dt} > 0\ ;\ \frac{dr}{dt} > 0\ .$$

Before the ceiling phase, the use of dirty coal energy should decline while the use of clean coal energy should expand, together with an ever increasing level of R&D efforts. Note however that $\lambda_Z$ being increasing, $q(t)$ has to decrease. The increased use of clean energy does not compensate for the declining rate of use of dirty energy, the aggregate use of energy being strictly decreasing with time before the ceiling is attained.

Next, assume that $\mathcal{T}^C \subset [\underline{t}_Z, \bar{t}_A)$. Now $x_d(t) = \bar{x}_d$ and $(x_c(t), r(t))$ are implicitly defined by (5.1) and (5.4) as functions of $\lambda_A$ only. Let $x_c(\lambda_A)$, $r(\lambda_A)$ be these functions. Denote:

$$\Delta_2 \ \equiv\ u''(q)(\lambda_A a_{rr} - c_r') - \lambda_A a_{cc}c_r' + \lambda_A^2\delta \ >\ 0\ .$$

Differentiating (5.1) and (5.4), we get:

$$\frac{dx_c(\lambda)}{d\lambda_A} = \frac{1}{\Delta_2}\left[\lambda_A a_{cr} a_r - a_c(\lambda_A a_{rr} - c_r')\right] > 0 \; ; \tag{5.13}$$

$$\frac{dr(\lambda_A)}{d\lambda_A} = \frac{1}{\Delta_2}\left[\lambda_A a_c a_{cr} - (u''(q) + \lambda_A a_{cc}) a_r\right] > 0 \; . \tag{5.14}$$

This shows that $x_c(\lambda_A)$ and $r(\lambda_A)$ are increasing functions of $\lambda_A$, and thus of time, also during the ceiling phase. This implies in turn that $a(\lambda_A) \equiv a(x_c(\lambda_A), r(\lambda_A))$ is also an increasing function of $\lambda_A$. Since $\lambda_A$ increases over time, $a(\lambda_A)$ increases over time, the accumulation of know-how through the combined effect of learning and R&D accelerates over time. Last, we conclude that $x_c(t)$ being increasing through time, the energy implicit price should permanently decrease during such a $\mathcal{T}^C$ time phase.

Let us denote by $\sigma \equiv a_c/a_r$ the marginal rate of substitution (MRS) between learning and R&D. We are going to show that during a combined tail, $\sigma$ is declining over time. Consider first the case $\mathcal{T}^C \subset [0, \underline{t}_Z)$. The MRS is defined implicitly as a function of $(\lambda_Z, \lambda_A)$ during $\mathcal{T}_c$. Let $\sigma(\lambda_Z, \lambda_A)$ be this implicit function:

$$\sigma(\lambda_Z, \lambda_A) = \frac{a_c(x_c(\lambda_Z, \lambda_A), r(\lambda_Z, \lambda_A))}{a_r(x_c(\lambda_Z, \lambda_A), r(\lambda_Z, \lambda_A))}.$$

Making use of (5.9)-(5.12), we get the following expressions of the partial derivatives of the function $\sigma(\lambda_Z, \lambda_A)$ with respect to $(\lambda_Z, \lambda_A)$:

$$\frac{\partial\sigma(\lambda_Z, \lambda_A)}{\partial\lambda_Z} = \frac{\zeta u''}{a_r \Delta_1}\left[\lambda_A \delta + c_r'(\sigma a_{rc} - a_{cc})\right] < 0 \tag{5.15}$$

$$\frac{\partial\sigma(\lambda_Z, \lambda_A)}{\partial\lambda_A} = \frac{\sigma c_r' u''}{\Delta_1}\left[\sigma a_{rc} - a_{cc}\right] < 0 \; . \tag{5.16}$$

Hence we conclude from $\dot\lambda_Z(t) > 0$ and $\dot\lambda_A(t) > 0$, $t \in [0, \underline{t}_Z)$, that:

$$\dot\sigma(t) = \frac{\partial\sigma}{\partial\lambda_Z}\dot\lambda_Z(t) + \frac{\partial\sigma}{\partial\lambda_A}\dot\lambda_A(t) < 0 \; .$$

In the case $\mathcal{T}^C \subset [\underline{t}_Z, \bar{t}_A)$, Appendix A.4 shows that $\dot\sigma < 0$. This property of combined tails does not translate to the other types of phases, a point we check below. The dynamics of $\sigma(t)$ has important implications over the dynamics of $x_c(t)$ and $r(t)$ during a combined phase.

**Proposition 6** *During any time phase accumulating know-how through both learning and R&D, either before the ceiling phase or either during the ceiling phase, the learning effort increases at a higher rate than the R&D effort.*

**Proof:** Since $\dot{\sigma}(t) < 0$, $t \in \mathcal{T}^C$:

$$\dot{\sigma}(t) < 0 \iff \frac{\dot{a}_c}{a_c} < \frac{\dot{a}_r}{a_r} \; .$$

This implies that in the $(x_c, r)$ plane, the optimal trajectory cuts lower an lower isoclines. Under our assumptions concerning the $a(x_c, r)$ function, the isoclines in the plane $(x_c, r)$ are increasing functions of $x_c$, describing lower and lower levels of $\sigma$ when moving in the east direction. Thus, the combined path cuts lower an lower rays $r/x_c$, the path bending more and more in the direction of experience with respect to R&D. In other words, while both $r$ and $x_c$ increase over time, $\dot{r}/r < \dot{x}_c/x_c$ during a combined learning and R&D phase. This phenomenon applies indifferently during the pre-ceiling phase or the ceiling phase.

The Figure 2 illustrates the corresponding dynamics in the $(x_c, r)$ plane.

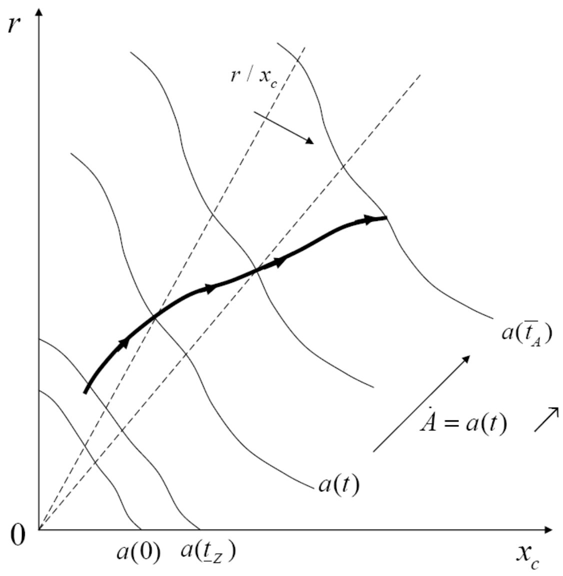


Figure 2: **Learning and R&D Dynamics in the $(x_c, r)$ Plane**

The difference between the respective dynamics of $x_c$ and $r$ may be explained by the fact that triggering the revolution through R&D relies upon costly efforts that will be recovered only when the revolution occurs. To the contrary, even if being more costly to use than dirty energy, clean coal energy generation generates a positive surplus when in use. This is the case before the ceiling constraint begins to be binding, the use of clean coal energy helping to alleviate the environmental burden upon the use of dirty energy. This is also the case during the ceiling phase, clean coal energy use allowing for an energy consumption increase above the constrained level $\bar{x}_d$. The consequence is that the economy should rely more and more over time upon learning with respect to research activity to trigger the technological revolution. The interesting aspect of this result is its degree of generality. It does not involve any kind of specific experience capital in learning or particular assumptions upon the relative productiveness of learning and R&D in boosting the acquisition of know-how.

*Pure R&D tails*

Let us now consider a time phase $\mathcal{T}^R$ involving only R&D activity without clean energy production and hence no learning on the CCS technology. Then (5.4) defines $r(t)$ as an implicit function of $\lambda_A$ and:

$$\frac{dr(\lambda_A)}{d\lambda_A} = \frac{a_r}{c'_r - \lambda_A a_{rr}} > 0 \; .$$

$\lambda_A(t)$ being increasing over time, $r(t)$ grows also over time. Since $a_{cr} > 0$, this implies that $\dot{a}_c(0,r) = a_{cr}\dot{r} > 0$ and hence that $\lambda_A a_c(0,r)$ is an increasing function of time. Assume first that $\mathcal{T}^R \subset [0, \underline{t}_Z)$. Then over the time interval $\mathcal{T}^R$:

$$\nu_{xc}(t) = \bar{c}_s - [\lambda_A(t) a_c(0, r(t)) + \zeta \lambda_Z(t)] \; .$$

It appears that $\nu_{xc}(t)$ is a strictly decreasing time function during a time interval $\mathcal{T}^R$. This implies that a corner path where $x_c = 0$ and $r > 0$ cannot follow an interior path where both $x_c > 0$ and $r > 0$. Furthermore we remark that $r(t)$ being increasing over time, $a_c(0,r)$ increases over time, since $a_{cr} > 0$, while $a_r(0,r)$ decreases over time, since $a_{rr} < 0$. Hence $\sigma = a_c/a_r$ increases over time during the time interval $\mathcal{T}^R$. Last, note that the border condition:

$$\zeta \lambda_Z = \bar{c}_s - \lambda_A a_c(0, r(\lambda_A))$$

defines an implicit relation between $\lambda_Z$ and $\lambda_A$ such that:

$$\left.\frac{d\lambda_A}{d\lambda_Z}\right|_{x_c=0} = -\frac{\zeta(c'_r - \lambda_A a_{rr})}{a_r a_{cr} \lambda_A + a_c(c'_r - \lambda_A a_{rr})} < 0 \; .$$

Assume now that $\mathcal{T}_r \subset [\underline{t}_Z, \bar{t}_A)$. Since $x_c = 0$, $u'(\bar{x}_d) = \bar{p}$ then $\nu_{xc} = \bar{c}_s - \lambda_A(t) a_c(0, r(\lambda_A)) - \bar{p}$, also a decreasing time function. Note that, as in the case $\mathcal{T}_r \subset [0, \underline{t}_Z)$, $\sigma(t)$ increases over time. The border condition is now:

$$\bar{p} = \bar{c}_s - \lambda_A a_c(0, r(\lambda_A)) \; .$$

This condition defines a unique level of $\lambda_A$, we denote by $\bar{\lambda}_A^R$. $\lambda_A(t) > \bar{\lambda}_A^R$ is incompatible with a policy performing only research activity without clean coal energy production.

*Pure learning tails*

Next, consider the case of a time phase $\mathcal{T}^L$. Note that this requires $c_r^0 > 0$. Assume first that $\mathcal{T}^L \subset [0, \underline{t}_Z)$. Then (5.1), (5.2) define implicitly $x_c$ and $x_d$ as functions of $(\lambda_Z, \lambda_A)$. Let $x_c(\lambda_Z, \lambda_A)$ and $x_d(\lambda_Z, \lambda_A)$ be the corresponding implicit functions. Denote by $\Delta_3 \equiv \lambda_A u''(q) a_{cc}(x_c, 0) > 0$. Then it is easily checked that:

$$\begin{aligned} \frac{\partial x_c}{\partial \lambda_Z} &= -\frac{\zeta u''(q)}{\Delta_3} > 0 \quad ; \quad \frac{\partial x_c}{\partial \lambda_A} = -\frac{a_c u''(q)}{\Delta_x} > 0 \ ; \\ \frac{\partial x_d}{\partial \lambda_Z} = \frac{\zeta(u''(q) + \lambda_A a_{cc})}{\Delta_3} &< 0 \quad ; \quad \frac{\partial x_d}{\partial \lambda_A} = \frac{u''(q) a_c}{\Delta_x} < 0 \ . \end{aligned}$$

Since $\lambda_Z(t)$ and $\lambda_A(t)$ are increasing time functions before $\underline{t}_Z$, this shows that $x_d(t)$ declines over time while $x_c(t)$ increases over time. Hence $a_c(x_c, 0)$ decreases over time, since $a_{cc} < 0$ and $a_r(x_c, 0)$ increases over time, since $a_{cr} > 0$. We conclude that $\sigma(t) = a_c(t)/a_r(t)$ decreases during a $\mathcal{T}^L$ type time interval. Furthermore $\nu_r(t)$ is given by:

$$\nu_r(t) \quad = \quad c_r^0 - \lambda_A(t) a_r(x_c(\lambda_Z(t), \lambda_A(t)), 0) \ .$$

$a_r(x_c, 0)$ being increasing over time, $\nu_r(t)$ decreases, implying that a time phase where $x_c > 0$ and $r = 0$ cannot follow a time phase where both $x_c > 0$ and $r > 0$. Next consider the border condition:

$$\lambda_A a_r(x_c(\lambda_Z, \lambda_A), 0) \quad = \quad c_r^0 \ .$$

This condition defines an implicit relationship between $\lambda_Z$ and $\lambda_A$ such that:

$$\left. \frac{d\lambda_A}{d\lambda_Z} \right|_{r=0} \quad = \quad -\frac{\zeta a_{rc}}{a_c a_{rc} - a_r a_{cc}} < 0 \ .$$

Next, assume that $\mathcal{T}^L \subset [\underline{t}_Z, \bar{t}_A)$. Then $x_d = \bar{x}_d$ and (5.1) defines an implicit function $x_c(\lambda_A)$ such that:

$$\frac{dx_c}{d\lambda_A} \quad = \quad -\frac{a_c}{u'' + \lambda_A a_{cc}} > 0 \ .$$

We thus conclude that $\lambda_A$ being an increasing time function, $x_c(t)$ should also grow over time. Hence $\lambda_A(t) a_r(x_c(\lambda_A(t)), 0)$ increases over time, implying that $\nu_r(t)$ should decrease over time. Once again, we have verified that a time phase during which $x_c(t) > 0$ and $r(t) = 0$ cannot follow a time phase where both $x_c > 0$ and $r > 0$. The border condition:

$$\lambda_A a_r(x_c(\lambda_A), 0) \quad = \quad c_r^0 \ ,$$

now defines a unique value of $\lambda_A$ we denote by $\bar{\lambda}_A^L$. If $\lambda_A > \bar{\lambda}_A^L$ a pure learning policy is no more optimal.

Taking stock, we can now describe the optimal know-how acquisition policies. Denote by $PR$ (Pure R&D) a time phase of the $\mathcal{T}^R$ type, where the know-how index increases thanks to only research efforts, by $PL$ (Pure Learning) a time phase of the $\mathcal{T}^L$ type and by $C$ (Combined) a time phase combining the use of research efforts and the clean coal energy technology. We have shown that $x_c$, $x_d$ and $r$ are defined as continuous functions of either both $\lambda_Z$ and $\lambda_A$ during the pre-ceiling phase, and that $x_c$ and $r$ are defined as continuous functions of $\lambda_A$ during the ceiling phase if $\underline{t}_Z < \bar{t}_A$. Since $\lambda_Z(t)$ and $\lambda_A(t)$ are continuous time functions over the time interval $[0, \bar{t}_A)$, that is to the exception of the revolution time $\bar{t}_A$, we conclude that $x_c$, $x_d$ and $r$ must be continuous time functions. This implies in turn that $\nu_{xc}$ and $\nu_r$ are also continuous time functions. A transition from a $PR$ phase to a $LR$ phase requires an upward jump down of $\nu_{xc}$ from zero to some strictly positive level since $\nu_{xc}$ will have to decrease strictly during the $LR$ phase. This cannot be optimal. The same argument applies to a transition from a $LR$ phase to a $PR$ phase, such a transition requiring an upward jump of $\nu_r$ at the transition time. It applies also to transitions from a $C$ phase to either a $PR$ phase or a $LR$ phase, the first transition requiring an upward jump of $\nu_r$ and the second one an upward jump of $\nu_{xc}$. Hence we conclude that a $PR$ phase or a $PL$ phase can only precede a combined $C$ phase. Of course, it remains possible that the optimal path begins with an inactive phase, during which the economy makes no efforts at all to trigger the technological revolution.

Let us first consider the case $\underline{t}_Z < \bar{t}_A$, that is the revolution occurs only strictly after the beginning of the ceiling phase. Then the optimal know-how active acquisition policy is one of the following scenarios:

(i) A $PR$ phase followed until $\bar{t}_A$, the revolution time;

(ii) A $PL$ phase followed until $\bar{t}_A$;

(iii) A $PR$ phase followed by a $C$ phase until $\bar{t}_A$;

(iv) A $PL$ phase followed by a $C$ phase until $\bar{t}_A$;

(v) A $C$ phase followed until $\bar{t}_A$.

The relevance of the preceding scenarios depends upon the models fundamentals, in particular the knowledge generation function. Let us briefly sketch the main features of these different scenarios.

*Scenario 1 : Pure R&D policies*

In this scenario, the research efforts are constantly increasing until the cost breakthrough. The energy implicit price is given by $c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ before $\underline{t}_Z$, growing exponentially while the use of dirty energy declines. Then the price stabilizes at $\bar{p}$ until the cost break occurs and clean energy is introduced inside the energy mix in combination with dirty energy.

*Scenario 2 : Pure learning policies*

Such a scenario requires a sufficiently high level of $c_r^0 > 0$ to prevent R&D to be profitable. The use of clean energy starts at time 0 and increases permanently until the technological revolution occurs. Before the ceiling phase, the aggregate energy production declines, the expansion in the use of clean energy does not compensate for the sharper decline of the use of dirty energy. After $\underline{t}_Z$, the continuous expansion of the use of clean energy makes decrease the energy price until the revolution occurs and the price stabilizes at the level $c_x + \underline{c}_s$.

*Scenario 3 : Pure research then combined policies*

These policies may be of two kinds, depending upon the ceiling beginning to bind during the pure R&D phase or during the combined phase. In the first case, the energy price increases up to $\bar{p}$, a level attained at the beginning of the ceiling phase. Then begins a first phase at the ceiling $[\underline{t}_Z, \bar{t}_r)$ during which only dirty coal energy is exploited. At $\bar{t}_r$, the economy begins to produce clean coal energy and the energy price decreases until $\bar{t}_A$, when the cost break occurs and the energy price stabilizes forever at the level $c_x + \underline{c}_s$. The research efforts increase over time until the technological revolution time. Thus the MRS between learning intensity and research first increases until $\bar{t}_r$ before decreasing during the time phase $[\bar{t}_r, \bar{t}_A)$.

In the second case, the clean coal energy option is introduced before the ceiling constraint begins to be binding, thus during a time phase where the energy price grows at the rate $(\rho + \alpha)$. During a first phase $[0, \bar{t}_r)$, only dirty energy is produced at a declining rate and the economy performs increasing R&D efforts. At $\bar{t}_r$, clean coal energy is introduced from a null level. Then clean coal energy production expands while the use of dirty energy decreases, the aggregate trend being a decreasing energy use until the ceiling is attained. During this second time phase $[\bar{t}_r, \underline{t}_Z)$, the research effort continues to increase while the MRS between learning and R&D decreases, implying a higher rate of growth of clean coal production than the research efforts growth rate. Then begins a first phase at the ceiling $[\underline{t}_Z, \bar{t}_A)$ until the cost revolution. Know-how accumulation accelerates also during this phase, learning expansion remaining higher than research efforts increases.

*Scenario 4 : Pure learning then combined policies*

As in the preceding scenario, the ceiling constraint may bind before or after the beginning of the combined phase of knowledge accumulation. In the first case, the learning process relies on a continuous expansion of the use of clean coal energy until $\underline{t}_Z$. The energy price increases exponentially during the time phase $[0, \underline{t}_Z)$, implying a declining energy consumption, the use of dirty energy being decreasing at a higher rate than the growth rate of use of clean energy. The MRS between learning and R&D declines during this time interval. Then begins a second phase $[\underline{t}_Z, \bar{t}_c)$, during which the economy does not perform R&D efforts, clean coal energy use continues to increase, the use of dirty energy is constrained at the level $\bar{x}_d$ and the energy price decreases. After this phase, the economy enter a combined regime of know-how accumulation based upon the use of clean coal energy and research activities. Such a scenario supposes that $c_r(0) = c_r^0$ be strictly positive and sufficiently high to prevent research activities before the ceiling has been attained. During the combined phase $[\bar{t}_c, \bar{t}_A)$ the energy price continues to decrease, the know-how accumulation accelerates, the MRS between learning and R&D decreases, the rate of growth of $x_c$ being larger than the rate of growth of $r(t)$. At the end of this phase, the cost revolution occurs and the energy use stabilizes to its optimal post-revolution level.

In the second case, the optimal path is composed of a first phase below the ceiling $[0, \bar{t}_c)$ without research activity but with a combined use of clean energy at an increasing rate and dirty energy at a declining rate. The aggregate energy use decreases while the energy price increases. Then begins a second phase below the ceiling $[\bar{t}_c, \underline{t}_Z)$ where the economy accumulates know-how both from learning and R&D activity. The energy price continues to increase and the MRS between learning and R&D being decreasing, $r(t)$ increases at a lower rate than $x_c(t)$. This phase is followed by a phase at the ceiling until the cost breakdown occurs, $[\underline{t}_Z, \bar{t}_A)$. The energy price now decreases during this phase, clean coal energy use and R&D activity continue to grow until $\bar{t}_A$ and the MRS being also decreasing, clean coal use grows at a higher rate than the research effort level.

*Scenario 5 : Combined Policies*

In this scenario, the economy accumulates know-how by using clean coal energy and an active R&D policy right from the beginning of time. During a first phase $[0, \underline{t}_Z)$, the energy price increases, the energy supply decreases but the use of dirty energy declines while the use of clean energy increases, together with the intensity of R&D efforts. Then the ceiling is reached and the economy enters a second phase of combined learning and R&D know-how accumulation, $[\underline{t}_Z, \bar{t}_A)$ until the technological revolution occurs. The energy price now decreases. Clean coal energy use continues to grow together with intensity of research activity. The MRS between learning and R&D being decreasing, the growth rate of $x_c(t)$ is higher than the growth rate of $r(t)$.

Figure 3 illustrates the shape of the energy price path if $\underline{t}_Z < \bar{t}_A$ for all scenarios expected a pure R&D policy followed until the cost break. In this last case, the energy price grows up to $\bar{p}$, attained at $\underline{t}_Z$. Then it stays constant at this level until the cost break occurs. At $\bar{t}_A$, the energy price jumps down from the level $\bar{p}$ to the level $c_x + \underline{c}_s$, last it stays permanently at this level, clean coal energy generation beginning after $\bar{t}_A$. The proof that the energy price should jump down at $\bar{t}_A$ is presented in the next subsection.

Figure 3 shows that the energy implicit price path combines in a straightforward way the features of the energy use dynamics exposed in the preceding sections. Before the ceiling constraint begins to bind, the energy price rises exponentially at the rate $(\rho + \alpha)$, and thus the aggregate supply of energy decreases. This means that even if clean coal energy is used during the pre-ceiling phase, its expansion over time does not compensate for the reduction of the use of dirty coal energy. During the first phase at the ceiling preceding the revolution, the use of dirty energy is constrained at the constant level $\bar{x}_d$ while the use of clean coal energy continues to expand. The result is a decreasing energy

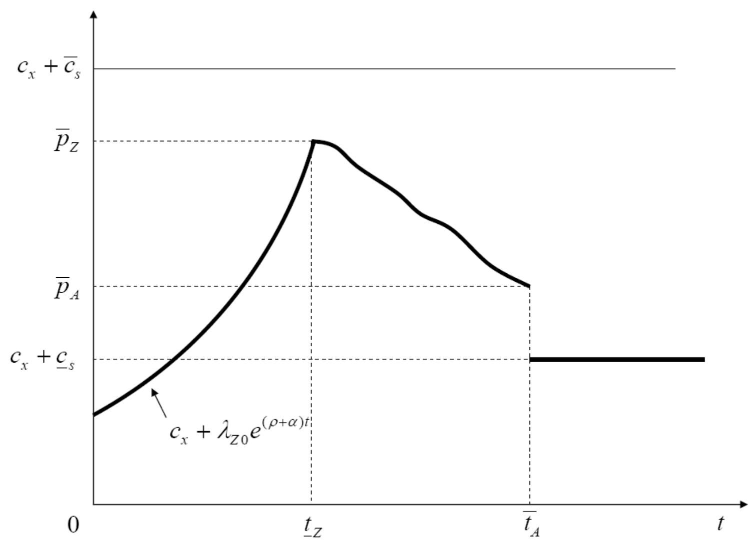


Figure 3: **Price Dynamics with Combined Learning and R&D Know-how Accumulation**

price until the revolution occurs. At the revolution time, as in the pure R&D case, the price jumps down while the use of clean energy jumps up. But because of the learning effect before the revolution, the price jump is no more equal to the cost gap $\bar{c}_s - \underline{c}_s$, as was the case in the pure R&D model. The jump is reduced thanks to learning.

As in the pure learning model, the implementation of the optimal policy requires to combine a carbon tax, its rate being given by $\zeta\lambda_Z(t)$, together with a subsidy to clean energy consumption, its rate being given by $\lambda_A(t)a_c(t)$. During the pre-ceiling phase, the tax rate must increase exponentially. The subsidy level must decrease over time if clean energy generation is put in operation. During the ceiling phase preceding the revolution, $[\underline{t}_Z, \bar{t}_A)$, the carbon tax must decrease while the subsidy must increase to sustain an increased use of clean energy during this time phase.

In the case of a technological revolution triggered just when the economy reaches the ceiling, that is $\underline{t}_Z = \bar{t}_A$, the already mentioned possible sequences of phase may be optimal. The only difference of course is that the sequence happens during the pre-ceiling phase.

To make progress in the determination of the optimal scenario, we need to describe the terminal conditions at $\bar{t}_A$. This discussion will alow to set conditions for the revolution to be triggered just when attaining the ceiling and for the reverse case $\underline{t}_Z < \bar{t}_A$.

### 5.2 Terminal condition at the revolution time $\bar{t}_A$

Thanks to Proposition 5, we know that two cases have to be considered, the case of a revolution happening during a ceiling phase, $\underline{t}_Z = \bar{t}_A$, and the case of a revolution happening just at the time at which the ceiling is attained, $\underline{t}_Z = \bar{t}_A$.

Let us compute the transversality condition at $\bar{t}_A$, the revolution time. The continuation value after the break, $\bar{V}$, keeps the same expression as before, being independent from the device triggering the breakthrough. Denote by $h^- \equiv \lim_{t\uparrow\bar{t}_A} h(t)$ and by $h^+ \equiv \lim_{t\downarrow\bar{t}_A} h(t)$ for any time function $h(t)$. Denote also by $\mu_Z$ the multiplier associated to the constraint $\bar{t}_A \geq \underline{t}_Z$. Then the condition reads:

$$u(q^-) - c_x q^- - \bar{c}_s x_c^- - C_r(r^-) + \lambda_A^- a(x_c^-, r^-) + \mu_Z \;=\; u(q^+) - c_x q^+ - \underline{c}_s x_c^+ \; .$$

Simplifying on both sides the $c_x \bar{x}_d$ term while taking into account (5.1) and $u'(q^+) = c_x + \underline{c}_s$, we obtain:

$$\Gamma(x_c^-) - C_r(r^-) + \lambda_A^- a(x_c^-, r^-) - \lambda_A^- a_c(x_c^-, r^-) x_c^- + \mu_Z \;=\; \Gamma(x_c^+) \; .$$

Since $x_c^+$ is independent from the devices used to trigger the revolution, the r.h.s is independent from what happens before $\bar{t}_A$. Denote it by $\bar{\Gamma} \equiv \Gamma(x_c^+)$. Then adding and subtracting $c_r(r^-)r^-$, remembering the expression of $\Gamma_r$ and taking (5.4) into account:

$$\begin{aligned}
\bar{\Gamma} - \Gamma(x_c^-) - \mu_Z \;&=\; c_r(r^-)r^- - C_r(r^-) \\
&\quad + \lambda_A \left[a(x_c^-, r^-) - a_c(x_c^-, r^-)x_c^-\right] - c_r(r^-)r^- \\
&=\; \Gamma_r(r^-) + \lambda_A^- \left[a(x_c^-, r^-) - a_c(x_c^-, r^-)x_c^- - a_r(x_c^-, r^-)r^-\right] \; .
\end{aligned}$$

Assume $\underline{t}_Z < \bar{t}_A$, then $\mu_Z = 0$. The *r.h.s.* is positive under our assumptions, showing that $x_c^+ > x_c^-$ and thus that the energy price should jump down at the revolution time. Figure 3 illustrates this feature of the energy price path. Furthermore, the transversality condition when $\underline{t}_Z < \bar{t}_A$ appears as an equation linking together $(x_c^-, r^-, \lambda_A^-)$ of the form:

$$\begin{aligned}
\Phi(x_c^-, r^-, \lambda_A^-) \;&\equiv\; u(\bar{x}_d + x_c^-) - c_x(\bar{x}_d + x_c^-) - \bar{c}_x x_c^- - C_r(r^-) + \lambda_A^- a(x_c^-, r^-) \\
&=\; \rho \bar{V} \; .
\end{aligned}$$

Differentiating we obtain:

$$d\Phi \;=\; \left[u'(q^-) - c_x - \bar{c}_s - \lambda_A a_c\right] dx_c + \left[\lambda_A a_r - c_r\right] dr - d\lambda_A^- a(x_c^-, r^-) \; .$$

Taking (5.1) and (5.4) into account, $d\Phi = d\lambda_A^- a(x_c^-, r^-) > 0$ if $x_c^- > 0$ and $r^- > 0$. The same applies if either $x_c^- > 0, r^- = 0$ or either $x_c^- = 0, r^- > 0$. Hence $\Phi = \rho\bar{V}$ defines a unique value of $\bar{\lambda}_A$ at $\bar{t}_A$, a value we denote by $\bar{\lambda}_A$. Furthermore, since $d\Phi/d\lambda_A > 0$, $\bar{\lambda}_A$ is an increasing function of $\bar{V}$, the continuation value. Let $\bar{\lambda}_A(\bar{V})$ be the corresponding implicit function.

Next, consider the case $\underline{t}_Z = \bar{t}_A$. As noticed before, the technological revolution may be triggered through any of the possible sequence of phases already described. It results that the revolution may occur from five possible paths before $\underline{t}_Z$. To these five possible paths correspond five possible terminal levels of $\lambda_A$, a vector $(\underline{\lambda}_A^R, \underline{\lambda}_A^L, \underline{\lambda}_A^C, \underline{\lambda}_A^{RC}, \underline{\lambda}_A^{LC})$. The conditions allowing to determine this vector are presented in Appendix A.5.

Under the constraint $\underline{t}_Z = \bar{t}_A$, only one of these scenarios is an optimum. To identify the optimal path, remark that the Hamilton-Bellman-Jacobi equation defines the value function from time 0 as $W = \rho H^*(0)$, where $H^*(0)$ is the optimized hamiltonian function at time 0. Differentiating and remembering that $\partial H^*/\partial x_d = \partial H^*/\partial x_c = \partial H^*/\partial r = 0$ we obtain:

$$\frac{dW}{\rho} = -d\lambda_{Z0}\dot{Z}(0) + d\lambda_{A0}\dot{A}(0) \ .$$

This shows that the value function is a decreasing function of $\lambda_Z$ and an increasing function of $\lambda_A$. Since $\lambda_Z(t)$ and $\lambda_A(t)$ are defined as exponentially increasing time functions at two different rates $(\rho + \alpha)$ and $\rho$, the $\{\lambda_Z(t), \lambda_A(t)$ trajectories never cross themselves during a pre-ceiling phase. From the fact that $\lambda_Z(\underline{t}_Z) = (\underline{c}_s - c_x)/\zeta$ in all scenarios, we conclude that the optimal scenario is the scenario giving the higher value of $\lambda_A$ at $\underline{t}_Z$. Let $\underline{\lambda}_A \equiv \max((\underline{\lambda}_A^R, \underline{\lambda}_A^L, \underline{\lambda}_A^C, \underline{\lambda}_A^{RC}, \underline{\lambda}_A^{LC})$.

Since $\bar{\lambda}_A(\bar{V})$ is an increasing function of $\bar{V}$, it appears that the constraint $\underline{t}_Z \leq \bar{t}_A$ does not bind if $\bar{\lambda}_A \leq \underline{\lambda}_A$ while it is binding in the reverse case $\underline{\lambda}_A < \bar{\lambda}_A$.

We are now in position of examining the relevance of the previously sketched scenarios. We proceed by considering the dual space $(\lambda_{Z0}, \lambda_{A0})$. In the next section we describe the optimal policy in the case $\bar{\lambda}_A < \underline{\lambda}_A$, that is we consider the optimal scenario in a situation where $\underline{t}_Z < \bar{t}_A$, the technological revolution occurs only after the beginning of the ceiling phase. Then we study the case of a technological revolution occurring just when the ceiling is attained.

### 5.3 Optimal policies triggering the revolution during the ceiling phase

To the terminal value $\bar{\lambda}_A$ corresponds a unique value of $\lambda_Z$, defined implicitly by: $\zeta\lambda_Z = \bar{c}_s - a_c(x_c(\bar{\lambda}_A), r(\bar{\lambda}_A))$. Let $\underline{\lambda}_Z$ be this value. Remark that because of the price jump at $\bar{t}_A$, $\lambda_Z$ will also jump down from $\underline{\lambda}_Z$ to the level $(\underline{c}_s - c_x)/\zeta$ after the break. In a situation where the cost breakthrough occurs only after the ceiling has been attained, we have shown previously that $\bar{\lambda}_A < \underline{\lambda}_A$.

To identify the domain of validity of the different scenarios in the $(\lambda_Z, \lambda_A)$ plane, we now take into account the definitions of the $x_c = 0$ and $r = 0$ borders. The border $x_c = 0$ defines an implicit relation between $\lambda_Z$ and $\lambda_A$ we denote by $\hat{\lambda}_A^x(\lambda_Z)$. Furthermore:

$$\frac{d\hat{\lambda}_A^x}{d\lambda_Z} = -\frac{\zeta(c_r' - \lambda_A a_{rr})}{a_r a_{cr}\lambda_A + a_c(c_r' - \lambda_A a_{rr})} < 0$$

On the other hand, the border $r = 0$ defines another implicit relation between $\lambda_Z$ and $\lambda_A$, a relation we denote by $\hat{\lambda}_A^r(\lambda_Z)$ and:

$$\frac{d\hat{\lambda}_A^r}{d\lambda_Z} = -\frac{\zeta a_{cr}}{a_r a_{cc} - a_c a_{cr}} < 0$$

The curves $\hat{\lambda}_A^x(\lambda_Z)$ and $\hat{\lambda}_A^r(\lambda_Z)$ cross themselves at $(\hat{\lambda}_Z^0, \hat{\lambda}_A^0)$ solution of:

$$\begin{aligned} \zeta\lambda_Z &= \bar{c}_s - \lambda_A a_c(0,0) \\ \lambda_A a_r(0,0) &= c_r^0 \end{aligned}$$

Note that is $c_x + \bar{c}_s < \bar{p}$, $\hat{\lambda}_Z^0 < \bar{c}_s/\zeta$ implies that $\hat{\lambda}_Z^0 < (\bar{p} - c_x)/\zeta \equiv \bar{\lambda}_Z$. Differentiating around the point $(\hat{\lambda}_Z^0, \hat{\lambda}_A^0)$, it is easily verified that:

$$\left|\frac{d\hat{\lambda}_A^x}{d\lambda_Z}\right|_{(\hat{\lambda}_Z^0, \hat{\lambda}_A^0)} > \left|\frac{d\hat{\lambda}_A^r}{d\lambda_Z}\right|_{(\hat{\lambda}_Z^0, \hat{\lambda}_A^0)} .$$

Three possibilities have to be considered:

(i) Either $\hat{\lambda}_Z^0 < (\underline{c}_s - c_x)/\zeta$ and the point $(\hat{\lambda}_Z^0, \hat{\lambda}_A^0)$ is located to the left of the vertical $\lambda_Z = (\underline{c}_s - c_x)/\zeta$.

(ii) Either $(\underline{c}_s - c_x)/\zeta < \lambda_Z < (\bar{p} - c_x)/\zeta$ and the point $(\hat{\lambda}_Z^0, \lambda_A^0)$ is located in between the vertical borders $\lambda_Z = (\underline{c}_s - c_x)/\zeta$ and the vertical border $(\bar{p} - c_x)/\zeta$.

(iii) Either $\hat{\lambda}_Z^0 > (\bar{p} - c_x)/\zeta$ and the point $(\hat{\lambda}_Z^0, \hat{\lambda}_A^0)$ is located to the right of the border $(\bar{p} - c_x)/\zeta$.

Note that the implicit energy price is at most equal to $\bar{p}$ in any optimal scenario. This means that $\lambda_Z(t) < (\bar{p} - c_x)/\zeta \equiv \bar{\lambda}_Z$. The vertical $\lambda_Z = \bar{\lambda}_Z$ defines the upper border of possible value of $\lambda_Z$ in all optimal scenario. The Figure 4 illustrates the three possible cases.

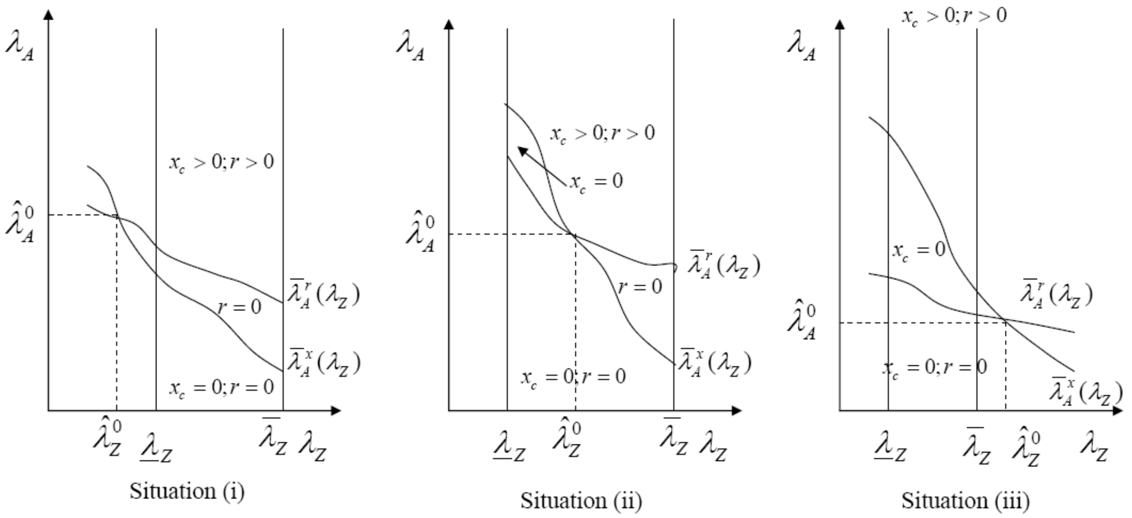


Figure 4: **Activity Constraints in the** $(\lambda_Z, \lambda_A)$ **Plane**

*Optimal combined policies*

Let us first consider the combined policies. The condition $\bar{x}_d = x_d(\lambda_Z, \lambda_A)$ defines an implicit relation between $\lambda_Z$ and $\lambda_A$ we denote by $\bar{\lambda}_A^c(\lambda_Z)$. Taking (5.7), (5.8) into account, it is immediately verified that: $d\bar{\lambda}_A^c(\lambda_Z)/d\lambda_Z < 0$. In the case $\underline{t}_Z < \bar{t}_A$, $\lambda_Z \in [\underline{\lambda}_Z, \bar{\lambda}_Z]$ defines the relevant domain of possible values of $\lambda_Z$. On one hand, $\bar{\lambda}_A^c(\underline{\lambda}_Z) = \bar{\lambda}_A$ in a combined scenario and on the other hand we denote by $\bar{\lambda}_A^c \equiv \bar{\lambda}_A^c(\bar{\lambda}_Z)$.

To a combined policy corresponds a $\{\lambda_Z(t), \lambda_A(t)\}$ trajectory initiating below the curve $\bar{\lambda}_A^c(\lambda_Z)$ at some point $(\lambda_{Z0}, \lambda_{A0})$. Before $\underline{t}_Z$, the trajectory is defined by $(\lambda_{Z0}e^{(\rho+\alpha)t}$, $\lambda_{A0}e^{\rho t})$. hence $\{\lambda_Z(t), \lambda_A(t)\}$ moves in the north east direction in the dual plane. At $\underline{t}_Z$, the trajectory hits the border $\bar{\lambda}_A^c(\lambda_Z)$. Then it follows this curve until $\bar{t}_A$ is reached, that is at the point $\lambda_Z(\bar{t}_A) = \underline{\lambda}_Z$, $\lambda_A(\bar{t}_A) = \bar{\lambda}_A$. The following Figure 5 illustrates this construction.

The combined scenario is optimal from any point $(\lambda_{Z0}, \lambda_{A0})$ located above the border $\max(\hat{\lambda}_A^x(\lambda_Z), \hat{\lambda}_A^r(\lambda_Z))$, that is the region where both $x_c > 0$ and $r > 0$.

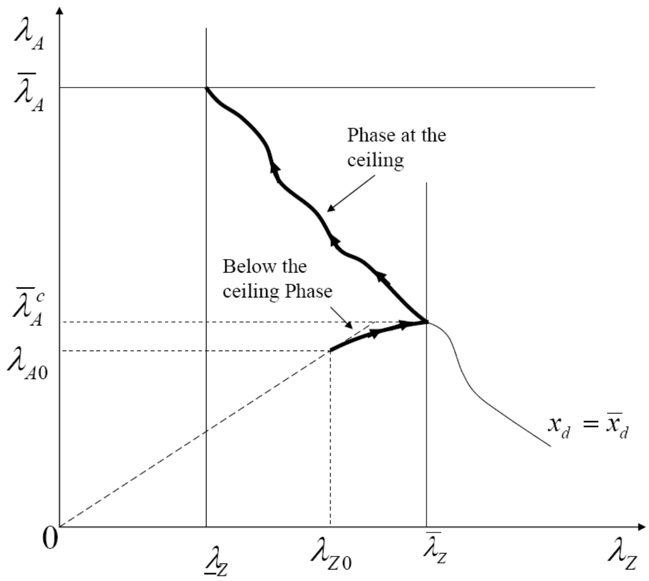


Figure 5: **Combined Policies**

*Initial R&D optimal policies*

Turn to the scenarios involving at least initially pure R&D policies. We call these policies *Initial R&D Policies.* Within a pure R&D tail, the ceiling constraint is defined by: $\zeta\lambda_Z = \bar{p} - c_x$, that is the vertical line $\lambda_Z = \bar{\lambda}_Z$. $\bar{\lambda}_A^c(\lambda_Z)$, the curve defining the ceiling constraint in a combined policy cuts the vertical $\lambda_Z = \bar{\lambda}_Z$ at a point where $x_c = 0$. Thus the curves $\bar{\lambda}_A^c(\lambda_Z)$ and $\hat{\lambda}_A^x(\lambda_Z)$ intersect themselves at $\lambda_Z = \bar{\lambda}_Z$, that is along the vertical $\lambda_Z = \bar{\lambda}_Z$ at the point $(\bar{\lambda}_A^c, \bar{\lambda}_Z)$. Furthermore, computing the derivatives $d\bar{\lambda}_A^c/d\lambda_Z$ and $d\hat{\lambda}_A^x/d\lambda_Z$ around the point $(\bar{\lambda}_Z, \bar{\lambda}_A^c)$ we observe that the ceiling border for a combined path is located above the $x_c = 0$ locus.

Next consider the situation $(i)$ depicted in Figure 4. The curve $\bar{\lambda}_A^r(\lambda_Z)$ is located above the curve $\bar{\lambda}_A^c(\lambda_Z)$ inside the whole domain $\underline{\lambda}_Z < \lambda_Z < \bar{\lambda}_Z$. Hence the curve $\bar{\lambda}_A^r(\lambda_Z)$ intersects the vertical $\lambda_Z = \bar{\lambda}_Z$ above $(\bar{\lambda}_Z, \bar{\lambda}_A^c)$. It results that it is impossible to follow an initial R&D policy in the relevant domain. In the situation $(ii)$, an initial R&D policy is the only optimal scenario from any point $(\lambda_{Z0}, \lambda_{A0})$ located above the curve $\bar{\lambda}_A^x$ and to the left of the vertical $\lambda_Z = \hat{\lambda}_Z^0$. In the situation $(iii)$, the border $x_c = 0$, that is the curve $\bar{\lambda}_A^x(\lambda_Z)$, is located above the $r = 0$ border, that is the curve $\bar{\lambda}_A^r(\lambda_Z)$ in the relevant domain $[\underline{\lambda}_Z, \bar{\lambda}_Z]$. Hence an initial R&D policy is the only optimal scenario in this situation.

Let us begin by describing the optimal scenario in the situation $(iii)$. Consider the

$\{\lambda_Z, \lambda_A\}$ trajectory where $\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}$, $\lambda_A(t) = \lambda_{A0}e^{\rho t}$ going through the point $(\bar{\lambda}_Z, \bar{\lambda}^c_A)$. Denote by $(S_r)$ this separating curve.

To complete the discussion we need to take into account the terminal condition. If $\bar{\lambda}_A < \bar{\lambda}^c_A$, the optimal scenario is a type 1 scenario. The optimal $\{\lambda_Z(t), \lambda_A(t)\}$ trajectory is located below the separatrix $(S_r)$. During the pre-ceiling phase, the trajectory moves in the north east direction until the ceiling is attained. At $\underline{t}_Z$, the trajectory hits the vertical $\lambda_Z = \bar{\lambda}_Z$. Then it moves upward along this vertical until $\bar{\lambda}_A$ is reached at $\bar{t}_A$.

If $\bar{\lambda}_A > \bar{\lambda}^c_A$, the optimal policy is a type 3 scenario. If $(\lambda_{Z0}, \lambda_{A0})$ is located below the separating curve $(S_r)$, then the economy hits the ceiling constraint while performing only R&D activity, that is $\underline{t}_Z < \bar{t}_r$. Then the trajectory follows the vertical $\bar{\lambda}_Z$ up to $\bar{\lambda}^c_A$. Next the optimal trajectory follows the curve corresponding to the ceiling constraint in a combined path, performing both research activity and clean coal energy production. If $(\lambda_{Z0}, \lambda_{A0})$ is located above the separating curve $(S_r)$, then the economy moves from a pure R&D regime to a combined regime before attaining the ceiling, that is $\bar{t}_r < \underline{t}_Z$. After crossing the $x_c = 0$ border, the optimal trajectory enters the combined regime zone until the ceiling border is reached. This border is then followed up to $(\underline{t}_Z, \bar{\lambda}_A)$. The Figure 6 illustrates this construction.

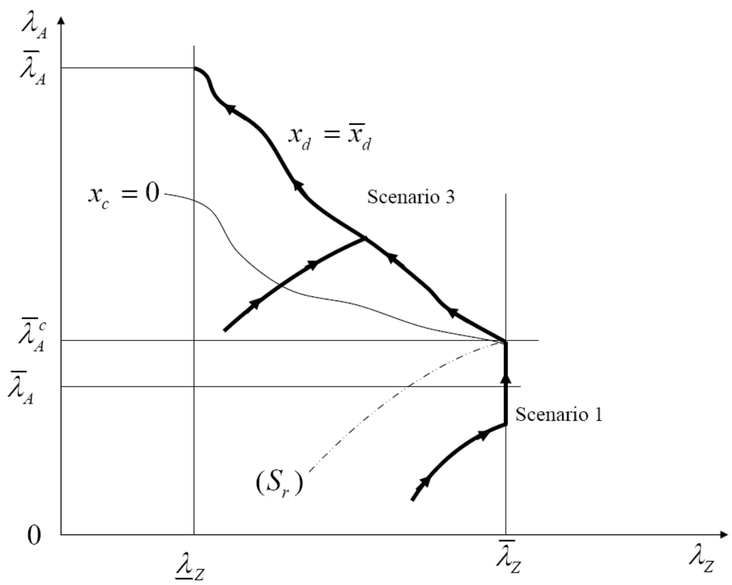


Figure 6: **Initial R&D Policies**

In the situation $(ii)$ the initial research optimal policies correspond to a type 3 scenario.

The economy starts performing only R&D before introducing the exploitation of clean coal before the ceiling is attained. Then the economy performs both R&D and clean coal energy production until the technological revolution. In the situation $(i)$, as noticed before, an initial R&D policy is not optimal.

*Initial learning policies*

The scenarios involving pure learning tails follow the same principle of construction. We call these policies *Initial Learning Policies.* The ceiling constraint along a pure learning tail defines an implicit relation between $\lambda_Z$ and $\lambda_A$, we denote by $\bar{\lambda}_A^L(\lambda_Z)$ and such that:

$$\frac{d\bar{\lambda}_A^L}{d\lambda_Z} = -\frac{\zeta(u'' + \lambda_A a_{cc})}{u'' a_c} < 0$$

It is easily checked that the curve $\bar{\lambda}_A^L(\lambda_Z)$ and the curve $\bar{\lambda}_A^c(\lambda_Z)$, corresponding respectively to the ceiling constraint in a pure learning regime and to the ceiling constraint in a combined R&D and learning regime, cross themselves along the locus $r = 0$, that is the curve $\hat{\lambda}_A^r(\lambda_Z)$. Let $(\bar{\lambda}_Z^L, \bar{\lambda}_A^L)$ be the intersection point of these three curves. It may also be verified that:

$$\left|\frac{d\hat{\lambda}_A^r}{d\lambda_Z}\right|_{(\bar{\lambda}_Z^L, \bar{\lambda}_A^L)} < \left|\frac{d\bar{\lambda}_A^c}{d\lambda_Z}\right|_{(\bar{\lambda}_Z^L, \bar{\lambda}_A^L)} < \left|\frac{d\bar{\lambda}_A^L}{d\lambda_Z}\right|_{(\bar{\lambda}_Z^L, \bar{\lambda}_A^L)}$$

In the situation $(i)$, an initial learning policy is the only optimal policy. In the situation $(ii)$, it is the optimal policy for $\lambda_{Z0} > \hat{\lambda}_Z^0$ and $\lambda_{A0} > \hat{\lambda}_A^r(\lambda_{Z0})$. In the situation $(iii)$, it cannot be an optimal policy.

Let us begin by the situation $(i)$. We have to consider the implications of the terminal condition. If $\bar{\lambda}_A < \bar{\lambda}_A^L$, then the optimal policy is a type 2 scenario. In the reverse case, it is a type 4 scenario. One can define a separating curve $(S_c)$ corresponding to the $\{\lambda_Z, \lambda_A\}$ trajectory going through $(\bar{\lambda}_Z^L, \bar{\lambda}_A^L)$, the intersection point between the three curves. Initiated below the separating curve $(S_c)$, the optimal path hits the ceiling border for a pure learning tail $\bar{\lambda}_A^L$ whence the ceiling is attained. Then it follows this curve until the point $(\bar{\lambda}_Z^L, \bar{\lambda}_A^L)$ is attained, a point at which a combined phase at the ceiling begins. This combined phase identifies in the $(\lambda_Z, \lambda_A)$ plane to a motion of the $\{\lambda_Z(t), \lambda_A(t)\}$ trajectory along the curve $\bar{\lambda}_A^c(\lambda_Z)$ until the cost breakthrough. Hence when starting from below the separatrix $(S_c)$, the economy begins to perform research activity only strictly after the ceiling has been attained, that is $\underline{t}_Z < \bar{t}_c$ in this scenario.

Initiated above $(S_c)$, the optimal path crosses the $r = 0$ border, that is the curve

$\hat{\lambda}^r_A(\lambda_Z)$, before the ceiling constraint begins to be binding. Then it enters a region of combined learning and R&D accumulation of know-how policies until it hits the ceiling curve for such combined policies. Hence $\bar{t}_c < \underline{t}_Z$ in this scenario. Last the optimal $\{\lambda_Z(t), \lambda_A(t)\}$ trajectory follows the curve $\bar{\lambda}^c_A(\lambda_Z)$ until $(\underline{\lambda}_Z, \bar{\lambda}_A)$ is attained at $\bar{t}_A$. The Figure 7 illustrates the construction.

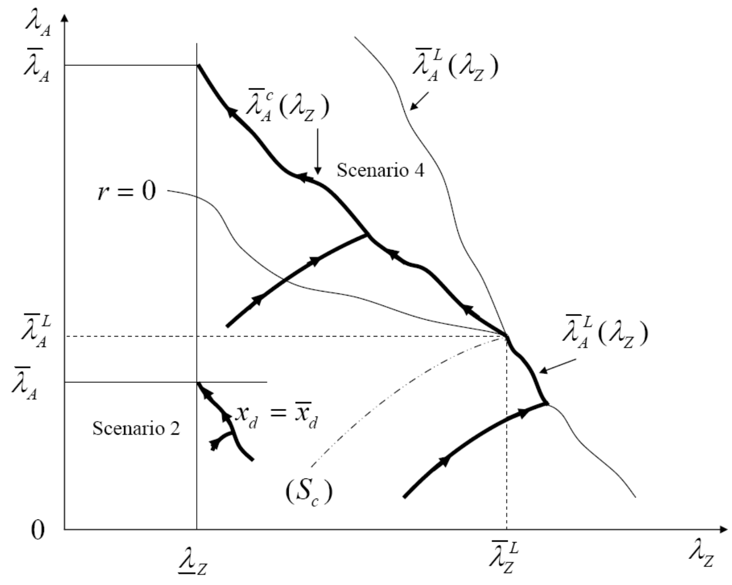


Figure 7: **Initial Learning Policies**

In the situation $(ii)$ only the type 4 scenario may be valid in a case where the combined process of know-how accumulation through both learning and R&D begins before the ceiling constraint is binding, that is $\bar{t}_c < \underline{t}_Z$. Last, as noticed before, an initial learning policy is never optimal in the situation $(iii)$.

### 5.4 Optimal policies triggering the revolution when the ceiling is attained

In the situation $(i)$, a pure research policy is not optimal. The optimal policy is the scenario of the possible type 2-5 giving the highest level of $\lambda_A$ when the vertical $\lambda_z = (\underline{c}_s - c_x)/\zeta$ is attained. In the situations $(ii)$ and $(iii)$, a pure learning policy is excluded, the optimal scenario among the types 1, 3,4, 5 is the one giving the highest level of $\lambda_A$ at $\underline{t}_Z$. Note that these active policies may be preceded by a time phase without any effort to trigger the technological revolution. The same applies to scenarios where $\underline{t}_Z < \bar{t}_A$.

## 6 Conclusion

Since Goulder, Matthai (2000), it is frequently advanced in the economic literature that carbon pricing policies should not be too stringent in order for R&D to have sufficient time to select and develop better abatement technological options. The present analysis invites to reconsider seriously such statements. Instead of an incremental process of technical improvement, we have modeled technical change as a drastic process able to trigger a technological revolution in the form of an abrupt cost break in pollution abatement, provided that a sufficiently high level of know-how has been previously accumulated. This framework allows for a much clearer view of the effects of an environmental policy upon the trend of efforts to trigger a technological improvement in abatement technologies. The Goulder-Matthai analysis proceeded by contrasting situations where technical progress resulted from learning-by-doing in pollution abatement techniques from situations where technical advances could be obtained only through dedicated R&D efforts. We have first followed this approach by studying the polar cases of a pure learning induced technological revolution and a pure R&D induced breakthrough.

When the revolution is triggered by learning-by-doing, the optimal policy implementation requires to combine a price upon carbon emissions and a subsidy to clean energy generation. This subsidy must grow over time during the learning period preceding the technological revolution, inducing a permanent rise of the use of clean energy. The optimal carbon tax should increase before the beginning of the learning process and decrease afterwards. We show also that a stricter environmental standard, here modeled in terms of a critical atmospheric carbon concentration not to be crossed over, has the effect of increasing the use of abatement technologies before the technical breakthrough, resulting in an earlier revolution. However, this does not mean that the optimal carbon tax corresponding to a stricter atmospheric concentration mandate needs to be increased. A stricter mandate has an ambiguous effect over the carbon tax level before the beginning of the clean energy generation phase and reduces this level during the learning phase.

In a R&D induced technological revolution framework, the optimal policy implementation no more requires specific subsidies, an optimal carbon price being a sufficient tool to induce the optimal level of R&D efforts. Under the reasonable assumption of increasing and convex costs of research, discounting favors delaying the R&D efforts, resulting in an increasing time pattern for such efforts until the technological breakthrough occurs. In all cases, it is not optimal to trigger the revolution strictly before the atmospheric concentration constraint begins to be binding. The optimal carbon tax should rise until the carbon concentration ceiling has been attained and then should be maintained at a constant level before jumping down at the technological revolution time. As for the learning induced technical revolution, a stricter environmental standard spurs more R&D efforts from the carbon abatement industry and reduces the time delay before the cost breakthrough. This is reminiscent of the Porter hypothesis.

However the initial cost level of the abatement technology has ambiguous effects upon the intensity of R&D efforts. This was not the case in the learning induced technical break. A higher initial pollution abatement cost reduces the use of clean energy before the revolution, slowing down the learning process and delaying the breakthrough. In a R&D induced technical break context, a higher initial pollution abatement cost widens the cost gap that can be achieved thanks to R&D, an incentive to increase R&D efforts. On the other hand, it also increases the cost of using clean energy before the break, an incentive to reduce the research efforts, in order to diminish the total costs of the energy policy.

A main drawback of the Goulder-Matthai analysis is that the pure R&D and the pure learning-by-doing technical change models are not really comparable. These extreme cases describe situations where the economy is constrained to rely upon only one of these devices to achieve a technological improvement of the pollution abatement technologies. A correct account of the effects of an environmental policy upon induced technical progress in abatement technologies requires a framework where both R&D and learning can contribute to technology advances. We thus turn to the study of such a combined process. We show that the R&D efforts should permanently increase before the technical revolution. This is a straightforward consequence of discounting and our increasing marginal cost of research assumption. The use of clean energy should also increase over time, meaning an accelerating learning process.

Even under the assumption of constant average and marginal costs of producing clean energy, it may now be the case that clean coal production starts before the atmospheric carbon concentration constraint begins to bind. But this has no qualitative consequences over the optimal time profile of the carbon price. The carbon price must increase before the atmospheric ceiling constraint is attained and decrease afterwards until the technological revolution occurs. As in the pure learning case, the carbon price tool has to be completed by a subsidy to the consumption of clean energy. The subsidy level rises all along the pre-revolution phase of clean energy production.

Concerning the priority that may be given to research with respect to learning in generating technical advances, we show that the growth rate of use of clean energy should be higher than the growth rate of research efforts. In a drastic technical progress framework, research activity bears only costs before the revolution, the prize in terms of cost cut being ripped only at the end of the research process. This is not the case for learning, since the use of the abatement technology, even at the high pre-revolution cost level, allows to generate some positive surplus. The consequence is that the economy gives more an more weight to learning with respect to R&D in achieving the technological breakthrough, independently of the respective marginal contributions of learning and research to the accumulation of know-how.

Technical progress paths combining research and learning are not the only optimal ones. We show also that an optimal policy may involve an initial period of only R&D activity before launching the use of the pollution abatement technology or an initial period based only upon learning, research being too costly to be justified until the time before the revolution be sufficiently short.

This work may be extended in several directions. The first one is to take explicitly into account the scarcity of fossil fuels. We assume an infinite supply of such resources, an assumption frequently made in the relevant literature. However fuel scarcity should result in Hotelling effects, affecting both the timing of the environmental policy and the timing of the technological investment policy before the breakthrough. The drastic technical progress framework is useful to obtain clear cut results concerning the relationships between an environmental policy and a technical development policy. It appears interesting to compare its conclusions with the results derived from incremental technical progress models. This should allow to shed some light on the various puzzles which have been identified in this literature. We focus primarily upon technical progress in pollution abatement technologies, but the analysis could be extended to technological competition between abatement techniques and clean energy generation process, like solar energy production for example.

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# APPENDIX

## A.1 Appendix 1: Proof that pure dirty and combined dirty and clean energy scenarios may be welfare equivalent

In the scenario under consideration, $p(\underline{t}_Z) = \bar{p}$. By a standard markovian argument nothing is changed in the scenarios comparison by assuming that $Z(0) = \bar{Z}$ and $\underline{t}_Z = 0$. Then the present value at time 0 of a policy using only dirty coal energy generation over $[0, \infty)$ is given by:

$$V_d = \frac{1}{\rho}\left[u(\bar{x}_d) - c_x \bar{x}_d\right]$$

The present value at 0 of a policy using both dirty and clean coal energy from some time $\underline{t}_c \geq 0$ is given by:

$$\begin{aligned} V_c &= \left[u(\bar{x}_d) - c_x \bar{x}_d\right]\left(\frac{1 - e^{-\rho \underline{t}_c}}{\rho}\right) \\ &\quad + \int_{\underline{t}_c}^{\bar{t}_A} \left[u(\bar{x}_d + x_c(t)) - c_x(\bar{x}_d + x_c(t)) - \bar{c}_s x_c(t)\right] e^{-\rho t} dt + \bar{V} e^{-\rho \bar{t}_A} \ . \end{aligned}$$

Let:

$$\int_{\underline{t}_c}^{\bar{t}_A} \left[u(\bar{x}_d + x_c(t)) - c_x(\bar{x}_d + x_c(t)) - \bar{c}_s x_c(t)\right] e^{-\rho t} dt \equiv \int_{\underline{t}_c}^{\bar{t}_A} \Phi(t) e^{-\rho t} dt \equiv I \ .$$

Integrating by parts:

$$I = -\left.\frac{\Phi(t) e^{-\rho t}}{\rho}\right|_{\underline{t}_c}^{\bar{t}_A} + \frac{1}{\rho}\int_{\underline{t}_c}^{\bar{t}_A} \dot{\Phi}(t) e^{-\rho t} dt \ .$$

Taking (3.1) into account, it is easily checked that:

$$\dot{\Phi}(t) = \left[u'(q(t)) - c_x - \bar{c}_s\right] \dot{x}_c(t) = -\lambda_{A0} \dot{x}_c(t) e^{\rho t} \ .$$

Remembering that $x_c(\underline{t}_c) = 0$ while $x_c(\bar{t}_A) = \underline{x}_c$ and making use of the previously computed expression of $\dot{\Phi}(t)$:

$$I = \left[u(\bar{x}_d) - c_x \bar{x}_d\right] \frac{e^{-\rho \underline{t}_c}}{\rho} - \left[u(\bar{x}_d + \underline{x}_c) - c_x(\bar{x}_d + \underline{x}_c) - \bar{c}_s \underline{x}_c\right] \frac{e^{-\rho \bar{t}_A}}{\rho} - \frac{\lambda_{A0} \underline{x}_c}{\rho} \ .$$

Remembering the expression of $\bar{V}$, $V_c$ simplifies to:

$$V_c = \frac{1}{\rho}\left[u(\bar{x}_d) - c_x \bar{x}_d\right] + \frac{1}{\rho}\left[(\bar{c}_s - \underline{c}_s) e^{-\rho \bar{t}_A} - \lambda_{A0}\right] \underline{x}_c \ .$$

Since $\lambda_{A0} e^{\rho \bar{t}_A} = \bar{c}_s - \underline{c}_s$ through the transversality condition at $\bar{t}_A$, we conclude that:

$$V_c \;=\; \frac{1}{\rho}\left[u(\bar{x}_d) - c_x \bar{x}_d\right] \;=\; V_d$$

The society is indifferent between sticking to the sole use of dirty coal energy once the ceiling constraint begins to be binding or follow some combined policy of dirty and clean coal energy generation started at any moment after the beginning of the ceiling phase.

## A.2 Appendix 2: Comparative dynamics in the pure learning case

Denote by:

$$\begin{aligned}
I_Z^Z &\equiv -\int_0^{\underline{t}_Z} \frac{e^{(\rho+2\alpha)t}}{u''(q(t))} dt \;>\; 0 \\
I_A &\equiv -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{e^{\rho t}}{u''(q(t))} dt \;>\; 0 \\
J_A^c &\equiv -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{dt}{u''(q(t))} \;>\; 0 \\
x_c^Z \equiv x_c(\underline{t}_Z) &\quad ; \quad x_c^A \equiv x_c(\bar{t}_A) \\
T_A \equiv \bar{t}_A - \underline{t}_Z &\quad ; \quad \pi_Z \equiv \zeta(\rho+\alpha)\lambda_{Z0} e^{\alpha \underline{t}_Z} + \rho \lambda_{A0}
\end{aligned}$$

Then after linearizing the set of conditions defining $(\lambda_{Z0}, \lambda_{A0}, \underline{t}_Z, \bar{t}_A)$, we get the following system in matrix form:

$$\begin{bmatrix} -\zeta^2 I_Z^Z & 0 & \zeta x_c^Z e^{\alpha \underline{t}_Z} & 0 \\ 0 & I_A & -x_c^Z & x_c^A \\ \zeta e^{\alpha \underline{t}_Z} & 1 & \pi_Z & 0 \\ 0 & 1 & 0 & \rho \lambda_{A0} \end{bmatrix} \begin{bmatrix} d\lambda_{Z0} \\ d\lambda_{A0} \\ d\underline{t}_Z \\ d\bar{t}_A \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} dZ^0 + \begin{bmatrix} e^{\alpha \underline{t}_Z} \\ \alpha T_A/\zeta \\ 0 \\ 0 \end{bmatrix} d\bar{Z}$$

$$+ \begin{bmatrix} 0 \\ J_A^c \\ e^{-\rho \underline{t}_Z} \\ e^{-\rho \bar{t}_A} \end{bmatrix} d\bar{c}_s + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -e^{-\rho \bar{t}_A} \end{bmatrix} d\underline{c}_s + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} d\bar{A} \,.$$

The determinant of the system, we denote by $\Delta$ is:

$$\Delta \;=\; x_c^A \zeta^2 \left[I_Z^Z \pi_Z + e^{2\alpha \underline{t}_Z} x_c^Z\right] - \rho \lambda_{A0} \zeta^2 \left[I_Z^Z (I_A \pi_Z + x_c^Z) + I_A x_c^Z e^{2\alpha \underline{t}_Z}\right] \,. \tag{A.2.1}$$

Note that $\dot{q} = \dot{x}_c = -\rho\lambda_{A0}e^{\rho t}/u''(q)$. Thus:

$$\begin{aligned} I_A &= -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{e^{\rho t}}{u''(q)}dt = -\frac{1}{\rho\lambda_{A0}} \int_{\underline{t}_Z}^{\bar{t}_A} \frac{\rho\lambda_{A0}e^{\rho t}}{u''(q)}dt \\ &= \frac{1}{\rho\lambda_{A0}} \int_{\underline{t}_Z}^{\bar{t}_A} \dot{x}_c(t)dt = \frac{x_c^A - x_c^Z}{\rho\lambda_{A0}} \ . \end{aligned} \tag{A.2.2}$$

Substituting for $I_A$ its expression (A.2.2) into (A.2.1) we obtain:

$$\begin{aligned} \Delta/\zeta^2 &= (x_c^A - \rho\lambda_{A0}I_A)\left[I_Z^Z\pi_Z + e^{2\alpha\underline{t}_Z}x_c^Z\right] - \rho\lambda_{A0}I_Z^Z x_c^Z \\ &= x_c^Z\left[I_Z^Z(\zeta(\rho+\alpha)\lambda_{Z0} + \rho\lambda_{A0}) + x_c^Z e^{2\alpha\underline{t}_Z} - \rho\lambda_{A0}I_Z^Z\right] \\ &= x_c^Z\left[\zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z e^{\alpha\underline{t}_Z} + x_c^Z e^{2\alpha\underline{t}_Z}\right] \ > 0 \ . \end{aligned}$$

Denote by $\Delta_0 = \zeta\left[\zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z + x_c^Z e^{\alpha\underline{t}_Z}\right]$, so that $\Delta = \zeta x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}$. Next, applying Cramer rule, we get first:

$$\begin{aligned} \frac{d\lambda_{Z0}}{dZ^0} &= -\frac{1}{\Delta}\left[-x_c^A\pi_Z + \rho\lambda_{A0}I_A\pi_Z + \rho\lambda_{A0}x_c^Z\right] \\ &= -\frac{1}{\Delta}\left[-x_c^A\pi_Z + (x_c^A - x_c^Z)\pi_Z + \rho\lambda_{A0}x_c^Z\right] \\ &= -\frac{x_c^Z}{\Delta}\left[-\zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z e^{\alpha\underline{t}_Z} - \rho\lambda_{A0} + \rho\lambda_{A0}\right] \\ &= \frac{\zeta(\rho+\alpha)x_c^Z\lambda_{A0}e^{\alpha\underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} \\ &= \frac{(\rho+\alpha)\lambda_{Z0}}{\Delta_0} > 0 \ . \end{aligned}$$

$$\begin{aligned} \frac{d\underline{t}_Z}{dZ^0} &= \frac{\zeta e^{\alpha\underline{t}_Z}}{\Delta}\left[\rho\lambda_{A0}I_A - x_c^A\right] \\ &= -\frac{\zeta x_c^Z e^{\alpha\underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} = -\frac{1}{\Delta_0} < 0 \ . \end{aligned}$$

$$\frac{d\lambda_{A0}}{dZ^0} = \frac{\zeta x_c^Z\rho\lambda_{A0}e^{\alpha\underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} = \frac{\rho\lambda_{A0}}{\Delta_0} > 0 \ .$$

$$\frac{d\bar{t}_A}{dZ^0} = -\frac{\zeta x_c^Z e^{\alpha\underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha\underline{t}_Z}} = -\frac{1}{\Delta_0} \ .$$

The computation shows that $d\underline{t}_Z/dZ^0 = d\bar{t}_Z/dZ^0$ and thus that $dT_A/dZ^0 = 0$. Furthermore:

$$\begin{aligned} \frac{d\bar{p}_Z}{dZ^0} &= -e^{\rho\underline{t}_Z}\left[\frac{d\lambda_{A0}}{dZ^0} + \rho\lambda_{A0}\frac{d\underline{t}_Z}{dZ^0}\right] \\ &= -\frac{e^{\rho\underline{t}_Z}}{\Delta_0}\left[\rho\lambda_{A0} - \rho\lambda_{A0}\right] = 0 \ . \end{aligned}$$

Turning to the effects of a higher $\bar{Z}$, we find:

$$\begin{aligned}
\frac{d\lambda_{Z0}}{d\bar{Z}} &= \frac{1}{\Delta}\left\{e^{\alpha \underline{t}_Z}\left[\rho\lambda_{A0}x_c^Z + \pi_Z(\rho\lambda_{A0}I_A - x_c^A)\right] - \frac{\alpha T_A}{\zeta}\left[-\zeta\rho\lambda_{A0}x_c^Z e^{\alpha \underline{t}_Z}\right]\right\} \\
&= \frac{x_c^Z}{\Delta}\left\{e^{\alpha \underline{t}_Z}\left[\rho\lambda_{A0} - \pi_Z\right] + \alpha T_A\rho\lambda_{A0}e^{\alpha \underline{t}_Z}\right\} \\
&= -\frac{\zeta(\rho+\alpha)\lambda_{Z0}e^{2\alpha \underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} + \alpha T_A\frac{\rho\lambda_{A0}x_c^Z e^{\alpha \underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} \\
&= -\frac{(\rho+\alpha)\lambda_{Z0}e^{\alpha \underline{t}_Z}}{\Delta_0} + \frac{\alpha T_A}{\zeta}\frac{\rho\lambda_{A0}}{\Delta_0}(?)\ .
\end{aligned}$$

$$\begin{aligned}
\frac{d\underline{t}_Z}{d\bar{Z}} &= \frac{1}{\Delta}\left\{-\zeta e^{2\alpha \underline{t}_Z}\left(\rho\lambda_{A0}I_A - x_c^A\right) - \frac{\alpha T_A}{\zeta}\rho\lambda_{A0}(-\zeta^2 I_Z^Z)\right\} \\
&= \frac{\zeta x_c^Z e^{2\alpha \underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} + \frac{\zeta^2\alpha\rho\lambda_{A0}T_A I_Z^Z}{\zeta^2 x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} \\
&= \frac{e^{\alpha \underline{t}_Z}}{\Delta_0} + \frac{\alpha\rho\lambda_{A0}T_A I_Z^Z}{x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} \ > \ 0\ .
\end{aligned}$$

$$\begin{aligned}
\frac{d\lambda_{A0}}{d\bar{Z}} &= \frac{1}{\Delta}\left\{\zeta e^{2\alpha \underline{t}_Z}(-\rho\lambda_{A0}x_c^Z) + \frac{\alpha T_A}{\zeta}\left[-\zeta^2 I_Z^Z\pi_Z - \zeta^2 x_c^Z e^{2\alpha \underline{t}_Z}\right]\right\} \\
&= -\frac{\zeta x_c^Z\rho\lambda_{A0}e^{2\alpha \underline{t}_Z}}{\zeta x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} - \frac{\zeta\alpha\rho\lambda_{A0}T_A\left[\pi_Z I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z}\right]}{\zeta x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} \\
&= -\frac{\rho\lambda_{A0}e^{\alpha \underline{t}_Z}}{\Delta_0} - \frac{\alpha\rho\lambda_{A0}T_A\left[\pi_Z I_Z^Z + x_c^Z e^{\alpha \underline{t}_Z}\right]}{x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}} \ < \ 0\ .
\end{aligned}$$

$$\begin{aligned}
\frac{d\bar{t}_A}{d\bar{Z}} &= \frac{1}{\Delta}\left\{\zeta x_c^Z e^{2\alpha \underline{t}_Z} + \alpha\zeta T_A\left[\pi_Z I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z}\right]\right\} \\
&= \frac{e^{\alpha \underline{t}_Z}}{\Delta_0} + \alpha T_A\frac{\pi_Z I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z}}{x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}}
\end{aligned}$$

$$\begin{aligned}
\frac{d\bar{p}_Z}{d\bar{Z}} &= \zeta e^{(\rho+\alpha)\underline{t}_Z}\left[\frac{d\lambda_{Z0}}{d\bar{Z}} + (\rho+\alpha)\lambda_{Z0}\frac{d\underline{t}_Z}{d\bar{Z}}\right] \\
&= \zeta e^{(\rho+\alpha)\underline{t}_Z}\left[\frac{\alpha\rho\lambda_{A0}T_A}{\zeta\Delta_0} + (\rho+\alpha)\lambda_{Z0}\frac{\alpha\rho\lambda_{A0}T_A I_Z^Z}{x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}}\right] \\
&= \frac{\alpha\rho\lambda_{A0}T_A e^{(\rho+\alpha)\underline{t}_Z}}{x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}}\left[x_c^Z e^{\alpha \underline{t}_Z} + \zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z\right] \\
&= \frac{\alpha\rho\lambda_{A0}T_A}{\zeta x_c^Z}e^{\rho \underline{t}_Z} \ > 0\ .
\end{aligned}$$

$$\begin{aligned}
\frac{d\bar{T}_A}{d\bar{Z}} &= \frac{d\bar{t}_A}{d\bar{Z}} - \frac{d\underline{t}_Z}{d\bar{Z}} \\
&= \frac{\alpha T_A}{x_c^Z\Delta_0 e^{\alpha \underline{t}_Z}}\left[(\pi_Z - \rho\lambda_{A0})I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z}\right] \\
&= \frac{\alpha T_A}{x_c^Z\Delta_0}\left[\zeta(\rho+\alpha)\lambda_{Z0}I_Z^Z + x_c^Z e^{\alpha \underline{t}_Z}\right] \\
&= \frac{\alpha T_A}{\zeta x_c^Z} \ > 0\ .
\end{aligned}$$

Next turning to the effects of a higher initial clean coal energy cost, we obtain:

$$\begin{aligned}\frac{d\lambda_{Z0}}{d\bar{c}_s} &= \frac{\zeta x_c^Z e^{\alpha \underline{t}_Z}}{\Delta}\left[x_c^A(e^{-\rho \underline{t}_Z} - e^{-\rho \bar{t}_A}) + \rho\lambda_{A0}(J_A^c - I_A e^{-\rho \underline{t}_Z})\right] \\ &= \frac{1}{\Delta_0}\left[(x_c^A - \rho\lambda_{A0} I_A)e^{-\rho \underline{t}_Z} - x_c^A e^{-\rho \bar{t}_A} + \rho\lambda_{A0} J_A^c\right] \\ &= \frac{1}{\Delta_0}\left[x_c^Z e^{-\rho \underline{t}_Z} - x_c^A e^{-\rho \bar{t}_A} + \rho\lambda_{A0} J_A^c\right] \qquad \text{(A.2.3)}\end{aligned}$$

This expression is of indeterminate sign. However taking into account the expression of $\dot{x}_c$:

$$\rho\lambda_{A0} J_A^c = -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{\rho\lambda_{A0} e^{\rho t}}{u''(q(t))} e^{-\rho t} dt = \int_{\underline{t}_Z}^{\bar{t}_A} \dot{x}_c(t) e^{-\rho t} dt$$

Integrating by parts:

$$\begin{aligned}\rho\lambda_{A0} J_A^c &= x_c(t)e^{-\rho t}\Big|_{\underline{t}_Z}^{\bar{t}_A} + \rho \int_{\underline{t}_Z}^{\bar{t}_A} x_c(t) e^{-\rho t} dt \\ &= x_c^A e^{-\rho \bar{t}_A} - x_c^Z e^{-\rho \underline{t}_Z} + \rho \int_{\underline{t}_Z}^{\bar{t}_A} x_c(t) e^{-\rho t} dt \ . \qquad \text{(A.2.4)}\end{aligned}$$

Inserting the expression (A.2.4) of $\rho\lambda_{A0} J_A^c$ into (A.2.3), we obtain:

$$\frac{d\lambda_{Z0}}{d\bar{c}_s} = \frac{\rho}{\Delta_0}\int_{\underline{t}_Z}^{\bar{t}_A} x_c(t) e^{-\rho t} dt \equiv \frac{\rho I_c}{\Delta_0} > 0$$

The effect of a higher $\bar{c}_s$ over $\lambda_{A0}$ is indeterminate. Next turning upon the impact over $\underline{t}_Z$ and $\bar{t}_A$, we obtain:

$$\begin{aligned}\frac{d\underline{t}_Z}{d\bar{c}_s} &= -\frac{\zeta^2 I_Z^Z}{\Delta}\left\{x_c^A(e^{-\rho \bar{t}_A} - e^{-\rho \underline{t}_Z}) + \rho\lambda_{A0}(I_A e^{-\rho \underline{t}_Z} - J_A^c)\right\} \\ &= -\frac{\zeta^2 I_Z^Z}{\Delta}\left\{x_c^A e^{-\rho \bar{t}_A} + (\rho\lambda_{A0} I_A - x_c^A)e^{-\rho \underline{t}_Z} - \rho\lambda_{A0} J_A^c\right\} \\ &= -\frac{\zeta^2 I_Z^Z}{\Delta}\left\{x_c^A e^{-\rho \bar{t}_A} - x_c^Z e^{-\rho \underline{t}_Z} - \left[x_c^A e^{-\rho \bar{t}_A} - x_c^Z e^{-\rho \underline{t}_Z} + \rho I_c\right]\right\} \\ &= \frac{\zeta^2 \rho I_Z^Z I_c}{\Delta} > 0\end{aligned}$$

Then:

$$\begin{aligned}\frac{d\bar{t}_A}{d\bar{c}_s} &= \frac{1}{\Delta}\left\{-\zeta^2 I_Z^Z\left[x_c^Z(e^{-\rho \bar{t}_A} - e^{-\rho \underline{t}_Z}) + \pi_Z(I_A e^{-\rho \bar{t}_A} - J_A^c)\right]\right. \\ &\quad \left. -\zeta^2 x_c^Z e^{2\alpha \underline{t}_Z}\left[I_A e^{-\rho \bar{t}_A} - J_A^c\right]\right\} \\ &= \frac{1}{\Delta}\left\{-\zeta^2 I_Z^Z x_c^Z(e^{-\rho \bar{t}_A} - e^{-\rho \underline{t}_Z}) - (I_A e^{-\rho \bar{t}_A} - J_A^c)\zeta^2\left[\pi_Z I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z}\right]\right\} \\ &= \frac{\zeta^2}{\Delta}\left\{I_Z^Z x_c^Z(e^{-\rho \underline{t}_Z} - e^{-\rho \bar{t}_A}) + (J_A^c - I_A e^{-\rho \bar{t}_A})\left[\pi_Z I_Z^Z + x_c^Z e^{2\alpha \underline{t}_Z}\right]\right\} \ .\end{aligned}$$

Since:

$$J_A^c - I_A e^{-\rho\bar{t}_A} = -\int_{\underline{t}_Z}^{\bar{t}_A} \frac{1}{u''(q(t))}\left[1 - e^{-\rho(\bar{t}_A - t)}\right] dt > 0 \ ,$$

while $\underline{t}_Z < \bar{t}_A$ implies that $e^{-\rho\underline{t}_Z} - e^{-\rho\bar{t}_A} > 0$, we conclude that $d\bar{t}_A/d\bar{c}_s > 0$.

This gives the following effects over $\bar{p}_Z$ and $T_A$:

$$\begin{aligned}
\frac{d\bar{p}_Z}{d\bar{c}_s} &= \zeta e^{(\rho+\alpha)\underline{t}_Z}\left[\frac{\rho I_c}{\Delta_0} + (\rho+\alpha)\lambda_{Z0}\frac{\rho\zeta^2 I_c I_Z^Z}{\Delta}\right] \\
&= \frac{\zeta^2 \rho I_c e^{(\rho+\alpha)\underline{t}_Z}}{\Delta}\left[x_c^Z e^{\alpha\underline{t}_Z} + \zeta(\rho+\alpha)\lambda_{Z0} I_Z^Z\right] \\
&= \frac{\zeta\rho I_c e^{(\rho+\alpha)\underline{t}_Z}\Delta_0}{\zeta x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}} \\
&= \frac{\rho I_c}{x_c^Z} e^{\rho\underline{t}_Z} > 0 \ .
\end{aligned}$$

$$\frac{dT_A}{d\bar{c}_s} = \frac{\zeta^2}{\Delta}\left\{I_Z^Z x_c^Z(e^{-\rho\underline{t}_Z} - e^{-\rho\bar{t}_A}) + (J_A^c - e^{-\rho\bar{t}_A} I_A)(\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z}) - \rho I_c I_Z^Z\right\} \ .$$

Since $\rho I_c = \rho\lambda_{A0} J_c^A - x_c^A e^{-\rho\bar{t}_A} + x_c^Z e^{-\rho\underline{t}_Z}$, the expression into brackets is equivalent to:

$$\begin{aligned}
\{\} &= I_Z^Z\left[x_c^Z(e^{-\rho\underline{t}_Z} - e^{-\rho\bar{t}_A}) + \pi_Z(J_A^c - e^{-\rho\bar{t}_A} I_A) - \rho\lambda_{A0} J_A^c + x_c^A e^{-\rho\bar{t}_A} - x_c^Z e^{-\rho\underline{t}_Z}\right] \\
&\quad + x_c^Z(J_A^c - I_A e^{-\rho\bar{t}_A})e^{2\alpha\underline{t}_Z} \\
&= I_Z^Z\left[(x_c^A - x_c^Z)e^{-\rho\bar{t}_A} + J_A^c(\pi_Z - \rho\lambda_{A0}) - \pi_Z I_A e^{-\rho\bar{t}_A}\right] + x_c^Z(J_A^c - I_A e^{-\rho\bar{t}_A}) \\
&= I_Z^Z\left[(x_c^A - x_c^Z)e^{-\rho\bar{t}_A} + \zeta(\rho+\alpha)\lambda_{Z0} e^{\alpha\underline{t}_Z}(J_A^c - I_A e^{-\rho\bar{t}_A}) - \rho\lambda_{A0} I_A e^{-\rho\bar{t}_A}\right] + x_c^Z(J_A^c - I_A e^{-\rho\bar{t}_A}) \ .
\end{aligned}$$

Since $\rho\lambda_{A0} I_A = x_c^A - x_c^Z$, we obtain after simplification:

$$\begin{aligned}
\frac{d\bar{T}_A}{d\bar{c}_s} &= \frac{\zeta^2 e^{\alpha\underline{t}_Z}}{\delta}\left[\zeta(\rho+\alpha)\lambda_{Z0} I_Z^Z + x_c^Z e^{\alpha\underline{t}_Z}\right](J_A^c - I_A e^{-\rho\bar{t}_A}) \\
&= \frac{\zeta e^{\alpha\underline{t}_Z}\Delta_0(J_A^c - I_A e^{-\rho\bar{t}_A})}{\zeta x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}} \\
&= \frac{J_A^c - I_A e^{-\rho\bar{t}_A} I_A}{x_c^Z} > 0 \ .
\end{aligned}$$

The effects of a higher $\underline{c}_s$ are the following.

$$\frac{d\lambda_{Z0}}{d\underline{c}_s} = \frac{\zeta x_c^Z e^{\alpha\underline{t}_Z} x_c^A e^{-\rho\bar{t}_A}}{\zeta x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}} = \frac{x_c^A}{\Delta_0} e^{-\rho\bar{t}_A} > 0 \ .$$

$$\frac{d\lambda_{A0}}{d\underline{c}_s} = -\frac{\zeta(\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z}) x_c^A e^{-\rho\bar{t}_A}}{x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}} < 0 \ .$$

$$\frac{d\underline{t}_Z}{d\underline{c}_s} = \frac{\zeta^2 I_Z^Z x_c^A e^{-\rho\bar{t}_A}}{\Delta} = \frac{\zeta x_c^A I_Z^Z}{x_c^Z \Delta_0 e^{\alpha \underline{t}_Z}} e^{-\rho\bar{t}_A} \quad > 0 \ .$$

$$\frac{d\bar{t}_A}{d\underline{c}_s} = \frac{\zeta^2 e^{-\rho\bar{t}_A}}{\Delta}\left[I_A(\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z}) + I_Z^Z x_c^Z\right] \ > 0 \ .$$

$$\frac{d\bar{p}_Z}{d\underline{c}_s} = \zeta e^{(\rho+\alpha)\underline{t}_Z}\left[\frac{d\lambda_{Z0}}{d\underline{c}_s} + (\rho+\alpha)\lambda_{Z0}\frac{d\underline{t}_Z}{d\underline{c}_s}\right] \ > 0 \ .$$

$$\begin{aligned}
\frac{dT_A}{d\underline{c}_s} &= \frac{\zeta^2 e^{-\rho\bar{t}_A}}{\delta}\left[I_A(\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z}) + I_Z^Z x_c^Z - x_c^A I_Z^Z\right] \\
&= \frac{\zeta^2 e^{-\rho\bar{t}_A}}{\delta}\left[I_Z^Z(\pi_Z I_A + x_c^Z - x_c^A) + x_c^Z I_A e^{2\alpha\underline{t}_Z}\right] \\
&= \frac{\zeta^2 e^{-\rho\bar{t}_A}}{\delta}\left[I_Z^Z(\zeta(\rho+\alpha)\lambda_{Z0} I_A e^{\alpha\underline{t}_Z} + \rho\lambda_{A0} I_A + x_c^Z - x_c^A) + x_c^Z I_A e^{2\alpha\underline{t}_Z}\right] \\
&= \frac{\zeta^2 I_A e^{-\rho\bar{t}_A} e^{\alpha\underline{t}_Z}}{\delta}\left[\zeta(\rho+\alpha)\lambda_{Z0} I_Z^Z + x_c^Z e^{\alpha\underline{t}_Z}\right] \\
&= \frac{\zeta I_A e^{\alpha\underline{t}_Z} e^{-\rho\bar{t}_A}\Delta_0}{x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}} \ = \frac{I_A}{x_c^Z} e^{-\rho\bar{t}_A} > 0 \ .
\end{aligned}$$

Last, the effects of a higher know-how target $\bar{A}$ are:

$$\frac{d\lambda_{Z0}}{d\bar{A}} = \frac{\zeta x_c^Z e^{\alpha\underline{t}_Z}}{\Delta}\rho\lambda_{A0} = \frac{\rho\lambda_{A0}}{\Delta_0} \ > 0 \ .$$

$$\frac{d\lambda_{A0}}{d\bar{A}} = -\frac{\zeta\rho\lambda_{A0}}{\Delta}\left[\pi_Z I_Z^Z + x_c^Z e^{2c\alpha\underline{t}_Z}\right] < 0 \ .$$

$$\frac{d\underline{t}_Z}{d\bar{A}} = \frac{\zeta^2\rho\lambda_{A0} I_Z^Z}{\Delta} \ > 0 \ .$$

$$\frac{d\bar{t}_A}{d\bar{A}} = \frac{\zeta^2}{\Delta}\left[\pi_Z I_Z^Z + x_c^Z e^{2\alpha\underline{t}_Z}\right] \ > 0 \ .$$

$$\begin{aligned}
\frac{d\bar{p}_Z}{d\bar{A}} &= \zeta e^{(\rho+\alpha)\underline{t}_Z}\left[\frac{d\lambda_{Z0}}{d\bar{A}} + (\rho+\alpha)\lambda_{Z0}\frac{d\underline{t}_Z}{d\bar{A}}\right] \\
&= \frac{\zeta\rho\lambda_{A0} e^{(\rho+\alpha)\underline{t}_Z}}{\Delta_0}\left[1 + \frac{\zeta(\rho+\alpha)\lambda_{Z0} I_Z^Z}{x_c^Z e^{\alpha\underline{t}_Z}}\right] \\
&= \frac{\zeta\rho\lambda_{A0} e^{(\rho+\alpha)\underline{t}_Z}}{x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}}\Delta_0 \\
&= \frac{\rho\lambda_{A0}}{x_c^Z} e^{\rho\underline{t}_Z} \ > 0 \ .
\end{aligned}$$

$$\begin{aligned}
\frac{dT_A}{dbarA} &= \frac{\zeta^2}{\Delta}\left[I_Z^Z(\pi_Z - \rho\lambda_{A0}) + x_c^Z e^{2\alpha\underline{t}_Z}\right] \\
&= \frac{\zeta^2}{\Delta}\left[\zeta(\rho+\alpha)\lambda_{Z0} I_Z^Z e^{\alpha\underline{t}_Z} + x_c^Z e^{2c\alpha\underline{t}_Z}\right] \\
&= \frac{\zeta\delta_0 e^{\alpha\underline{t}_Z}}{\zeta x_c^Z \Delta_0 e^{\alpha\underline{t}_Z}} \ = \frac{1}{x_c^Z} \ > 0 \ .
\end{aligned}$$

## A.3 Appendix 3: Comparative dynamics in the pure R&D model

The relative independency of the R&D policy with respect to the energy policy results in a pair of two dimensional linearized systems, the first one describing the effects over $(\lambda_{Z0}, \underline{t}_Z)$ while the second one describes the effects over $(\lambda_{A0}, \bar{t}_A)$. Denote by:

$$
\begin{aligned}
& I_Z^Z \equiv \int_0^{\underline{t}_Z} \frac{e^{(\rho+2\alpha)t}}{u''(q(t))} dt > 0 \;\; ; \;\; I_A \equiv \int_0^{\bar{t}_A} \frac{e^{\rho t}}{c_r'(r(t))} dt > 0 \\
& \bar{r} = r(\bar{t}_A) \;\; ; \;\; r(0) = r_0
\end{aligned}
$$

Then the systems are expressed as:

$$
\begin{bmatrix} -\zeta^2 I_Z^Z & \zeta x_c^Z e^{\alpha \underline{t}_Z} \\ 1 & (\rho+\alpha)\lambda_{Z0} \end{bmatrix} \begin{bmatrix} d\lambda_{Z0} \\ d\underline{t}_Z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} dZ^0 + \begin{bmatrix} -e^{\alpha \underline{t}_Z} \\ 0 \end{bmatrix} d\bar{Z} + \begin{bmatrix} 0 \\ e^{-(\rho+\alpha)\underline{t}_Z}/\zeta \end{bmatrix} d\bar{c}_s
$$

$$
\begin{aligned}
\begin{bmatrix} I_A & \bar{r} \\ 1 & \rho\lambda_{A0} \end{bmatrix} \begin{bmatrix} d\lambda_{A0} \\ d\bar{t}_A \end{bmatrix} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} d\bar{A} + \begin{bmatrix} 0 \\ -\frac{\alpha(\bar{c}_s - \underline{c}_s)}{\zeta \bar{r}} e^{-\rho \bar{t}_A} \end{bmatrix} d\bar{Z} \\
&+ \begin{bmatrix} 0 \\ \frac{\bar{x_c}}{\bar{r}} e^{-\rho \bar{t}_A} \end{bmatrix} d\bar{c}_s - \begin{bmatrix} 0 \\ \frac{\underline{x}_c}{\bar{r}} e^{-\rho \bar{t}_A} \end{bmatrix} d\underline{c}_s
\end{aligned}
$$

The determinant of the first system is $-\zeta(\zeta(\rho+\alpha)\lambda_{Z0} I_Z^Z + x_c^Z e^{\alpha \underline{t}_Z}) \equiv -\Delta_0 < 0$. Applying Cramer rule, we obtain:

$$
\begin{aligned}
& \frac{d\lambda_{Z0}}{dZ^0} = \frac{(\rho+\alpha)\lambda_{Z0}}{\Delta_0} \;\; ; \;\; \frac{d\lambda_{Z0}}{d\bar{Z}} = \frac{(\rho+\alpha)\lambda_{Z0} e^{\alpha \underline{t}_Z}}{\Delta_0} \;\; ; \;\; \frac{d\lambda_{Z0}}{d\bar{c}_s} = \frac{x_c^Z e^{-\rho \underline{t}_Z}}{\Delta_0} \\
& \frac{d\underline{t}_Z}{dZ^0} = -\frac{1}{\Delta_0} \;\; ; \;\; \frac{d\underline{t}_Z}{d\bar{Z}} = -\frac{e^{\alpha \underline{t}_Z}}{\Delta_0} \;\; ; \;\; \frac{d\underline{t}_Z}{d\bar{c}_s} = \frac{\zeta I_Z^Z e^{-(\rho+\alpha)\underline{t}_Z}}{\Delta_0}
\end{aligned}
$$

By construction: $d\lambda_{Z0}/d\bar{A} = d\underline{t}_Z/d\bar{A} = 0$ and $d\lambda_{Z0}/d\underline{c}_s = d\underline{t}_Z/d\underline{c}_s = 0$. The features of the energy policy before the cost breakthrough do not depend of the know-how requirement to trigger the break or of the clean energy additional cost after the break.

Next turn to the second system. The determinant of this system is: $\rho\lambda_{A0} I_A - \bar{r}$. Since $\dot{r}(t) = \rho\lambda_{A0} e^{\rho t}/c_r'(r)$, we obtain:

$$
\rho\lambda_{A0} I_A = \rho\lambda_{A0} \int_0^{\bar{t}_A} \frac{e^{\rho t}}{c_r'(r(t))} dt = \int_0^{\bar{t}_A} \dot{r}(t) dt = \bar{r} - r_0
$$

Hence: $\rho\lambda_{A0} I_A - \bar{r} = -r_0 < 0$. Remember that the strict negativity of the determinant is a consequence of the assumption $\lambda_{!A0} > c_r(0)$, which implies that $r_0 > 0$. Applying

Cramer rule, we then get:

$$\begin{aligned}
\frac{d\lambda_{A0}}{d\bar{A}} &= -\frac{\rho\lambda_{A0}}{r_0} \;;\; \frac{d\lambda_{A0}}{d\bar{Z}} = -\frac{\alpha(\bar{c}_s - \underline{c}_s)e^{-\rho\bar{t}_A}}{\zeta r_0} \\
\frac{d\lambda_{A0}}{d\bar{c}_s} &= \frac{x_c^- e^{-\rho\bar{t}_A}}{r_0} \;;\; \frac{d\lambda_{A0}}{d\underline{c}_s} = -\frac{\underline{x}_c e^{-\rho\underline{t}_A}}{r_0} \\
\frac{d\bar{t}_A}{d\bar{A}} &= \frac{1}{r_0} \;;\; \frac{d\bar{t}_A}{d\bar{Z}} = \frac{\alpha(\bar{c}_s - \underline{c}_s)I_A e^{-\rho\underline{t}_A}}{\zeta r_0\bar{r}} \\
\frac{d\bar{t}_A}{d\bar{c}_s} &= -\frac{x_c^- I_A e^{-\rho\bar{t}_A}}{r_0\bar{r}} \;;\; \frac{d\bar{t}_A}{d\underline{c}_s} = \frac{\underline{x}_c I_A e^{-\rho\bar{t}_A}}{r_0\bar{r}}
\end{aligned}$$

## A.4 Appendix 4. Proof that $\dot{\sigma} < 0$

We get from (5.4): $\lambda_A = c_r/a_r$. Denote $\sigma = a_c/a_r$, then (5.1) becomes: $u'(q) = c_x + \bar{c}_s - \sigma c_r$ during the time interval $[\underline{t}_Z, \bar{t}_A)$. Time differentiating, we obtain:

$$\dot{\sigma} c_r = -\left[\sigma c_r' \dot{r} + u''(q)\dot{x}_c\right]$$

Taking (5.13) and (5.14) into account, this is equivalent to:

$$\begin{aligned}
\dot{\sigma} c_r &= -\frac{\dot{\lambda}_A}{\Delta_1}\left\{u''\left[\lambda_A(a_{cr}a_r - a_c a_{rr}) + a_c c_r'\right]\right. \\
&\quad \left. +\sigma c_r'\left[\lambda_A(a_{cr}a_c - a_r a_{cc}) - u'' a_r\right]\right\} \\
&= -\frac{\dot{\lambda}_A}{\Delta_1}\left\{\lambda_A\left[u''(a_{cr}a_r - a_c a_{rr}) + \sigma c_r'(a_{cr}a_c - a_r a_{cc})\right]\right. \\
&\quad \left. +u'' a_c c_r' - u''\frac{a_c}{a_r}a_r c_r'\right\} \\
&= -\frac{\rho\lambda_A^2 a_r}{\Delta_1}\left[u''(a_{cr} - \sigma a_{rr}) + \sigma c_r'(a_{cr}\sigma - a_{cc})\right]
\end{aligned}$$

Let: $P(\sigma) \equiv c_r' a_{cr}\sigma^2 - \sigma(u'' a_{rr} + c_r' a_c c) + u'' a_{cr}$, then:

$$\dot{\sigma} = -\frac{\rho\lambda_A^2 a_r}{\Delta_1 c_r}P(\sigma)$$

$c_r' a_{cr} > 0$ and $u'' a_{cr} < 0$ imply that $P(\sigma)$ has two real roots of opposite signs. Denote by $\hat{\sigma}$ the positive root. Then:

$$\dot{\sigma} \gtreqless 0 \iff P(\sigma) \lesseqgtr 0 \iff \sigma \lesseqgtr \hat{\sigma}$$

We conclude that during the time interval $[\underline{t}_Z, \bar{t}_A)$, $\sigma(t)$ must be a monotonous time function, either constantly increasing or either constantly decreasing. Then denoting by

$\bar{p}_Z = u'(q(\underline{t}_Z))$ and by $\bar{p}_A = u'(q^-(\bar{t}_A))$, $\sigma_Z \equiv \sigma(\underline{t}_Z)$ and $\sigma_A \equiv \sigma^-(\bar{t}_A)$ may be expressed as:

$$\sigma_Z = \frac{\bar{p}_Z - c_x - \bar{c}_s}{c_r(r(\underline{t}_Z))} \quad \text{and} \quad \sigma_A = \frac{\bar{p}_A - c_x - \bar{c}_s}{c_r(r^-(\bar{t}_A))}$$

And $\bar{p}_A < \bar{p}_Z$ together with $r(\underline{t}_Z) < r^-(\bar{t}_A)$, that is $c_r(r(\underline{t}_Z)) < c_r(r^-(\bar{t}_A))$ imply that $\sigma_A < \sigma_Z$. We thus conclude that $\sigma(t)$ having to be monotonous, $\sigma(t)$ is a decreasing time function over the interval $[\underline{t}_Z, \bar{t}_A)$.

## A.5 Appendix 5.

*Pure research paths*

Let $\underline{t}^R$ be the date of both the arrival at the ceiling and the revolution: $\underline{t}_Z = \bar{t}_A = \underline{t}^R$ in a scenario where the revolution is triggered only through R&D efforts. Assume that initially $r(0) > 0$. Then $(\lambda_{Z0}, \lambda_{A0}, \underline{t}^R)$ are solution of the following system of conditions:

$$\begin{aligned} \bar{Z}e^{\alpha \underline{t}^R} &= Z^0 + \int_0^{\underline{t}^R} x_d(\lambda_{Z0}e^{(\rho+\alpha)t})e^{\alpha t}dt \\ \bar{A} &= \int_0^{\underline{t}^R} a(0, r(\lambda_{A0}e^{\rho t}))dt \\ \underline{c}_s &= c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}^R} \end{aligned}$$

Let $\underline{\lambda}_A^R \equiv \lambda_{A0}e^{\rho \underline{t}^R}$.

*Pure learning phases*

Let $\underline{t}^L$ be the common date of arrival at the ceiling and the revolution when the policy scenario involves only learning-by-doing. Assume that initially $x_c(0) > 0$, then $(\lambda_{Z0}, \lambda_{A0}, \underline{t}^L)$ are solution of:

$$\begin{aligned} \bar{Z}e^{\alpha \underline{t}^L} &= Z^0 + \int_0^{\underline{t}^L} x_d(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t})e^{\alpha t}dt \\ \bar{A} &= \int_0^{\underline{t}^L} a(x_c(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}), 0)dt \\ \underline{c}_s &= c_x + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}^L} \end{aligned}$$

Let $\underline{\lambda}_A^L \equiv \lambda_{A0}e^{\rho \underline{t}^L}$.

*Combined phases*

Let $\underline{t}^C$ be given by $\underline{t}^C = \underline{t}_Z = \bar{t}_A$ in a scenario involving both learning and R&D to trigger the technological breakthrough. In a case where initially $x_c(0) > 0$ and $r(0) > 0$,

$(\lambda_{Z0}, \lambda_{A0}, \underline{t}^C$ are solution of:

$$\begin{aligned}
\bar{Z}e^{\alpha \underline{t}^C} &= Z^0 + \int_0^{\underline{t}^C} x_d(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t})e^{\alpha t}dt \\
\bar{A} &= \int_0^{\underline{t}^C} a(x_c(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}), r(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}))dt \\
\underline{c}_s &= c_x + \zeta \lambda_{Z0}e^{(\rho+\alpha)\underline{t}^C}
\end{aligned}$$

Let $\underline{\lambda}_A^C \equiv \lambda_{A0}e^{\rho \underline{t}^C}$.

*Pure R&D then combined phases*

Let $\underline{t}^{RC} = \underline{t}_Z = \bar{t}_A$ in a two phases scenario during which the economy performs only research during a time interval $[0, \bar{t}_r)$ and next both clean coal production and research during a time phase $[\bar{t}_r, \underline{t}^{RC})$. Assume that initially $r(0) > 0$, then $(\lambda_{Z0}, \lambda_{A0}, \bar{t}_r, \underline{t}^{RC})$ are defined by the following set of conditions:

$$\begin{aligned}
\bar{Z}e^{\alpha \underline{t}^{LC}} &= Z^0 + \int_0^{\bar{t}_r} x_d(\underline{\lambda}_{Z0}e^{(\rho+\alpha)t}dt + \int_{\bar{t}_r}^{\underline{t}^{RC}} x_d(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t})e^{\alpha t}dt \\
\bar{A} &= \int_0^{\bar{t}_r} a(0, r(\lambda_{A0}e^{\rho t})dt + \int_{\bar{t}_r}^{\underline{t}^C} a(x_c(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}), r(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}))dt \\
\bar{c}_s &= \lambda_{A0}a_c(0, r(\lambda_{A0}e^{\rho \bar{t}_t})e^{\rho \bar{t}_r} + \zeta \lambda_{Z0}e^{(\rho+\alpha)\bar{t}_r} \\
\underline{c}_s &= c_x + \zeta \lambda_{Z0}e^{(\rho+\alpha)\underline{t}^C}
\end{aligned}$$

Let $\underline{\lambda}_A^{RC} \equiv \lambda_{A0}e^{\rho \underline{t}^{RC}}$.

*Pure learning then combined phases*

Let $\underline{t}^{LC} = \underline{t}_Z = \bar{t}_A$ in a two phases scenario where the economy increases the know-how level only through learning during a first time phase $[0, \bar{t}_l)$ and then through a combination of learning and R&D during the phase $[\bar{t}_l, \underline{t}^{LC})$. Assume that initially $x_c(0) = 0$, then $(\lambda_{Z0}, \lambda_{A0}, \bar{t}_l, \underline{t}^{LC})$ are solutions of the following system of conditions:

$$\begin{aligned}
\bar{Z}e^{\alpha \underline{t}^{LC}} &= Z^0 + \int_0^{\bar{t}_l} x_d(\underline{\lambda}_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}))dt + \int_{\bar{t}_l}^{\underline{t}^{LC}} x_d(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t})e^{\alpha t}dt \\
\bar{A} &= \int_0^{\bar{t}_l} a(x_c(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}, 0)dt + \int_{\bar{t}_l}^{\underline{t}^{LC}} a(x_c(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}), r(\lambda_{Z0}e^{(\rho+\alpha)t}, \lambda_{A0}e^{\rho t}))dt \\
\bar{c}_s &= \lambda_{A0}a_c(x_c(\lambda_{Z0}e^{(\rho+\alpha)\bar{t}_l}, \lambda_{A0}e^{\rho \bar{t}_l}), 0)e^{\rho \bar{t}_l} + \zeta \lambda_{Z0}e^{(\rho+\alpha)\bar{t}_l} \\
\underline{c}_s &= c_x + \zeta \lambda_{Z0}e^{(\rho+\alpha)\underline{t}^C}
\end{aligned}$$

Let $\underline{\lambda}_A^{LC} \equiv \lambda_{A0}e^{\rho \underline{t}^{LC}}$.